Building Thinking Classrooms

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Abstract In this chapter I first introduce the notion of a thinking classroom and then present the results of over ten years of research done on the development and maintenance of thinking classrooms. Using a narrative style I tell the story of how this research began and led first to the notion of a thinking classroom and then to a research project designed to find ways to help teacher build such a classroom. Results show that there are a number of relatively easy to implement teaching practices that can bypass the normative behaviours of almost any classroom and begin the process of developing a thinking classroom.

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Motivation

My work on the research presented in this chapter began over 10 years ago when I spent a three days observing June try to implement problem solving in her grade 8 mathematics class. What I observed was not good. The students gave up almost as soon as a problem was presented to them. There was some work attempted when June was close by and encouraging the students, but as soon as she left the trying stopped. At the end of the three days June and I agreed that our efforts were not working for this class. But I wanted to understand why. So, I stayed on for a week and just watched June teach her mathematics classes using her normal practice.

After three days of observing June’s normal classroom routines I began see what was going on. That the students were lacking in effort was immediately obvious, but what took time to manifest was the realization that what was missing in this classroom was that the students were not thinking. More alarming was that June’s teaching was predicated on an assumption that the students either could not, or would not, think. The classroom norms (Yackel & Rasmussen, 2002) that had been established in June’s class had resulted in, what I now refer to as, a non-thinking classroom. Once I realized this I proceeded to visit other mathematics classes – first in the same school and then in other schools. In each class I saw the same basic behaviour – an assumption, implicit in the teaching, that the students either could not or would not think. Under such conditions it was unreasonable to expect that students were going to be able to spontaneously engage in problem solving.

What was missing for these students, and their teachers, was a central focus in mathematics on thinking. The realization that this was absent in so many classrooms that I visited motivated me to find a way to build, within these same classrooms, a culture of thinking, both for the student and the teachers. I wanted to build, what I now call, a *thinking classroom* – "a classroom that is not only conducive to thinking but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together, and constructing knowledge and understanding through activity and discussion" (Liljedahl, 2016a, p.364).

Early Efforts

Classroom norms, once established, are difficult to change (Yackel & Rasmussen, 2002). My early efforts to build thinking classrooms revealed that even when a teacher is motivated to get their students to think, their initial efforts to do so are rarely rewarded by comparable changes in their behaviour. Quite the opposite, many of the teachers I was working with were met with resistance and complaints when they tried to make changes to their practice.

From these experiences I realized that if I wanted to build thinking classrooms – to help teachers to change their classrooms into thinking classrooms – I needed a set of tools that would allow teachers to bypass any existing classroom norms. These tools needed to be easy to adopt and have the ability to provide the space for students to engage in thinking unencumbered by their rehearsed tendencies and approaches when in their mathematics classroom.

This realization moved me to begin a program of research that would explore both the elements of thinking classrooms and the traditional elements of classroom practice that block the development and sustainability of thinking classrooms. I wanted to find a collection of teacher practices that had the ability to break students out of their classroom normative behaviour – practices that could be used by teacher that had previously entrenched the classroom norms that now needed to be broken.

In Pursuit of Thinking Classrooms

My research to find the elements and teaching practices that foster, sustain, and impeded thinking classrooms has been ongoing for over ten years. I initially explored my own teaching, as well as the practices of more than 40 classroom mathematics teachers. From this emerged a set of 11 elements that were found to permeate mathematics classroom practice – elements that account for most of whether or not a classroom is a thinking or a non-thinking classroom. These 11 elements of mathematics teaching became the focus of my research. They are:

1. the type of tasks used, and when and how they are used;
2. the way in which tasks are given to students;
3. how groups are formed;
4. student work space while they work on tasks;
5. room organization, both in general and when students work on tasks;
6. how questions are answered when students are working on tasks;
7. the ways in which hints and extensions are used while students work on tasks;
8. the autonomy students have while working on tasks;
9. when and how a teacher levels[1] their classroom during or after tasks;
10. the ways in which students record notes;
11. and assessment, both in general and when students work on tasks.

June's class, as determined earlier, was not a thinking classroom:

1. practice tasks were given after she had done a number of worked examples;
2. students either copied these from the textbook or from a question written on the board;
3. students had the option to self-group to work on the homework assignment when the lesson portion of the class was done;
4. students worked at their desks writing in their notebooks;
5. students sat in rows with the students' desk facing the board at the front of the classroom;
6. students who struggled were helped individually through the solution process, either part way or all the way;
7. there were no hints, only answers, and an extension was merely the next practice question on the list;

[1] Levelling (Schoenfeld, 1985) is a term given to the act of closing of, or interrupting, students' work on tasks for the purposes of bringing the whole of the class (usually) up to certain level of understanding. It is most commonly seen when a teacher ends students work on a task by showing how to solve the task.

8. students had little to no autonomy in how they engaged in tasks, usually work sheets or work out of the textbook;
9. when “enough time” time had passed June would demonstrate the solution on the board, sometimes calling on “the class” to tell her how to proceed;
10. students wrote down what June wrote on the board at the front of the room;
11. and assessment was always through individual quizzes and test.

Each of these elements were something that needed exploring and experimentation. Many were steeped in tradition and classroom norms (Yackel & Rasmussen, 2002). As such, research into each of these was done using design-based methods (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Design-Based Research Collective, 2003) within my own teaching practice as well as the practices of more than 400 teachers participating in a variety of professional development opportunities. This approach allowed me to vary the teaching around each of the elements, either independently or jointly, and to measure the effectiveness of that method for building and/or maintaining a thinking classroom. Results fed recursively back into teaching practice, each time leading either to refining or abandoning what was done in the previous iteration.

The challenge, however, was to figure out how to shift a teaching practice when it was determined that a particular teaching method needed to be abandoned. Early results indicated that small shifts in practice, in these circumstances, did little to shift the behaviours of the class as a whole. Larger, more substantial shifts were needed. These were sometimes difficult to conceptualize. In the end, a contrarian approach was adopted. That is, when a teaching method around a specific element needed to be abandoned, the new approach to be adopted was, as much as possible, the exact opposite to the practice that had shown to be ineffective for building or maintaining a thinking classroom. For example, when sitting showed to be ineffective, we tried making the students stand. When leveling to the top failed we tried levelling to the bottom. When answering questions proved to cause learned helplessness we stopped answering questions. Each of these approaches then needed further refinement through the iterative design-based research approach, but it gave good starting points for this process.

Results

Through this process a number of results eventually began, at first slowly, to emerge. In what follows I will present, in brief, the results of the research done on each of these eleven elements and discuss how they hold together as a framework to build and maintain thinking classrooms.

1. The type of tasks used

Lessons need to begin with good problem solving tasks. In the beginning of the school year, or when first attempting to transform a classroom, these tasks are highly engaging, non-curricular, collaborative tasks that drive students to want to talk with each other as they try to solve them (Liljedahl, 2008). After a period of time (usually 2-3 weeks) these should gradually be replaced with curricular problem solving tasks that permeate the entirety of the lesson and emerge rich mathematics (Schoenfeld, 1985) that can be linked to the curriculum content to be ‘taught’ that day. These curricula tasks can simply be questions from the textbook provided they are new to the students and present something that is problematic for them.

2. How tasks are given to students

As much as possible, tasks need to be given orally. If there are data, diagrams, or long expressions needed these can be provided on paper or projected on the wall, but the instructions pertaining to the activity of the task need to be given orally. This very quickly drives the groups to discuss what is being asked, focuses groups on the mathematics, and reduces the urge to individually decode instructions on a page.

3. How groups are formed

Grouping and regrouping needs to be frequent and visibly random. Ideally, at the beginning of every class a visibly random method is used to create groups of 2-3 students who will work together for the duration of the class. These groups will work together on any assigned problem solving tasks, sit together or stand together during any group or whole class discussions. Frequent randomization will fundamentally transform the social structure of the classroom within three weeks (Liljedahl, 2016a, 2016b, 2014) and build the type of community needed to autonomously maintain a thinking classroom.

4. Student work space

The work on these aforementioned tasks needs to be done with groups standing and working on vertical non-permanent surfaces such as whiteboards, blackboards, or windows. This makes visible all work being done, not just to the teacher but to the groups doing the work. To facilitate discussion, there is only one felt pen or piece of chalk per group. The use of vertical non-permanent surfaces will increase eagerness to start, increase discussion, participation, and perseverance amongst the group members, and facilitate the mobility of knowledge between groups (Liljedahl, 2016a, 2016b).

5. Room organization

The classroom needs to be de-fronted. The teacher must let go of one wall of the classroom as being the designated teaching space that all desks are oriented towards. The teacher needs to address the class from a variety of locations within the room and, as much as possible, use all four walls of the classroom. It is best if desks are placed in a random configuration around the room, and away from the walls.

6. How questions are answered

It turns out that students only ask three types of questions: (1) proximity questions – asked when the teacher is close; (2) stop thinking questions – most often of the form “is this right” or "will this be on the test"; and (3) keep thinking questions – questions that students ask so they can get back to work. Only the third of these types should be answered. The first two need to be acknowledged, but not answered.

7. How hints and extensions are used

Once a thinking classroom is established, it needs to be nurtured. Student engagement should be maintained through the teacher's judicious and timely use of hints and extensions (Liljedahl, 2016a, 2016c, in press). Flow (Csíkszentmihályi 1996, 1990) is a good framework for thinking about this. Hints and extensions need to be given so as to keep students in a perfect balance between the challenge of the current task and their abilities in working on it. If their ability is too high the risk is they get bored. If the challenge is too great the risk is they become frustrated.

8. Student autonomy

Providing of hints and extensions in a timely fashion is difficult when there are 10-12 groups in the class. If students have autonomy to interact with other groups, however, they will manage much of this on their own as they use each other to provide help when they are stuck and to seek increased challenge when they are done (Liljedahl, in press). Simply providing this autonomy is not enough, however. Students need to be shown that this autonomy exists, and feel its value. As such, the teacher needs to build autonomy by deliberatively push students towards other groups when they are stuck or need an extension.

9. When and how a teacher levels their classroom

Levelling needs be done to the bottom. When every group has passed a minimum threshold the teacher should pull the students together to debrief what they have been doing. At this time the teacher will either go over one or more of the students' solutions or work through a new problem together with the class as a whole. This helps reify and formalize the work the students have been doing and should constitutes the ‘lesson’ for that particular class.

10. Student notes

After the levelling has occurred students need to write some notes for themselves. These notes should be based on the work that is already existing on the boards and can come from their own work, another group's work, or a combination of work from many groups. As part of the levelling process teachers can highlight particular parts of the work that is on the boards, but it is important that the students select themselves, and synthesize and reorganize notes on their own. Students younger than grade 8 will need guidance as to what to write down.

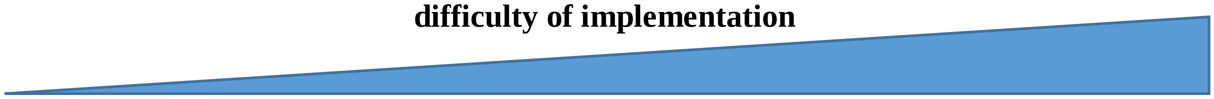
11. Assessment

Assessment in a thinking classroom needs to be mostly about the involvement of students in the learning process through efforts to communicate with them where they are and where they are going in their learning. It needs to honour the activities of a thinking classroom through a focus on the processes of learning more so than the products, and it needs to include both group work and individual work (Liljedahl, 2010).

Taken Together

This research also showed that these are not all equally impactful or purposeful in the building and maintenance of a thinking classroom. Some of these are blunt instruments capable of leveraging significant changes while others are more refined, used for the fine-tuning and maintenance of a thinking classrooms. Some are necessary precursors to others. Some are easier to implement by teachers than others while others are more nuanced, requiring great attention and more practice as a teacher. And some are better received by students than others. From the whole of these results emerged a three tier hierarchy that represent, not only the bluntness and ease of implementation, but also an ideal chronology of implementation (see table 1).

Table 1: Eleven elements as chronologically implemented

Stage One	Stage Two	Stage Three
• begin lessons with tasks • form visibly random groups • use vertical non-permanent surfaces	• use oral instructions • defront the classroom • answer only keep thinking questions • build autonomy	• level to the bottom • use hints and extensions to manage flow • use assessment as communication • use mindful notes
bluntness 		
difficulty of implementation 		

These stages can be envisioned as a set of cycles working in sequence and together to build a thinking classroom (see figure 1).

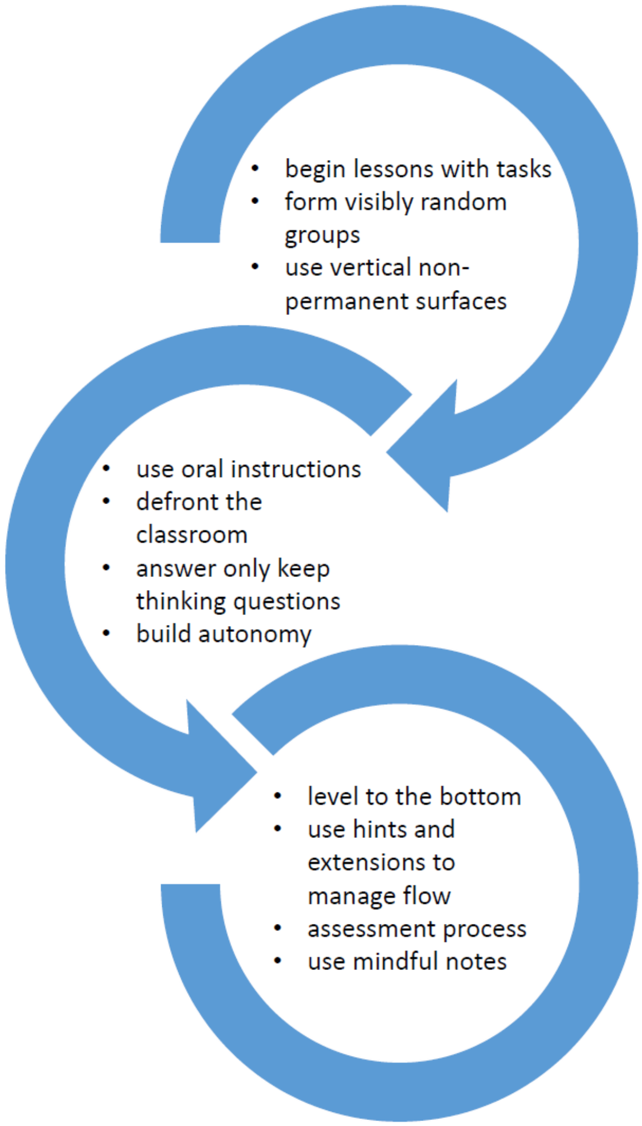


Figure 1: The eleven tools organized into discrete cycles

Since their emergence, these eleven tools and the aforementioned stages, have been used to successfully build thinking classrooms in over 600 mathematics classrooms from kindergarten to grade 12 (Liljedahl, 2016a).

References

Cobb, P., Confrey, J. diSessa, A., Lehrer, R. and Schauble, L. (2003). Design Experiments in Educational Research. *Educational Researcher, 32*(1) 9-13.

Csíkszentmihályi, M. (1996). *Creativity: Flow and the psychology of discovery and invention*. New York, NY: HarperCollins Publishers.

Csíkszentmihályi, M. (1990). *Flow: The Psychology of Optimal Experience*. New York, NY: Harper & Row.

Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher, 32*(1), 5-8, 35-37.

Liljedahl, P. (2016a). Building thinking classrooms: Conditions for problem solving. In P. Felmer, J. Kilpatrick, & E. Pekhonen (eds.), *Posing and Solving Mathematical Problems: Advances and New Perspectives.* (pp. 361-386). New York, NY: Springer.

Liljedahl, P. (2016b). Classroom Practices for Supporting Problem Solving. Paper Presented in TSG #19 (Problem Solving) at ICME 13. Hamburg, Germany.

Liljedahl, P. (2016c). Flow: A Framework for Discussing Teaching. C. Csíkos, A. Rausch, & J. Szitányi (eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education, Vol. 3*. (pp. 203-210). Szeged, Hungary: PME.

Liljedahl, P. (in press). On the edges of flow: Student problem solving behavior. In S. Carreira, N. Amado, & K. Jones (eds.), Broadening the scope of research on mathematical problem solving: A focus on technology, creativity and affect. New York, NY: Springer.

Liljedahl, P. (2014). The affordances of using visibly random groups in a mathematics classroom. In Y. Li, E. Silver, & S. Li (eds.), *Transforming Mathematics Instruction: Multiple Approaches and Practices*. (pp. 127-144). New York, NY: Springer.

Liljedahl, P. (2010). The four purposes of assessment. *Vector*, *2010*(2), 4-12.

Liljedahl, P. (2008). The AHA! Experience: Mathematical Contexts, Pedagogical Implications. Saarbrücken, Germany: VDM Verlag.

Schoenfeld, A. (1985). *Mathematical Problem Solving*. Orlando, FL: Academic Press.

Yackel, E. & Rasmussen, C. (2002). Beliefs and Norms in the Mathematics Classroom. In G. Leder, E. Pehkonen, & G. Törner (eds.) *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 313-330). London: Kluwer Academic Publishing.

For Further Reading

Fenstermacher, G. (1986). Philosophy of research on teaching: Three aspects. In M.C. Whittrock (Ed.), *Handbook of Research on Teaching* (3rd ed.) (pp. 37-49). New York, NY: Macmillan.

In this paper Fenstermacher introduces the idea of studenting as a description of what students do in learning settings.

Fenstermacher, G. (1994, revised 1997). On the distinction between being a student and being a learner. *Paper presented at the annual meeting of the American Educational Research Association*, New Orleans, LA.

In this paper Fenstermacher revises his idea of studenting to include the non-learning behaviours students engage in in learning settings.

Liljedahl, P. & Allan, D. (2013a). Studenting: The case of "now you try one". In Lindmeier, A. M. & Heinze, A. (Eds.). *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, pp. 257-264. Kiel, Germany: PME.

In this paper Darien Allan and I look closely at student behaviour while working on a task. This is a stark and descriptive look at a non-thinking classroom.

Liljedahl, P. & Allan, D. (2013b). Studenting: The Case of Homework. *Proceedings of the 35th Conference for Psychology of Mathematics Education – North American Chapter*. Chicago, USA.

In this paper Darien Allan and I look closely at student behaviour while doing homework. This is also a stark and descriptive look at a non-thinking classroom.

Mason, J. (2002). *Researching Your Own Practice: The Discipline of Noticing*. New York, NY: Routledge.

This is a very useful read for any teacher who wants practical ways to examine their own practice.

Mason, J., Burton, L., & Stacey, K. (1982). *Thinking mathematically*. London, England: Addison-Wesley Publishing Company.

This is a great resource of problem solving tasks as well as a powerful framework for developing student problem solving skills.