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Embodied experience of velocity and acceleration: a narrative

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Abstract

Constructing a link between what a student is learning and personal experience is an important, and sometimes difficult task. I present here a narrative of my own experience as a mathematics and physics teacher trying to create an embodied sense of motion in my students by actually putting them in motion. I use the story to present the difficulty of teaching motion in the absence of the embodiment of motion as well as the tension that is created between an embodied sense of motion and the static representations used to describe it. © 2002 Elsevier Science Inc. All rights reserved.

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1. Introduction

There is an abundance of literature on the power of building connections between content and context. Students learn better when they are able to link what is being taught to personal experience and in the absence of relevant personal experience, the lesson needs to provide not only the content but also the context. Years before I was formally introduced to these concepts through the literature, however, I was a beginning high school mathematics and physics teacher discovering them on my own. This article is a narrative of one such experience.¹

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¹ Since the events occurred prior to my introduction to the world of education research, I have not done any analysis on the events other than to reflect on how they demonstrate the power of an embodied experience on learning. The data and discussion here are related, however, to two papers by [Speiser and Walter \(1994, 1996\)](#), which show how linear representations of motion and embodied experience also play important roles. See also [Sherin \(2000\)](#).

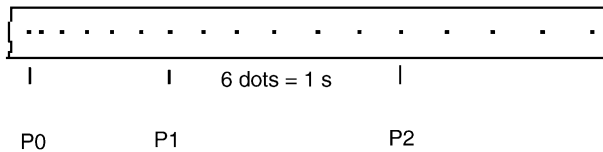


Fig. 1. Sample ticker tape.

For my part the story began in my second year teaching a physics course for grade 11 students in a high school in British Columbia, Canada. The first portion of the curriculum consisted entirely of the topic of kinematics (the study of motion). Although physics is taught as a portion of the more general science course taken by all grade 8, 9, and 10 students this is the first time they have encountered this particular topic. I knew from experience that the early parts of kinematics often presented the students with difficulty; largely due to a specific piece of experimental equipment we used to explore the subject matter.

At the school where I taught we used a piece of apparatus called a ticker tape machine. The operating principle is based on the use of an oscillating needle to make marks on ticker tape that is being pulled through the machine. Its usefulness hinges on the fact that these oscillations are set at a very consistent speed of 360 cycles per minute, thereby making exactly six dots per second. Thus, ticker tape pulled through the machine will be marked with a series of dots which, when read correctly, will produce data for both the time and the distance traveled by whatever mechanism that was used to pull the tape (see Fig. 1) — a very useful fact when it comes to calculating velocity. I suspect, however, that the persistent use of this apparatus in our school over the years was more due to financial reasons than for the simple elegance of the data it produced. The alternative, at the time, was to purchase a class set of graphing calculators with attachments for motion sensor equipment — not an inexpensive alternative.

Regardless of the simplicity of this instrument the application of it by my students had in the past proven to be quite cumbersome. The multivariable representational ability of the ticker tape seemed to confuse the students. My sense was that they were having great difficulty correlating their idea of time as something measured with a watch during the experiment and the ability of ticker tape to present it as a number of dots on paper to be counted and analyzed after the experiment. So, with trepidation I gave careful instruction and ample time for them to familiarize themselves with the equipment through a series of ‘warm-up’ activities. Although they were quite proficient at using the equipment in the introductory activities, and extracting time measurements procedurally during these early activities the robustness of their primary belief as to how time should be measured were once again exemplified in their efforts to produce distance vs. time graphs. Any inroads I made into their understanding of the graphs were marginal and strictly procedural.

Then, 2 weeks into the course, I had an eye-opening experience. A student, delayed in returning from summer vacation, joined the class. In trying to familiarize the new addition to the class with what he had missed over the last 2 weeks, I began to see the events unfolding in my classroom through the eyes of a student. There was ticker tape everywhere. Gaining an

understanding of velocity and acceleration had been reduced to the decoding of static dots on reams of paper. Yes, there had been motion at one time, but the wind up car and the rolling cart that produced the data had now been stationary for several classes. It was at that moment that I realized that the embodied ideas of motion that I wanted to achieve within my students was being handicapped by the oversimplified elegance of the ticker tape machine.

2. My solution

Rather than go onto the even more complicated task of reading velocity off the ticker tape (see Fig. 2), I scrapped the whole thing and started over. This time I was determined to create the embodied sense of motion through actual motion. When my students came to class the next day, we headed for the parking lot with a stopwatch, an extra long tape measure (brought from home), a box full of beanbags (borrowed from the physical education department), and my car keys. The gist of the task at hand was quite simple, and somewhat familiar to many of my students. While one of the students steered the car across the length of the parking lot, under the impetus of three of my stronger pupils pushing with all they had, a fifth student was dropping beanbags out the window at regular intervals measured off with the stopwatch. Quite simply, I had constructed a rather large, somewhat clumsy, but very effective ticker tape machine.

I'm not sure where my inspiration for this came from, but I'm relatively certain that I would not have acted on it had I not owned a rather beat-up old Honda left over from my days as a university student. I know that I had wanted my students to physically experience the sense of motion and changing motion, and this seemed like a good way for this to be achieved. The idea to drop beanbags was no doubt influenced by my exposure to the ticker tape machine as I valued the principle of how it operated as a useful way to mark out motion. I was not sure how this would translate into time for my students, or even if they would gain any greater insight into the measurement and representation of motion, but I felt that I had nothing to lose. What transpired still fills me with wonder, and every time I reflect upon the few days we spent working with the data produced by our foray into the parking lot, I am amazed by the depth of understanding that was achieved through the simple act of embodying the experience of motion.

Initially the plan had been for the dropper of the beanbags to make a drop every 5 s, but the students concluded rather quickly that this was not providing enough data points as the whole

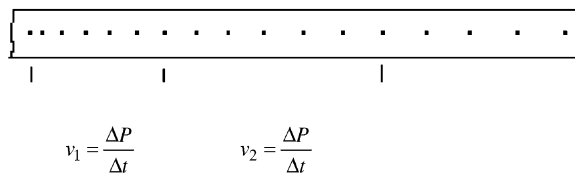


Fig. 2. Reading velocity of ticker tape.

ordeal took somewhere in the neighborhood of 20–30 s, depending on the motivation of the pushers and the choice of the passengers. The solution offered by my pupils was to shorten the intervals to 2 s and to put another student in the car to keep track of the time as to allow the dropper to focus on the task at hand (so to speak). Upon reflection, I'm not entirely certain that they were looking at the beanbags as representing any specific time interval as much as they represented just regular intervals and there simply weren't enough of them. The addition of a passenger also resulted in the addition of another pusher, which made for a somewhat more crowded bumper and the awkwardness that accompanied it. However, the embodied sense of motion was now being experienced by seven members of my class, which by my accounts was approximately seven more than had been realized in the confines of the classroom.

The time allotted for classes at my school was 1 h and 20 min. This was fortunate, for we spent the first 20 min discussing how it was going to be done, deciding on significant personnel, checking driving credentials, doing test runs, fine tuning the apparatus, changing personnel, rechecking driving credentials, increasing the frequency of the drops, etc. We were then ready to make our first real run and collect our first set of data.

This would probably be a good time to mention an unforeseen consequence of this particular embodied experience. The bodies were males. To be more specific, the students that were pushing the car were 16- and 17-year-old males. As this activity was remotely related to speed, an atmosphere of performance began to consume the group. Had there been enough students to produce a comparable second team I have no doubt that this desire for performance would have turned into competition. As it was, however, the 'pushers' settled on a goal of covering the length of the parking lot in the shortest time possible. Not wanting to spoil the spontaneity of the moment, I was willing to accommodate the bravado that was developing with the stipulation that all the data would be collected.

As the whole affair was as new to me as it was to my students, the desire by my students to turn the experiment into a sort of race was unforeseen. Likewise, my decision to allow it to continue spawned even more unforeseen richness. The immediate consequence was that the sense of competition somehow became a unifying class goal and as a result all sorts of new worker bee positions popped up. We ended up with more pushers designated strictly to the task of returning the car to its initial position now clearly marked with a blue beanbag (I wouldn't allow them to turn the car around in the confines of the staff parking lot). In addition, the measuring, recording and collecting of the beanbags became a well-orchestrated task consuming at least half a dozen people. Nearly all the students were now somehow physically involved in the task.

For the remainder of the period my car traveled back and forth across the parking lot generating in its wake a line of beanbags. However, the watch that one of my students was using to time each pass of the car still held the focus of the group. I mentioned earlier that I am relatively certain that my students were viewing the beanbags only as a measure of distance traveled between intervals, almost skirting around its inherent association with time. The reason I believe this to be the case is that during this quest for speed, my students were beginning to make estimates as to the relevant time of one particular pass to another. They were beginning to see the relationship between the time it took to push the car from one end of

the parking lot to the other and the number of beanbags that were being ejected from the car. This progressed to the point where the students doing the recording were making predictions as to the total time of the pass based on the total number of beanbags and the frequency with which they were dropped. This seemed quite natural to me and as a result I did not pay or call any significant attention to them. It wasn't until we were in the classroom the next period that I became aware of the significance of some students attempts at prediction.

Two days later, armed with reams of data, we began our analysis. It quickly became apparent to me that the use of the beanbags to mark out time was not as obvious to all my students as it was to me. Those students who had been making the predictions based on the number of beanbags had no difficulty in creating tables of position vs. time. Those students more directly involved in the competitive nature of the activity (the pushers) who had been relying on the clock to measure time were less willing to use the dual properties of the beanbags to measure both position and time. Unlike my earlier attempts to use ticker tape the hesitation was not difficult to overcome, but intervention was still necessary to make them see the connection.

There was another unforeseen consequence of this experiment. It came with the analysis of the data. There was a tension between the students embodied experience and the graphical representation of the motion of the car. The natural course of analysis, and the one adopted by every physics textbook I have seen, was to first create a position vs. time graph. In the case of our car, this produced a graph (see Fig. 3) that appeared to be incongruent with the experience of pushing the car. My initial insistence on producing this graph first was squashing any sense of embodied motion that the experience with the car had produced.

When I realized what was happening I asked them to tell me how they would describe their experience. The response was unanimous, even from those students who were never in contact with the car: "The car starts off slowly, then it begins to pick up speed until it can't go any faster, and then it stays at that speed until we get too tired to keep going." The above paraphrase was accompanied with hand gestures that basically traced out the velocity–time graph (see Fig. 4). Shifting the focus of the data analysis to this later graph immediately resolved the tension between the students' shared experience and the mathematical representation of the situation.

The data we had gathered provided a base for all of our discussion in this unit. However, not all these discussions flowed directly from the link between the embodied experience and the velocity vs. time graph. Although a clear understanding of the position vs. time graph developed naturally from the sharp memory of what had transpired on that one day in the

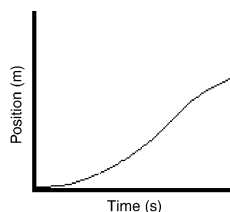


Fig. 3. Position–time graph of the motion of the car.

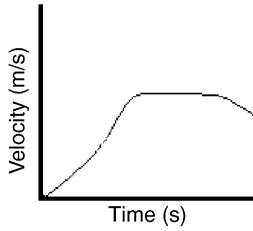


Fig. 4. Velocity–time graph of the motion of the car.

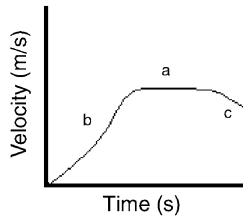


Fig. 5. Velocity–time graph.

parking lot, it was only able to strengthen discussions on constant (and maximum) velocity (see a in Fig. 5), acceleration (see b in Fig. 5), and deceleration (see c in Fig. 5). The embodied experience of these topics could be seen in the obvious representations of the slopes in the graphical representation of the events.

The position vs. time graph, although easily generated from the data, required more exploration and discussion before a clear link to the motion of the car was established. Once solidified, however, the idea of average velocity evolved naturally from the embodied experience and could be seen as $\Delta position / \Delta time$ which corresponds to the slope of the secant line of the position vs. time graph (see Fig. 6). The most difficult link to establish, however, was the relationship between the two graphs. This link was necessary in order to see the connection between the slope of the position vs. time graph and the instantaneous velocity as indicated on the velocity vs. time graph (see Fig. 7).

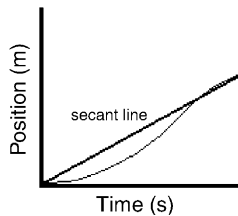


Fig. 6. The slope of the secant line of the position–time graph.

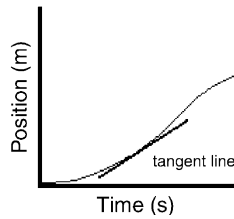


Fig. 7. The slope of the tangent line of the position–time graph.

When I look back on what happened in my class for that one unit, with my years of teaching experience and my immersion into the field of educational research, I'm not surprised by the events. The need for building students' connection to the material being learned through experience is well documented. However, the surprise I experienced over the students natural representation of the motion of the car (in terms of speed) and the tension that it produced with the formal representation I was teaching still pleases me. Here was a clear case of the importance of attending to students internal and natural representations rather than those dictated by the content.

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