CHAPTER 1.1.2: COMPONENTS OF MATHEMATICS TEACHER TRAINING

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With
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1. INTRODUCTION

Initial mathematics teacher education is primarily concerned with knowledge—the acquisition of knowledge required for the teaching of mathematics. Opinions as to what exactly comprises this knowledge and how it is best delivered and best learned varies widely across different contexts. In what follows, we will look more closely at this concept of teacher knowledge and how it plays itself out in the context of initial mathematics teacher education.

2. KNOWLEDGE AND BELIEFS FOR THE TEACHING OF MATHEMATICS

Teacher knowledge is most often discussed as being comprised of three strands: content knowledge, pedagogical knowledge, and didactical knowledge (Durand-Guerrier & Winsløw, 2005). Shulman (1987) refers to these same categories, respectively, as subject matter knowledge (SMK), pedagogical knowledge (PK), and pedagogical content knowledge (PCK). In the context of mathematics education, content knowledge pertains to mathematical concepts, use of mathematical techniques, mathematical reasoning, proof, and so forth. PK is subject independent and deals with general principles of education such as theories of learning; sociological, psychological, and ethical aspects of education and its functions (Durand-Guerrier & Winsløw, 2005); and classroom management and assessment. Didactical knowledge is the knowledge regarding the conditions and ways of mathematics teaching and learning (Bloch, 2005; Brousseau, 1997; Durand-Guerrier & Winsløw, 2005) and "captures both the link and the distinction between knowing something for oneself and being able to enable others to know it" (Rowland, Thwaites, & Huckstep, 2005). In general, the three strands can be seen as knowing the mathematics, knowing teaching, and knowing how to teach mathematics.

This is not to say that this is the only way in which teacher knowledge can be partitioned. Bergsten and Grevholm (2005) speak of teacher knowledge as being comprised of disciplinary knowledge and PK. Disciplinary knowledge is the substantive knowledge of facts, procedures, concepts, and so forth, as well as knowledge of mathematics as a discipline. PK, on the other hand, is PCK and curriculum knowledge as well as knowledge of general issues in education, such as learning, developmental psychology, and socialisation. Adler and Davis (2005) view the acquisition of teacher knowledge as learning to teach and learning mathematics for teaching. Rowland et al. (2005) introduce the notion of the Knowledge Quartet, which is "a tool for thinking about the ways that a teacher's subject knowledge comes into play in the classroom" (p. 2). This quartet is comprised of "foundation (teachers' knowledge, beliefs, and understandings acquired 'in the academy'), transformation (teachers' knowledge in action as demonstrated both in planning to teach and in the act of teaching itself), connection (binds together certain choices and decisions that are made for the more or less discrete parts of mathematics education), and contingency (witnessed in classroom events that are almost impossible to prepare for)" (paraphrased from Rowland et al., 2005, p. 2) .

Variation and further partitioning of each of the aforementioned knowledge strands allow for more fine-grain discussion of the particularities of teachers' knowledge. Didactical knowledge, for example, has been extensively elaborated on to account for the specific knowledge that is needed for the teaching of specific mathematical concepts (García & Sánchez, 2005). Such elaborations start with a topic and work outwards to encompass specific strategies, tasks, and assessment instruments that will facilitate the learning of that topic. An altogether different elaboration of didactical knowledge is task knowledge (Liljedahl, Chernoff, & Zazkis, 2007), which refers to teachers' knowledge of the mathematical and pedagogical affordances that exist within a given mathematical task.

With respect to PK, constructivism in its many forms (Bruner, 1966; Dewey, 1916; Piaget, 1951; Wertsch, 1985) are still foundational elements of most initial teacher education programs. However, more recently conceived theories are also starting to have a presence in these programs. Situated learning (Lave & Wenger, 1991), for example, and its discourse on communities of practice as it pertains to both the classroom context and the teacher education context is beginning to have an influence on teacher education. This discourse is re-casting what it means to be a participant in these communities (see García & Sánchez, 2005). Although not as overtly present in teacher education curriculum as constructivism and situated learning, a number of contemporary theories are having local influences on what is imparted as PK in various initial teacher education programs. Imaginative education (Egan, 2005), with its descriptive emphasis on cognitive tools and prescriptive emphasis on capitalizing on students’ propensity for accessing particular cognitive tools at different developmental stages, is recasting what it means for a learner to develop. Likewise, theories on learning as communicating (Sfard, 2001) and classrooms as complex (learning) systems (Davis & Simmt, 2003) are beginning to take hold in some initial teacher education contexts (see Proulx, 2005).

The content knowledge considered relevant to teacher education is also not immune to the variance of context. Although the broad brush strokes of curriculum have not changed much over recent decades, the finer details show greater variability through time. Subsequently, the specific content knowledge required for the teaching of mathematics has also changed. Surprisingly, however, the content knowledge that is required of teachers hasn't changed much. This is primarily due to adherence to the traditions of mathematics teacher education and the structures they have entrenched. This will be discussed in greater details in the Structures of Initial Mathematics Teacher Education section, which follows.

However, discussions of teachers' knowledge cannot be strictly limited to these objective forms—teachers' subjective knowledge is also important. "It has become an accepted view that it is the [mathematics] teacher's subjective school-related knowledge that determines for the most part what happens in the classroom" (Chapman 2002, p. 177). One central aspect of subjective knowledge is beliefs (Op't Eynde, De Corte, & Verschaffel, 2002). In fact, Ernest (1989) suggests that beliefs are the primary regulators for mathematics teachers' professional behaviour in the classrooms. These beliefs do not develop within the practice of teaching, however.

Prospective elementary teachers do not come to teacher education believing that they know nothing about teaching mathematics (Feiman-Nemser, Mcdiarmid, Malnick, & Parker, 1987). "Long before they enrol in their first education course or math methods course, they have developed a web of interconnected ideas about mathematics, about teaching and learning mathematics, and about schools" (Ball, 1988). These ideas are more than just feelings or fleeting notions about mathematics and mathematics teaching. During their time as students of mathematics they first formulated, and then concretized, deep-seated beliefs about mathematics and what it means to learn and teach mathematics (Lortie, 1975). These beliefs often form the foundation on which they will eventually build their own practice as teachers of mathematics (Skott, 2001). Unfortunately, these deep-seated beliefs often run counter to contemporary research on what constitutes good practice. As such, it is the role of teacher education programmes to reshape these beliefs and correct misconceptions that could impede effective teaching in mathematics (Green, 1971).

This distinction between knowledge and beliefs is a false dichotomy, however. At the level of teachers' action the distinction between knowledge and beliefs is not so clear. In general, knowledge is seen as an "essentially a social construct" (Op 'T Eynde, De Corte, & Verschaffel, 2002). That is, the division between knowledge and belief is the evaluation of these notions against some socially shared criteria. If the truth criterion is satisfied then the conception is deemed to be knowledge. However, when teachers operate on their knowledge the distinction between what is true and what they believe to be true is not made. Leatham (2006) articulates this argument nicely:

> Of all the things we believe, there are some things that we "just believe" and other things we "more than believe—we know." Those things we "more than believe" we refer to as knowledge and those things we "just believe" we refer to as beliefs. Thus beliefs and

knowledge can profitably be viewed as complementary subsets of the things we believe (p. 92).

3. STRUCTURES OF INITIAL TEACHER EDUCATION

Initial teacher education is largely aimed at developing an integrated proficiency with the aforementioned forms of knowledge and beliefs. How this is achieved varies greatly across different contexts. Our goal here is not to go into the minutiae of these differences, but rather to highlight some of the general commonalities. First, however, we introduce some terminology to create distinction between teachers in their varying stages of development. In particular, we are interested in drawing a distinction between what we refer to as prospective teachers and pre-service teachers. Pre-service teachers are teacher candidates working within a teacher education program. Prospective teachers, on the other hand, are individuals who have decided that they would like to become teachers and have begun the process of acquiring some of the necessary prerequisite knowledge and/or experiences (in the form of courses and/or required volunteer experience) to be accepted into a teacher education program. We make this distinction because both the nature of their experiences and their expectations of these experiences are very different.

In general, initial teacher education is separated into generalist teacher education and specialized mathematics teacher education. This divide is most often facilitated along a divide between elementary teacher education and secondary mathematics teacher education.[1] Elementary teacher education programs, for the most part, require very little mathematical content knowledge of their prospective teachers. The knowledge they do require, however, is usually very specific and consists primarily of elementary mathematics content knowledge (Flowers, Rubenstein, Grant, & Kline, 2005). At the same time, these programs may require their candidates to acquire some PK, either of the general form (e.g., psychology of learning) or the specialized form (e.g., learning disabilities). Secondary mathematics teacher education programmes, on the other hand, often require highly specialized mathematics content knowledge. The nature of this knowledge is quite different, however. Whereas prospective elementary teachers are required to obtain mathematical knowledge relevant to the teaching of elementary mathematics, prospective secondary teachers are often required to obtain mathematical knowledge that is of a more academic nature (Moreira & David, 2005; Opolot-Okurut, 2005). That is, they are required to become proficient in the university-level mathematics taught to a wide spectrum of mathematics, engineering, and science students. Other than this requirement of university-level mathematical knowledge there are rarely any other requirements placed on them.

[1] The exception to this is in settings where elementary mathematics is taught by a specialist mathematics teacher.

This discrepancy between what is required of elementary teachers and secondary mathematics teachers is primarily due to a confluence of pragmatics and tradition. Elementary teachers are, for the most part, generalist teachers having to be proficient in the teaching of all subjects. Thus, to require prospective elementary teachers to obtain a large amount of university-level content knowledge in all of the school subjects prior to entering a teacher education program is rather unrealistic. Instead, these programs opt for proficiency with school-level content knowledge in all of the subject areas as a requirement.

Such pragmatics does not extend to the requirements of prospective secondary teachers, however. As already mentioned, prospective secondary mathematics teachers are often required to obtain mathematical content knowledge of a form that is not obviously relevant to secondary mathematics. The reason for this is tradition—the tradition of what it means to be a mathematics teacher. Since the classical period, to be a mathematics teacher meant that one was first a mathematician. This thinking has changed very little in the last 2,500 years. Prospective mathematics teachers must first become mathematicians.

Once accepted into a teacher education program, pre-service teachers of both a generalist (elementary) and a specialized (secondary) type are subjected to very similar experiences. Through courses and seminars their repertoire of content knowledge, PK, and didactical knowledge is expanded, and through case studies and practicum experience these discrete forms of knowledge are integrated (for more information, see Subtheme 3).

Teacher education is a unique enterprise. The reason for this is that the what is also the how. That is, what we teach is also how we teach. As such, pre-service teachers have a unique experience. What they are learning is also how they are learning. Through their experiences as student teachers they are both student and teacher, and through the constant shifting between student and teacher they are given the opportunity to not only acquire the knowledge that they will require to become effective teachers, but also are given the opportunity to recast their initial (pre-conceived) beliefs about what it means to be a teacher, what it means to teach, what it means to learn, and even what it means for something to be mathematics. Through this recasting process they begin to form an identity of who they are as a teacher, and what it is that they teach as a subject.

4. RESEARCH IN INITIAL TEACHER EDUCATION: PAST, PRESENT, AND FUTURE

Research in initial teacher education is vast, contributing to all its aspects from prospective teachers' initial beliefs (Liljedahl, 2005) to teacher identity (Lerman, 2001) to the effectiveness of a specific teacher education method (Chapman, 2005). This research can be viewed in terms of two dimensions: how it contributes to the domain of knowledge that teachers need for teaching and how best to help teachers acquire this knowledge. In keeping with the recurring theme of knowledge for teaching in this chapter, we will focus only on the first of these.

The domain of knowledge that teachers need for teaching possesses a duality within mathematics education—a duality that can be encapsulated as the tension between ‘has’ and ‘should have’. That is, there is a constant tension in the literature between the knowledge that a teacher ‘has’ and the knowledge that a teacher ‘should have’. In many ways this is a product of the constant confluence of theory, research, and practice within the field of mathematics education and cannot be, and should not be, resolved by the exclusion of one or the other. Our understanding of what knowledge is needed for the teaching of a specific mathematical concept is informed by the knowledge possessed by teachers who are effectively (or not effectively) teaching that concept (Ball, 2005). This emerging understanding, in turn, informs our work in pre-service and in-service teacher education as we work to develop the necessary knowledge within teachers.

For example, research into students' misconceptions and analysis of student thinking (Flowers et al., 2005) about specific mathematics concepts can be seen as a informing the discourse of what teachers need to know in order to teach those specific concepts. Also informing this discourse is research into teachers' practice of teaching these same concepts. The results of both of these forms of research coupled with mathematical analysis of students' understanding and/or teachers’ practice contributes greatly to the didactical knowledge base.

Similarly, research into prospective and teachers' knowledge of subject matter helps to extend the discourse on content knowledge. However, this extension of discourse should be viewed more as a focusing rather than an expansion. For example, extensive work has been done on pre-service teachers' understanding of elementary number theory (see Zazkis & Campbell, 1996). The fine-grain analysis associated with this research has alerted us to subtle variations and the developmental nature of pre-service teachers' understanding of this content. As such, it focuses our own understanding of this particular SMK, as well as gives insights into what and how to teach elementary number theory to pre-service teachers.

5. CONCLUDING REMARKS

Initial teacher education is primarily concerned with developing proficiency with a number of different dimensions of teacher knowledge, from teachers' knowledge of mathematical content to teachers' knowledge of pedagogy and didactics. Although much of initial teacher education deals with these different dimensions discretely, a significant portion is often devoted to treating these dimensions in an integrated manner. As pre-service teachers progress through the initial teacher education experience, these different forms of knowledge are wound tighter and tighter together until the content of their experience can best be described as knowledge needed for teaching. That is, initial teacher education can be viewed as the beginning of a braid (see Fig. 1.1.2). At the beginning stages the different dimensions of teacher knowledge are represented by individual and discrete strands. As teacher education progresses, these strands are braided together to form a tighter experience in which, although still distinguishable from one another, the different strands are integrated. In ideal circumstances this braid tightens towards the end of the initial teacher experience to form a unified fibre, the content of which is teacher knowledge.

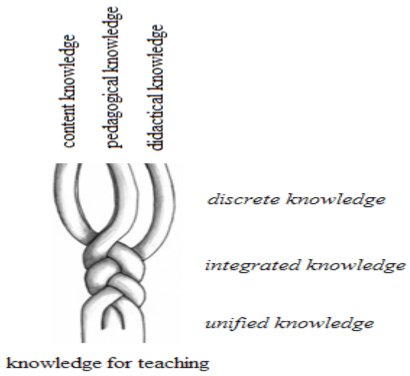

Figure 1: Braiding Knowledge for Teaching

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