

Lesson Play – A vehicle for multiple shifts of attention in teaching

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Motivation

As University professors working in mathematics teacher education, the ‘methods’ courses for prospective teachers are part of our usual teaching assignment. ‘Planning for instruction’ is one of the topics that are explored through these courses in different settings. It has been the students’ expectation that the ‘methods’ courses will provide them with an opportunity to develop a variety of ‘lesson plans.’ These plans would have a dual purpose: first, to satisfy some of the requirements of the course, and later to be used as already prepared modules in their teaching.

Traditional lesson planning is based on a linear sequence that starts with a declaration of behavioural objectives and ends with a description of assessment techniques that should determine to what degree the objectives have been met. Despite the contemporary prevalence of student-centred approaches advocated in teacher education, the traditional teacher-centred mode of instructional planning, as taught in teacher education programs, has not changed.

Following prior experience with the ‘general’, that is, not ‘subject matter specific’ methods of instruction, many prospective teachers interpret creating plans for a lesson as filling in boxes of predetermined rubrics, such as SWAT (students will be able to), goals and objectives, assessment practices, teacher/student activities and explicit timeline. Having examined a variety of lesson plans created according to standard templates, we felt uneasy. On one hand, we recognized many worthwhile features in the presented plans, such as the use of manipulatives and other teaching aids, appropriate choice and sequence of activities, and different modes of engagement, such as lecture, group work, or individual student work). However, we felt that in most of the lesson plans, even well crafted ones, the main ingredient in preparing for instruction was missing – that of interaction with students and attention to students’ difficulties. As such, we sought a vehicle that would direct prospective teachers to what Mason (1998) refers to as “a fundamental question of teaching,”(p. 247) namely, “What are students attending to?” Simultaneously, we were seeking an answer to a related question for teacher education, again, as formulated by Mason, “How can we enable teachers to see what students are attending to?”(ibid.)

With these questions in mind, we suggest a technique for implementation in teacher education, which can either enhance or replace the traditional approach to planning. We refer to it as ‘lesson play,’ which involves presenting part of a lesson in the form of a play or dialogue, which is set in a classroom, where the characters/players are the teacher and

the student(s). In Zazkis, Liljedahl and Sinclair (2009) we provided a detailed analysis of the ‘lesson play’ construct as “a novel juxtaposition to the traditional planning framework as a method of preparing to teach a lesson” and discussed its pedagogical affordances. Here we use Mason’s (1998) classification of three different forms of awareness to document our own evolution of the task design, as well as that of the prospective teachers.

Awareness-in-action is the ability to act in the moment. This corresponds to Vergnaud’s notion of ‘theorem-in-action’, the term that describes students’ acting according to a certain rule or constrain, without being able to state it explicitly. An individual may not be explicitly conscious of his/her own awareness-in-action, but it can be recognized by others through the individual’s actions. A student may demonstrate this kind of awareness by performing an action such as adding fractions, but his skill may be limited to ‘knowing how’ rather than ‘knowing why.’ This level of awareness in teaching is recognized when a teacher poses a certain question, corrects a mistake, suggests an answer or selects a task, but is unable to justify or explain his/her choice.

Awareness-in-discipline is awareness of awareness-in-action. This awareness is essential in order to articulate awareness-in-action for others. According to Mason, the one important distinction between the two kinds of awareness involves the ability “to do” in contrast with the ability to instruct others. Teachers who possess awareness-in-discipline are able to articulate the choices they make in instructional situations.

Awareness-in-council is awareness of awareness-in-discipline. This awareness is essential in order to articulate awareness-in-discipline for others. It describes one’s sensitivity to what others require for building or enhancing their awareness.

In what follows, we present the evolution in working with “lesson plays” in our teaching and research. This evolution represents our own developing awareness of how to enable teachers to direct their attention to different aspects of mathematics and pedagogy.

Lesson Play – first steps

As one of the group assignments in the methods course, we asked prospective teachers to design and write a ‘lesson play’– a script for a lesson that described an interaction between a teacher and a group of students. The topic for the lesson was left wide open, with a shared understanding that it should relate to the school curriculum. Prospective teachers could further choose the age group in which their imagined lesson would take place and the size of the group with which the teacher communicated.

Not surprisingly, we say in retrospect, our first steps were doomed to failure. The produced lessons looked like a capture of a lecture with limited interruptions. That is to say, the plays included long monologs by a teacher character, with occasional questions for students. In most cases, students provided correct answers to the posed questions. The teacher character praised them and followed up with further explanations and some

additional questions. In a limited number of cases, where an imaginary student suggested an incorrect answer, the teacher immediately addressed the class, asking whether someone had a “different idea,” and then a desired solution was put forward in the script.

With this first attempt we felt that the attention of prospective teachers was mainly focused on self-delivery of the content, rather than on their students. In a way, they were acting out a ‘standard’ lesson plan. Their responses served for us, using Mason’s (2002) phrase, as a “form of disturbance which starts things off” (p. 10) that triggered the next phases of development. We needed to re-design the lesson play task in order to help move the prospective teachers from their awareness-in-action, which seemed to be prompting them to re-create familiar interactions with students, to an awareness-in-discipline.

Lesson Play – Second Steps (Shift of attention to a possible problematic)

In our second attempt to engage prospective teachers in writing lesson plays, in order to foster interaction with students and students’ difficulties, we added a requirement that the lesson should address a specific problem that a learner may encounter with the content. Yet again, the choice of the problem, as well as the choice of the content, was left open. In this iteration of the task, prospective teachers were able to identify a problem, but their ways of approaching problems were rather limited. In most scripts, the way to deal with an imagined problem was first to get a correct answer and then to turn “back to the basics,” that is, to re-teach the content or restate the ‘rule’ rather than attend to a specific naïve conception causing the problem.

The following is an illustrative example:

Teacher: Today we will practice multiplication of decimals. What is 0.2×0.3 ?

Gary: 0.6

Teacher: Does someone have a different answer?

Annie: 0.06

Teacher: This is correct. Recall that our answer has to have the same number of digits after the decimal as the numbers in question. So in this case our answer will have 2 digits after the decimal.

Susan: So can’t it be 0.60

Teacher: No, because 0.60 is the same as 0.6. Remember, decimals are also fractions. So $60/100$ is the same as $6/10$. So in fractions the answer is $6/100$.

Susan: So can we write the answer as $6/100$

Teacher: No, since we are working with decimals, we write this as 0.06. Now, let’s do a few more examples.

We recognize in this excerpt that the prospective teacher is able to predict a possible confusion, but is not dealing with it in a skillful manner. She explained not only the ‘rule,’ but also a motivation for the rule by drawing a connection with a common fraction. In terms of shift of attention, we recognize that the teacher’s attention is focused on helping students get a correct answer, rather than on attempting to understand and

address their difficulties. In terms of awareness, we suggest that the teacher's response indicates that she has not yet developed awareness-in-discipline, and her instructional behaviour is an attempt to transform self-awareness-in-action, that is, ability to perform calculations correctly, to students' awareness-in-action.

Looking at the collection of plays produced by teachers at this phase, we further found that their personal 'example space' of possible mistakes was very limited, and most of the examples of mistakes generated by imaginary students dealt with fractions or incorrect column subtraction. The latter example was previously discussed in class when the topic of deliberation was 'buggy algorithms' (Ashlock, 2005). This choice of topics is hardly surprising as personal example spaces are said to be "triggered by current task, cues, and environment, as well as by recent experience" (Watson and Mason, 2005, p.76). Yet again, a more specific direction was needed, and it emerged in our next phase.

Lesson Play – Third Steps (Shift of attention to a specific problematic)

As the third iteration of the task, we engaged prospective teachers in writing and presenting scripts for lesson plays, based on the provided prompts of interaction. Each prompt presented a common error in a student's reasoning. These prompts included the following:

- There are 20-25 students in the classroom. They are working on the following problem:
Once upon a time there were two melon farmers; John and Bill. John's farm was 200m by 600m and Bill's farm was 100m by 700m. Who grew the most melons?
S/he sees that the student has written:
They both grew the same amount.
- There is a conversation between the teacher and a student. There are 20-25 other students in the room.
T: Why do you say that 91 is prime?
S: Because it is not in our times table.
- There are 20-25 students in the classroom. They are working on the following problem:
A toy train has 100 cars. The first car is red, the second is blue, the third is yellow, the fourth is red, the fifth is blue and sixth is yellow and so on.
(a) What is the colour of the 80th car?
(b) What is the number of the last blue car?
The teacher is moving through the room observing how the students are progressing. S/he stops and points at one student's work.
T: Why is the 80th car red?
S: Because the 10th car is red. So, the 20th car, the 30th car, the 40th car, and so on, will be red.
- There are 20-25 students in the classroom. They are working on the following problem:
A toy train has 100 cars. The first car is red, the second is blue, the third is yellow, the fourth is red, the fifth is blue and sixth is yellow and so on.
(a) What is the colour of the 80th car?
(b) What is the number of the last blue car?

The teacher is moving through the room observing how the students are progressing. S/he stops and points at one student's work.

T: Why is the 80th car red?

S: Because the 4th car is red, and 80 is a multiple of 4.

The prospective teachers were asked to identify a possible source of the error, consider instructional treatments that attend to the error—without immediately correcting it—and, as in the previous iterations, present those in a dialogue form. These detailed requirements prompted an explicit focus of attention on the students.

The provided prompts were drawn from our 'expert' example space – they included commonly known errors (drawn from conventional example space), such as confusion between perimeter and area, and also difficulties identified in our prior research related to prime numbers and division with remainder (Liljedahl, 2004; Zazkis & Campbell, 1996). They presented prospective teachers with a variety of choices for their assignment, while ensuring that the errors they had to confront were significant enough to prompt thoughtful treatment.

Lesson Play – Fourth Steps (Shift of attention to language and articulation)

The fourth iteration was similar to the third one, but with two important variations. While all the previous 'lesson plays' were designed and written by groups of 3-4 students, this time we asked prospective teachers to submit their plays individually. They could plan and discuss possible approaches to the presented prompts in their groups, but the write-up had to be completed separately. We believed that this would help in developing a personal voice and – though the approach could have been shared by group members – the individual write-up would create a further shift of attention on the language – both imagining and imitating students' language and choosing personal language in a teacher's response. Given the importance of language in mathematical thinking, we wanted every prospective teacher to have to choose the specific words they would use to convey ideas, offer definitions, and respond to student-generated statements. Indeed, when the prospective teachers moved from their group discussions, which generated statements such as "I would just show her ..." or "I'd simply get her to explain what she did...", to the crafting of their play, they found that articulating those simple strategies was not a straightforward task.

The second variation was that we asked students in this iteration to include a diagnosis (written in regular, paragraph form) for the presented error, that is, their speculations of what could have caused the imaginary student's erroneous response. The diagnosis, which required consideration of how an error had been made, and how it might be taken up, would force a more explicit awareness-in-discipline.

Excerpts of 4th generation Lesson Plays

In what follows we present excerpts from several lesson plays and discuss shifts of attention that are present in these plays. We acknowledge that a teacher's attention is

directed simultaneously to a variety of issues, and therefore focusing our analysis on one particular shift does not mean ignoring other aspects.

We invite the reader to consider the following excerpt in terms of shifts of attention that various teacher characters undertake (and that, of necessity, what the prospective teachers writing the lesson play have imagined).

Excerpt 1: pedagogical and mathematical shifts of attention

- Mrs. Green: Why is 80th car red?
Andrew: Because the 4th care is red and 80 is a multiple of 4.
Mrs. Green: I see, 80 is definitely a multiple of 4. How did you reach your answer?
Andrew: Because we just skip-counted by 4's and we called out 80 so that must mean 80 is a multiple of 4. Since the 4th car is red, the 80th car will be red.
Mrs. Green: I see. So what you're saying is that all multiples of 4 will be red?
Andrew: Yes!
Mrs. Green: OK, Let's test your theory and break it down into smaller numbers. Using the blocks, arrange them into the pattern of red-blue-yellow and find out what colour the 8th block is.

Mrs. Green walks away while Andrew finds the box of blocks and arranges them into the pattern. She circulates among other students while Andrew works on his own. After a few minutes she returns.

- Mrs. Green: So Andrew, what did you discover?
Andrew: I put the blocks into patterns but the 8th car isn't red.
[...]
Mrs. Green: Well. So far, what do we know?
Andrew: That the 4th car is red, the 8th car is blue and the 12th car is yellow
Mrs. Green: Right, and these numbers are all multiples of what:
Andrew: 4
Mrs. Green: What are you thinking now: Has anything changed?
Andrew: I don't think the 80th car is red anymore, But I still don't know what color it is. What do I do?
Mrs. Green: I don't know the answer, I was hoping you would find that for me. Let's take a look at another pattern. What happens if we add another colour, green, to the train? So the pattern is red, blue, yellow, green?

Mrs. Green circulates and returns in a few minutes.

(In what follows Andrew makes a connection between the suggested pattern of 4 colours and a multiples of 4, understands that he has to look at multiples of 3 when considering the original problem, skip counts up to 78 and draws a conclusion about the 80th car, continuing the repeating pattern from 79.)

We recognize a variety of pedagogical shifts of attention in this excerpt. We consider as ‘general pedagogy’ a teacher’s move that is not specific to the task. That is, a move that can be applied in a different situation, not necessarily mathematical. This includes requests to clarify (“How did you reach your answer”), reflection/rephrasing and extenuation (“what you’re saying is that all multiples of 4 will be red?”), directing a student to use manipulatives, putting the student in charge (by suggesting “I don’t know the answer, I was hoping you would find that”), and walking away to allow the student to make progress on his own. In this last action, by removing herself from the discussion, the teacher fosters exploration, discovery and self-reliance.

We further recognize shifts of attention that are mathematical in nature and directed towards the specific problem. We consider those as ‘mathematical pedagogy’ (Mason, 2007). To help Andrew discover his error, Mrs. Green suggests to “break it down into smaller numbers.” This may be an implicit enacting of Polya’s powerful problem-solving strategy: think of a simpler but similar problem (Polya, 1945/1988). While Mrs. Green skillfully left it open for the student to decide what smaller number to focus on, in other excerpts we found the teacher character explicitly instructing the student to consider the 8th car or leaving the specific but a-priori choice for a student, asking “give me a multiple of 4 under 25,” in order to continue the exploration with the student’s choice.

Moreover, attending to mathematics and to Andrew’s mathematics, Mrs. Green’s intention is not only for Andrew to discover that considering multiples of 4 is unhelpful, but also to exemplify in what case looking at multiples of 4 is appropriate. She achieves this by inviting Andrew to consider a train with a unit of repeat consisting of 4 colours (by adding a green car). This move, which demonstrates an explicit choice of action to promote learning, can be seen as an example of her awareness-in-discipline. It is likely that the writer of this play saw major importance in this move, given that the character was named “Mrs. Green.” In a play written by another prospective teacher, a similar intention was exemplified by drawing her student’s attention to a pattern with a repeating unit RBYR, in order to distinguish from the given sequence RBY.

Excerpt 2: shift of attention toward language

- T: Why do you think the 80th car is red?
Zach: Well, the red car is the 4th car, and we need to know what colour the 80th car will be so we divided 80 by 4 and it worked.
T: What do you mean it worked? 80 divided by 4 = red car?
Zach: Yeah, well no, like 4 divides into 80 evenly, so it must be red because the fourth car is red in the pattern.
T: OK. So you are saying every fourth car in the pattern is going to be red, and 80 is a multiple of 4, so the 80th car must be red. Why

don't you check your theory on some other multiples of four by writing out the pattern.

[...]

Zach: Red is wrong, a yellow car comes every three cars now, look!

red blue yellow red blue yellow red blue yellow red

1 2 3 1 2 3 1 2 3 1

Hailey: So the red car is actually the first car, we can't divide by 1

Zach: So let's divide 80 by 3, what do we get?

Hailey: Um, it equals 26.6666 Great, now what

Zach and Hailey's hands go up

Zack: We decided that in the pattern every third car is yellow, so we divided 80 by 3, which is 26.6666. So we do not know what to do.

T: You are on the right path, I'll give you a hint. Put away your calculators and divide it by yourself and think of it in terms of remainders...

Hailey: Are you good at long division Zach?

[...]

Peter: Can I pretend the question asked me about 78th car?

Zack: What for?

Peter: if 3 goes into 78, 26 times evenly then the 78th car is yellow, and that is a multiple of 3, so to get to 80 you just add two more to the pattern, a red and a blue. So the 80th car is Blue.

Hailey: Oh my gosh, your right. You're so smart.

In the beginning of this excerpt we notice the writer's awareness of the language of mathematics. While Zach claims "like 4 divides into 80 evenly", the teacher picks up his idea, introducing appropriate mathematical expression "80 is a multiple of 4". Her mathematical-pedagogical suggestions are to "write out the pattern" and her hint is to "put away your calculator". In approaching the task, this nameless teacher's character has older or more sophisticated students than those of Mrs. Green. While in the previous excerpt the solution is found by modeling the train with manipulatives and then counting up by multiples of 3, in this play the solution is found by more efficient means: writing out the pattern and then carrying out the division. While the error that we introduced as the motivation for the play was observed with different age groups, including prospective teachers (Liljedahl, 2004), in the produced plays there are implicit assumptions about the students' prior knowledge and the approach is chosen to be appropriate for the students' level of mathematical sophistication. Further, in Hailey's question, "Are you good at long division Zach?" the writer acknowledges her pedagogical awareness of the fact that many students consider long division as troublesome.

This writer also demonstrates her pedagogical awareness of different ideas students may bring to the task by introducing into the play some group work and collaboration among students. Further, the powerful idea of looking at a multiple of 3 close to 80 is voiced by

a student. We note that not only does this idea lead to the desired solution, but also this student uses both the language accessible to his classmates – “3 goes into 30 26 times evenly” – and also the language modeled by his teacher – “that is a multiple of 3”. We consider this delicate choice of wording as the writers’ awareness-in-discipline. It appears intentional that the “smart” student’s character is named Peter. ☺

Excerpt 3: shift of attention toward the learner

Novice teachers tend to go through a phase in which they focus on the curriculum at the exclusion of students’ needs. Often this can be seen as a drive towards a specific learning outcome as opposed to working with the mathematics the student brings to a situation. In what follows we see a teacher who actually attempts to follow through with the students’ mathematics.

Teacher: Why is the 80th car red?
Stacy: Because the 10th car is red. So, the 20th car, the 30th car, the 40th car, and so on will be red.
Teacher: Alright, so you are thinking that the patterns will repeat for each set of 10?
Stacy: Yes.
Teacher: Ok, well why don’t you continue this pattern for a while and see another row until you reach 20 and then let me know if your theory is working out.

(5 minutes later)

Stacy: It didn’t work ... the 20th car is blue!
Teacher: Hmm ... okay. So are you still sure that the 80th car is going to be red?
Stacy: Not any more. If the 20th car is different than the 10th car, then I’m not sure what the 80th car is.
Teacher: So, what are you going to do?
Stacy: I’m going to keep going to see what colour the 30th car is?

(1 minute later)

Teacher: Well ...
Stacy: The 30th car is yellow. So, I’m thinking that the 40th car will be red.
Teacher: Why is that?
Stacy: Because that is the pattern we have – red, blue, yellow.
Teacher: Well, try it and see.
Stacy: It is red. So that means the 50th will be blue, the 60th will be yellow, the 70th will be red, and the 80th will be blue.
Teacher: Good ... now why don’t you go share your discovery with Aaron. I’m sure that he has something he wants to share with you as well.

We appreciate here the teacher’s attempt to pursue the students’ initial ideas in focusing on multiples of 10. However, while in the previous excerpts there was an attempt to have students understand the pattern, and not only to discover the pattern, in this excerpt the discovery about repeating pattern of colours within multiples of 10 remains unexplained. Recognizing that there are more efficient and transferable strategies the teacher mobilizes

some of the knowledge that exists within the classroom by having Stacy work with Aaron – who, as it turns out, is using a more standard strategy of looking at the length of the repeating block. This is a valuable pedagogical shift of attention.

While in many examples the play writers exhibited a variety of shifts of attention, both mathematical and pedagogical, we still received a number of lesson plays where the lesson involved what we call “simple telling.” In these imaginary lessons the teacher character simply corrected the student’s error and explained how the problem should be approached. We suggest that the lesson plays prospective teachers write provide researchers with a window on mathematical and pedagogical knowing, and especially on their awareness-in-action, thus giving teacher educators a better opportunity to develop their own awareness-in-council. Our ongoing research analyses this mathematical and pedagogical knowing, thereby providing a novel contribution to the growing body of research on mathematics-for-teaching.

Lesson play: towards ‘real teaching’

“The key notions underlying real teaching are the structure of attention and the nature of awareness” (Mason, 1998, p. 244). However, much of the preparation for ‘real teaching’ is done in University/college classrooms. In attempting to make this preparation more effective, teacher educators continuously endeavour to design suitable tasks and experiences for prospective teachers. For example, Silver, Clark, Ghouseini, Charalambous, and Sealy (2007) advocate for ‘practice-based professional development’ and illustrate its key element, the ‘professional learning task.’ The goal of such a task is to engage prospective teachers in activities that resemble the daily work of practicing teaching. Professional learning tasks include examination of curriculum materials, video or narrative records of classroom teaching episodes and consideration of students’ work. They “create opportunities for teachers to ponder pedagogical problems and their potential solutions through processes of reflection, knowledge sharing and knowledge building” (p. 262).

With the specific goal of developing teachers’ reflective practice, Peng (2007) describes the task of ‘lesson explaining’ that has been designed in China. The task requires teachers “to explain how the content unfolds in the lesson, and the nature of mathematical challenges it offers” (p. 290). It includes explanation of mathematical content, justification of the chosen method and explanation of the ‘teaching procedure’, where the latter focuses on the development of students’ ability, attending to learner’s cognitive foundation and individual differences.

Biza, Nardi, and Zachariades (2007) developed tasks with “situation specific contexts,” where they present prospective teachers with specific student response and seek their comments. The student response they chose was fictional, erroneous, and yet plausible. They suggest that such a task offers “an opportunity to explore and develop teacher’ sensitivity to student difficulties and needs as well as ability to provide adequate (pedagogically sensitive and mathematically precise) feedback to the student” (p. 303). They further acknowledge that since the engagement is not in the classroom and not in

“real time” it provides teachers with the opportunity to think about their reaction and be reflective.

The task of “lesson play” fits well with the intentions and goals developed by the above-mentioned authors. It can be considered as a “professional learning task” (Silver et al, 2007) in creating resemblance to the work of practicing teachers. It includes components of ‘lesson explaining’ (Peng, 2007) and it starts with presenting fictional, yet plausible situations that are context specific (Biza et al, 2007) and seeks their resolution. Moreover, the task of designing a lesson play adds an important component in creating a situation of “*imagining* the real teaching”, rather than simply discussing it. This is in accord with Watson and Mason (2007) view that “the fundamental issue in working with teachers is to resonate with their experience so that they can *imagine* [our italics] themselves ‘doing something’ in their own situation” (p. 208). With this imagination, attention and awareness are developed in “slow motion,” having a complete control of the situation and ability to replay or redress it, rather than “thinking on one’s feet” and making in-the-moment decisions.

The task of creating a lesson play shifts prospective teachers’ attention from general curricular objectives to specific teaching incidents, and invites them to imagine such incidents in a very detailed manner. Mason uses the phrase ‘shift of attention’ to focus on different mathematical aspect of a given problem. We adopted the phrase, but considered shifts that take place in teaching, which, in addition to shifts on various mathematical components of the task, include shifts to pedagogy, to didactics, to students’ difficulties, and to language. Lesson plays provide an opportunity for prospective teachers to develop their awareness-in-discipline, to put in action their mathematical pedagogy, while attending to different demands of the complex act of teaching. We believe that creating these plays equips teachers with a repertoire of responses that they will be able to call upon in their ‘real teaching’.

Afterthought

In a recent (March, 2008) meeting in Rome, celebrating the centennial of ICMI, Hyman Bass posed the following question to the participants of the working group on teacher education: “What does/should teacher educator know that an experience teacher doesn’t know”. It was not a simple question to answer and there was a feeling of unease in the group. After some discussion, Hyman Bass suggested that an experienced teacher should/does know how to design instructional task and the expertise of a teacher educator is in guiding teachers in their task design.

John Mason’s opus, and specifically his work on awareness and shifts of attention, suggests a different answer. The difference between expert-teacher and expert-teacher-educator is not at the level of knowledge, but at the level of awareness. Awareness-in-discipline is a characteristic of an expert teacher and “what constitutes the practice of an expert” (1998, p. 260). Awareness-in-counsel is what guides the practice of teacher-educator. Designing tasks for students and guiding teachers in task design are just one example of these different levels of awareness in play. The lesson play task, in its various

iterations, enriched our awareness-in-counsel while helping prospective teachers develop their awareness-in-discipline.

After-Afterthought

In Hebrew “to pay attention” is “lasim lev”, a phrase that is translated literally as “to put a heart”. This harmonizes nicely with Mason’s (1998) suggestion that “we are our attention and we are where our attention is” (p. 251). It also gives enriched meaning to John Mason’s suggestion that the significant products of mathematics education research are the “transformations in the being of the researchers” (Mason, 1998, p. 357). Indeed.

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