

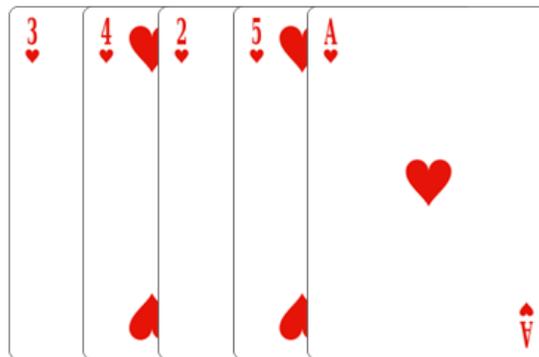
Bootstrapping Thinking: The Role of Engaging Mathematical Tasks

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Mathematical thinking is not too difficult to achieve among a class of students who are already accustomed to thinking mathematically. But it is very difficult to achieve within a whole class setting when this way of thinking and being is foreign to their day-to-day classroom experience. So, how then do we introduce mathematical thinking into such non-accustomed classrooms? One way is through specifically designed or selected tasks that have the ability to spark, not just interest and curiosity, but also mathematical thinking. I call such tasks bootstrapping tasks because they are able to achieve so much thinking where there previously wasn't much. One class of bootstrapping tasks are mathematical card tricks. That is, card tricks that have a mathematical solution or explanation for why they work.

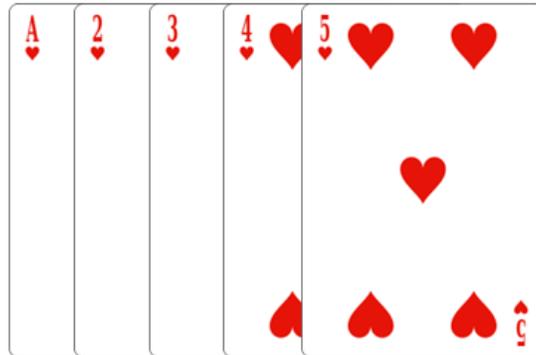
My anecdote regarding such a task comes from some work I did in Hazelton, BC a few years ago. Hazelton has an aboriginal population of more than 50% and, at the time, an unemployment rate of over 90%. This meant that for an average student taking Math 11 in their grade 12 year (very common in this community) their prospects for the future were glum. This dark outlook was reflected in students' lack of engagement and amotivation in their mathematics classes. I visited such a classroom one morning and did a number of mathematical activities with them, one of which was a mathematical card trick. Later that afternoon, after school, I returned to the school to meet with some of the teachers I had visited earlier in the day. The meeting was scheduled to be in the library. When I walked in I saw four grade 12 boys from the class I had visited in the morning working on the card trick. The librarian and the vice-principal told me that they had been in there all afternoon working non-stop on the problem. Their teacher later shared with me that this was the first time she had seen any of these boys putting any effort into something they had learned in class – in class or outside of class.

The trick is a simple one and has more to do with the arrangement of cards than more traditional card tricks. Consider the following special arrangement of cards:



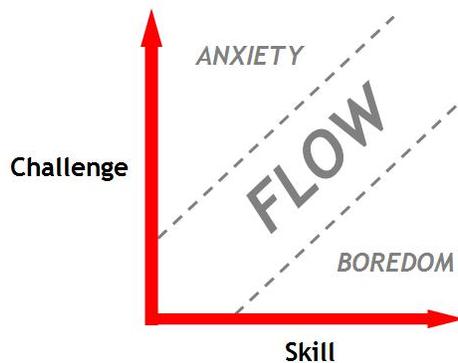
Holding the cards in this order take the top card and place it face up on the table. Take the next card and move it to the bottom of the pile of cards in your hand. Repeat this process, each time placing the top card on the table (to the right of the previous one) and moving the next card to the bottom until all the

cards are arranged on the table. What makes the above arrangement special is that once the above recursive process is complete the cards on the table are arranged in an ascending order.



The task is now to arrange the cards from ace to six so that the same ascending order is produced when following the iterative process of placing one card on the table and placing the next card on the bottom of the pile. Then ace to seven ... and so on ... until the whole deck can be arranged to produce the desired result.

So why is this trick so engaging? A very rudimentary explanation is that it is simple for students to start, it provides early success, but then it gets progressively harder. I have used this trick many times, with students from as young as eight all the way up to adults, and it always has the same effect. The students (or adults) are immediately intrigued and then very quickly become fully absorbed in the task to the point that it is difficult to pull them away from it. Csíkszentmihályi (1990) refers to this phenomenon as the *flow* experience - the experience of being lost in your work where time and distractions fall away. This experience occurs when the challenge of a task is in perfect balance with the skill level of the student.



Flow Channel (Batista, 2010)

As the diagram above shows, this balance is very delicate. If the challenge exceeds the skill then anxiety or frustration ensues. Likewise, if the skill exceeds the challenge the student gets bored. But there is more to this than simply finding a task that is well suited for a student's skill level. This is a dynamic process wherein the skill of the student increases as they engage in a task. And if the challenge that the task offers does not keep pace with this evolving skills the student will soon find him or herself bored by

the task. This is why the above card trick is so engaging. Solving it for six cards is harder than solving it for five, and so on. At each stage the student's skill increases, but so too does the challenge that the task offers. This can continue step by step, as in the anecdote above, for a very long time.

This is not to say that the trick can't be solved in other ways. Some students may eventually decide to solve it by working backwards – by starting with the cards arranged in the right order and then picking them up through a process that reverses the iterative process described above. Other may solve it by thinking of the arrangement as a circle of eligible spaces to put cards into and recognizing that the cards need to be placed, in ascending order, into every second empty space. But, neither of these insightful solutions fit the model of flow as described by Csíkszentmihályi. There is no evolving complexity. There is no prolonged engagement. There is no *flow* experience. There is just the solving of a problem through deductive reasoning (as opposed to the more inductive problem solving process described above). It is interesting that such elegant solutions are not the ideal solution.

References:

Batista, E. (2010). Happiness, Excellence and Boundaries: A Framework for Leaders. Retrieved from: <http://www.edbatista.com/2010/09/happiness.html>.

Csikszentmihalyi, M. (1990). Flow: The Psychology of Optimal Experience. New York: Harper and Row.