TEACHERS' INSIGHTS INTO THE RELATIONSHIP BETWEEN BELIEFS AND PRACTICE

Peter Liljedahl

Simon Fraser University

Canada

A set of complex relationships exist between teachers' espoused mathematical beliefs, their plans for teaching mathematics, and their actual teaching of mathematics. Many of these relationships have been explored in previous research. In this study I look more closely at one of the relationships that has not received much attention. In particular, I examine the possible disjunctive relationship between teachers' espoused beliefs and their intentions of practice. In so doing, I challenge the research that assumes a correlative relationship between these two aspects. The results indicate that this disjunctive relationship exists and that there are very rational reasons for it to exist. As such, I add to the growing body of research that links teachers' beliefs to teachers' practice. Finally, I introduce a research methodology that proves to be a very effective method for gaining insight into the complex domain of teachers' beliefs and practice.

INTRODUCTION

Much has been written about the inconsistency between teachers' beliefs and teachers' practice. Mostly this has been documented as a difference between preservice or novice teachers' espoused beliefs and their classroom actions (c.f. Vacc & Bright, 1999; Wilson & Cooney, 2002). As a result, Wilson & Cooney (2002) warn us not to consider teachers' espoused beliefs as predictors of practice. However, they also warn us not to discount these professed beliefs as unauthentic, but rather to think of them as representative of their intentions of practice (Feiman-Nemser, McDiarmid, Malnick, and Parker, 1987; Fosnot, 1989). That is, the espoused beliefs are indicators of how the preservice teachers envision themselves behaving in their imagined classroom environments and encounters. Conversely, teachers’ articulated intentions of practice "can provide significant insight into what teachers value and the relative importance they assign to different aspects of mathematics or the teaching of mathematics" (Wilson & Cooney, 2002, p. 131). At the same time, there is a large body of literature that details the disjunction between novice teachers' intentions of practice and actual actions. This disjunction is often attributed to the challenges that novice teachers must contend with in their early years of practice (Cooney, 1985; Karaagac & Threlfall, 2004; Noyes, 2004; Skott, 2001). That is, they fail to realize their intentions in the face of a reality that does not match the envisioned environments and encounters for which their intentions were constructed.

Together, this aforementioned research can be represented in a triangular relationship with espoused beliefs, intentions of practice, and actual practice at the vertices and the disjunctive (→←) and correlative (↔) relationships represented by arrows along the edges (see figure 1).

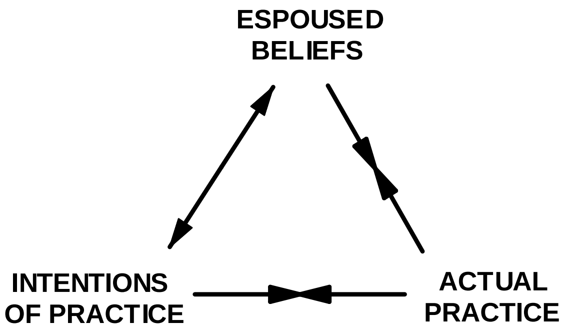


Figure 1: Triangular representation of relationships as seen in the research

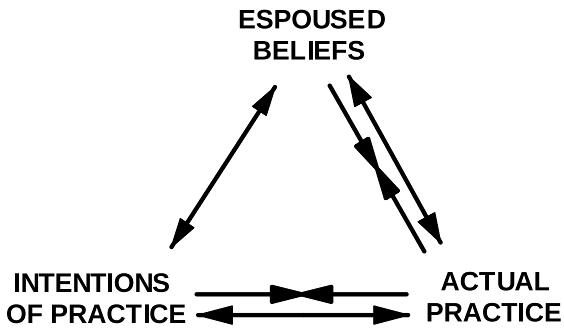


Figure 2: Modified triangular representation of relationships

What is neglected in much of the aforementioned small scale research and case studies, however, are the contrary relationships that also exist. That is, as much as there are instances of disjunction (→←) between professed beliefs and classroom practice (or intentions of practice and actual practice) there are also many instances of correlation (↔) (Fosnot, 1989; Millsaps, 2000; Skott, 2001; Uusimaki & Nason, 2004). However, these correlative relationships are to be expected and, as such, are seen as "uninteresting" and unworthy of the close scrutiny devoted to the disjunctive relationships. Nonetheless, these relationships do exist and should be included in the triangular representation (see figure 2).

Figure 2 highlights an obvious omission – the possible disjunctive relationship between espoused beliefs and intentions of practice. In this study I look more closely at this relationship to establish 1) if such a disjunction can/does exist, and if so 2) what the nature of this disjunction is.

RELEVANT PRIOR WORK

Ernest (1991) describes three philosophies of mathematics called instrumentalist, Platonist and problem solving. Similarily, Dionne (1984) suggests that mathematics can be seen as one of (or a combination of) three basic components called the *traditional* perspective, the *formalist* perspective, and the *constructivist* perspective. Törner and Grigutsch (1994) refer to these three aspects as the *toolbox* aspect, the *system* aspect and the *process* aspect. For Törner and Grigutsch (1994) seeing mathematics as a *toolbox* encompasses beliefs about mathematics as a set of rules, formulae, skills and procedures, and beliefs about mathematical activity as calculating as well as using rules, procedures and formulae. A *systems* view of mathematics is characterized by logic, rigorous proofs, exact definitions and a precise mathematical language, and doing mathematics consists of accurate proofs as well as of the use of a precise and rigorous language. In the *process* aspect mathematics is considered as a constructive process where relations between different notions and sentences play an important role. Here the mathematical activity involves creative steps, such as generating rules and formulae, thereby inventing or re-inventing the mathematics.

Beliefs about mathematics and mathematical activity are not confined to one of these discrete aspects, however (Dionne, 1984; Törner and Grigutsch, 1994). Instead, an individual's beliefs about mathematics can be seen as a combination of these three with preference potentially being given to some aspects more than others. Dionne explored this combination of aspects by allowing his subjects to assign weight to each aspect with the stipulation that the total of the three weightings be 30. Törner and Grigutsch (1994) extended this methodology by allowing their subjects to represent this weighting not only as a series of numbers summing to 30, but also as a point within an equilateral triangle whose vertices are the three dimensions of mathematical beliefs (see figure 3). Within this graphical representation proximity to a vertex demonstrates dominance of a specific aspect. For example, in the particular case displayed in figure 3, the individual possesses a dominant belief that mathematics is a *system* with lesser views of mathematics as a *toolbox* or as a *process*. Of course, in such a construct it is possible to demonstrate complete rejection of one or two of the dimensions by placing a point on one of the edges of the triangle or at a vertex.

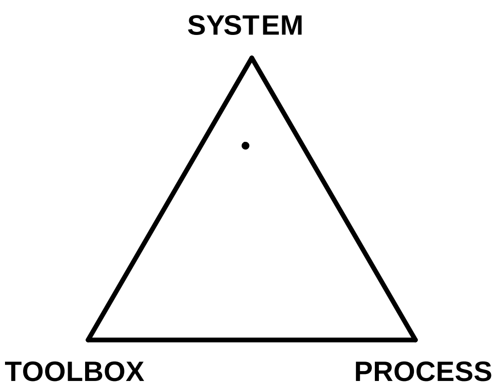


Figure 3: Graphical representation of three aspects of mathematical beliefs

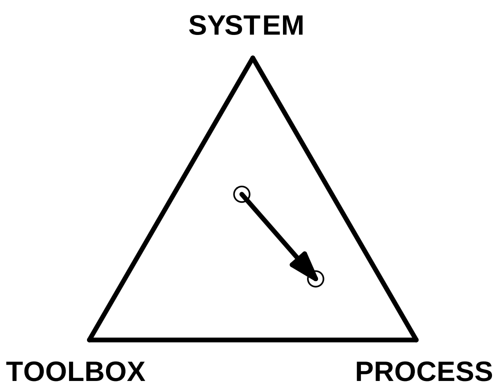


Figure 4: Vector representation of difference between *real* and *ideal* teaching

Törner and Pehkonen (1998), in a case study examining different self-estimation methods, used and extended this methodology even further by first asking teachers to represent their beliefs about teaching as opposed to their beliefs about mathematics. In fact, within the context of their study they felt that "it appears problematic to distinguish between beliefs on mathematics and beliefs on the teaching of mathematics" (p. 3) because "in school, mathematics appears solely in the form of mathematics taught by teachers to pupils" (p. 3). Second, they used an extension of the methodology by asking teachers to identify both their real teaching of mathematics and their ideal teaching of mathematics vis-à-vis the three aspects of *toolbox*, *system*, and *process* using both, what they refer to as, the Dionne method of assigning weighting as well as the graphical representation developed by Törner and Grigutsch (1994). Finally, they plotted the difference between participants' visually demonstrated *real teaching* and *ideal teaching* as vectors whose lengths were calculated and compared (see figure 4).

METHODOLOGY

The study presented here is part of a larger, ongoing, study investigating the relationship between teachers' beliefs and their professional growth. This larger scale study involves preservice teachers, novice teachers, and inservice teachers. The data used in this particular study comes from a group of inservice secondary school mathematics teachers (n = 13) enrolled in a Secondary Mathematics Education Masters program. However, the methodology used concerns both the inservice secondary teachers as well as a group of preservice elementary school teachers (n = 36) enrolled in an Elementary Mathematics Methods course.

The preservice elementary school teachers were given a questionnaire that is a variation of the one used by Törner and Pehkonen (1998). This questionnaire was given in two parts, three hours apart. First, based on the definitions of the three aspects of *toolbox*, *system*, and *process* the participations were asked to locate their **view of mathematics** on the equilateral triangle provided. Second, based on the assumption that in a given school year they would dedicate 90 hours to the teaching of mathematics, they were asked to articulate how many hours they would dedicate to the **teaching of mathematics** according to each of the three aforementioned aspects. Following Törner and Pehkonen (1998) these two tasks were seen as related and thus affording the possibility to explore the relationship between espoused beliefs and intentions of practice.

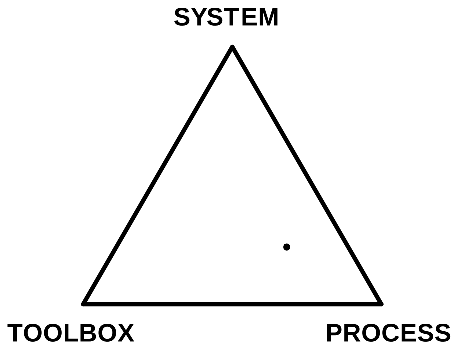
Initial analysis[2] of the data revealed that, although there was general agreement between the two tasks for most of the participants there was also a number of participants who demonstrated an obvious[3] disjunction between their responses to the two tasks. To gain insight into this disjunction two exemplars (see figure 5 and 6) were created and given to the inservice secondary mathematics teachers for consideration. Working in pairs, the participants were asked to hypothesize as to why a teacher, or a group of teachers, may respond to the two tasks in such a fashion –

[2] A detailed analysis has since been done. Results correspond with the initial analysis – but will not be reported here.

[3] On such a task as presented here disjunction is first a matter of degree, and second a matter of subjective judgement. Detailed analysis can not change this. Having said that, however, using a simple categorization of *clear correlation*, *clear disjunction*, and *partial correlation/disjunction* some results were extracted from the data. Of the 36 responses, 19 were classified as *clear correlation*, 8 as *clear disjunction*, and 9 as *partial correlation/disjunction*.

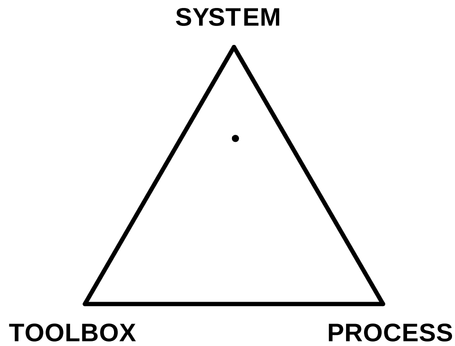
surprisingly, they had no difficulty doing so, often coming up with several possible reasons. The data consists of field notes of in-class presentations of their conjectures as well as individual responses to a subsequent writing prompt.

The data was sorted according to trends that emerged in the participants' responses. This involved an iterative process of identifying themes, coding for themes, identifying more themes, recoding for the new themes, and so on. This was followed by a collapsing together of themes that were subsumed by another theme.



Aspect	**Hours**
toolbox	70
system	10
process	10
total	**90**

Figure 5: Disjunction A



Aspect	**Hours**
toolbox	10
system	20
process	60
total	**90**

Figure 6: Disjunction B

RESULTS

In the end, there were a total of 13 rationalizations for disjunction A (see figure 5) offered and 17 rationalizations for disjunction B (see figure 6). These 30 rationalizations collapsed into five emergent themes, each of which gives insight into the disjunction between espoused beliefs and intentions of practice. In what follows I first summarize the distribution of these rationalizations across the disjunctive exemplars and the emergent themes (see table 1), then I briefly exemplify each of the themes with data.

A reminder is needed before the results are presented, however. Although the exemplified disjunctions comes from the preservice elementary school teachers the actual data for this study comes from the inservice secondary mathematics teachers reactions and rationalizations about the disjunctions. It is important to keep this in mind when attending to the presented results that follow.

Theme	Disjunction A	Disjunction B	totals
Theoretical vs. Practical	3	5	8
Ideal vs. Real	4	3	7
Outcome vs. Process	4	5	9
Mathematics vs. Teaching	1	3	4
Context vs. Context	1	1	2
totals	13	17	30

Table 1: Distribution of rationalizations

Theoretical vs. Practical

An interesting, and wholly unexpected, theme to emerge is one that clearly articulates the disjunctions as a difference between teachers' theoretical beliefs and practical beliefs. This can be seen in David and Wanda's explanation to the class.

> Maybe these teachers have different beliefs for different situations. When they are in a class here at the university they believe that mathematics is one thing and when they are in the classroom they believe that mathematics is another thing. For example, if we look at disjunction A, we can think of the teacher as believing that mathematics is all about creating knowledge and connections and discovering relationships when they are sitting in a theoretical course about teaching. However, when they are actually in the classroom they believe that mathematics is really all about skills and facts which is probably something they picked up when they were a student themselves.

Steve and Colleen offer a slightly different view of this in their presentation to the class. They articulate the disjunction as the difference between the nature of *'real'* mathematics and school mathematics.

> Both of these disjunctions can be seen as a difference of the way the teachers are looking at mathematics between real mathematics and school mathematics. Real mathematics is the mathematics that mathematicians do and it can either be about proof or about discovery depending on how you look at it. School mathematics, on the other hand, is the mathematics done in school and it can either be a lot of memorized math facts and algorithms or it can be relationships and discovered patterns.

Ideal vs. Real

This theme is similar in nature to the first theme in that it deals with the tension between an unrealized (theoretical) view and a realized (practical) view. The difference here, however, is that both responses refer to the teaching of mathematics as opposed to the beliefs of mathematics as in the previous theme. That is, there were seven rationalizations made regarding the disjunctions that articulated the differences as a difference between how the subjects would want to teach versus how they are teaching. Sam's written response about disjunction A demonstrates this nicely.

> One obvious way to look at this inconsistency is to think about it as the difference between how they are teaching and how they want to teach. The breakdown of how they spend their time is how they are currently teaching and is indicative of them being trapped in a traditional practice. The triangle, on the other hand, shows how they want to be a more progressive teacher.

For disjunction B Angela makes the same observation as Sam with regards to the positioning of the idealized view of teaching and the current way of teaching, but turns the tables on what is and what is not desired.

> If we think of the triangle as showing how the teachers want to teach and the table as how they are teaching we can make sense of disjunction B. Perhaps this teacher is trapped in a curriculum where they are having to use a fuzzy math[4] approach to teaching but at heart they don't believe in it. They maybe come from a mathematics background and long for a style of teaching that focuses on proofs and definitions.

Outcome vs. Process

The most common rationalization for the exemplified disjunctions was an articulation that this is the difference between outcome and process. That is, the view of mathematics is an idealized view of how mathematics should be, but that a process needs to be followed in the teaching of mathematics to achieve this. Within the context of disjunction A this was uniformly articulated as a rationalization that the subjects believed that a focus on the toolbox aspect will achieve the idealized outcome of a more constructed view of mathematics. This can be seen in Keith and Darleen's explanation to the class:

> One way to think about this is to assume that the preservice teachers have bought into the reform idea of mathematics but they see that it requires a lot of prerequisite knowledge and skills in order to start 'constructing' mathematics.

Within the context of disjunction B, however, the notions of outcome and process are articulated completely differently. Here the outcome is seen as a rigorous and precisely defined view of mathematics and the process that is followed to achieve it is a reform view of teaching. This is articulated in what Richard wrote in his journal.

> This is sort of a hybrid position. The teachers who produced this data could have bought into the notion of the reform movement as a way of teaching, but still held to the idea that mathematics is really all about precise definitions and rigorous proof. This really describes my own journey through this program. I came in with a very rigorous view of mathematics and this view has remained quite unchanged. What has changed, however,

[4] This is a phrase that has been used and popularized within the local media to describe reform methods of teaching mathematics.

> is my teaching practice – from the teaching of rules and facts to the co-construction of understanding.

Richard is not the only one who expressed such thoughts. Given that the participants were secondary teachers, many came to mathematics teaching through degrees in mathematics and, as such, were exposed to a very analytic perspective of mathematics – a perspective that they often held onto even as they reformed their teaching

Mathematics vs. Teaching

One of the least commonly articulated rationalization for the presented disjunctions is seen as a difference between mathematics and teaching. This is different from the previous theme in that here the differences between the two exemplars are seen as differences in views of mathematics and views of the possible realities of teaching mathematics. One participant, Meagan, commented on both cases at the same time.

> Really, this can simply be seen as the difference between mathematics for mathematicians and the teaching of mathematics. It is certainly possible to view mathematics as either a system or a process. In fact, this describes my own growth. I used to see mathematics as more of a system, but now I view it more as a process [..]. It is also easy to view teaching as either a toolbox or a process depending where the teachers are in their own growth. I'm sure that most preservice elementary school teachers come to PDP[5] with experiences of having been taught mathematics as a toolbox. When they get into PDP they are exposed to the ideas of the reform movement.

It is interesting to note that the very assumption that Törner and Pehkonen (1998) make regarding the lack of distinction between mathematics and teaching surfaces as a possible rationalization for the presented disjunctions.

Context vs. Context

Lastly, and least common, is the theme that explains the disjunction as a difference between contexts. Keith picks up on this idea in his written journal.

> It is not too difficult to imagine that teachers have different beliefs for different situations. I know that my own beliefs about mathematics changes depending on if I am teaching a grade 8 class or an AP Calculus class. So, why can't teachers have different beliefs about mathematics when they are in school and when they are out of school. After all, many of us come from a background of university mathematics and are now teaching high-school mathematics. These are two very different mathematics.

Keith's ideas resonate with the theme of theoretical vs. Practical in that it speaks to the situatedness of beliefs. That is, beliefs are context dependent.

[5] This is the acronym for the local teacher education program (PDP - Professional Development Program).

CONCLUSIONS

There are several conclusions that are drawn from this study. First, however, I wish to once again remind the readers who the participants in this study are. Although the exemplars come from preservice elementary teachers, the research participants are inservice secondary mathematics teachers. Their insights into the exemplified disjunctions between beliefs and intentions of practice may provide insights into the possible disjunctions that preservice elementary teachers demonstrate. However, I argue that the results speak more about the disjunctions that may have been present within the secondary teachers' own professional growth. At a very rudimentary level, this is most easily seen in the many 'subject errors' they make in their comments. That is, in the errors regarding who they are discussing and what their context is. There are three types of such errors. First, although they know that the exemplars are drawn from preservice teachers they often slip into a discussion about practicing teachers. This can be seen in Angela and Sam's comments. Second, although they know the exemplars are drawn from an elementary context they often port the discussion over to the secondary context. Keith does this in his discussion on the situatedness of beliefs and Angela implies it when discussing a background in mathematics that included a lot of rigour and proof. Finally, many of the participants explicitly discuss the disjunctions within the contexts of their own experiences. We saw this from Keith, Meagan and Richard in the data presented above.

The results of this study clearly show that, contrary to some of the literature (Feiman-Nemser, McDiarmid, Malnick, and Parker, 1987; Fosnot, 1989; Wilson & Cooney, 2002) there is not always a correlative relationship between teachers' espoused beliefs and their intentions of practice. This fact was first introduced in the disjunctions that presented themselves in the data collected from the preservice elementary teachers and then elaborated on in great detail when these disjunctions were rationalized by the inservice secondary teachers. Further, the nature of this disjunctive relationship has proven to be rather complex. The data has shown that the disjunction can exist due to five different themes, each of which speaks to reasonable[6] ways in which teachers may think about mathematics or mathematics teaching that do not correlate with ways in which they may plan for, or think about, teaching. As such, we can no longer assume that a strong connection exists between teachers' espoused beliefs and their intentions of practice. This result adds another refinement to the graphically represented relationships seen in figures 1 and 2 (see figure 7).

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[6] As such, this research adds to the recent literature on teachers' beliefs as sensible (Leatham, 2006).

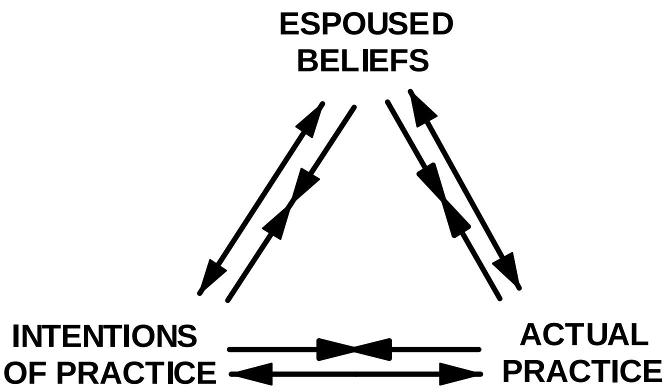

Figure 7: The correlative and disjunctive relationships that exist between teachers' espoused beliefs, their intentions of practice, and their actual practice.

Finally, this study makes a methodological contribution to the research on teachers' mathematical beliefs. Having inservice teachers hypothesize and comment about preservice teachers' demonstrated disjunctions provided a window, not only into the preservice teachers' beliefs and how these beliefs impact on practice, but also into the inservice teachers' beliefs, their professional growth, and their subsequent practice. This methodology can be utilized within a large number of contexts within this research domain and others.

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