Noticing rapid and profound mathematics teacher change

Peter Liljedahl

Abstract Change in mathematics teachers' practice is often characterized as something that takes time and sustained intervention. In this article, I present the results of research that highlights a different kind of change—a profound change that takes place very quickly. Based on the analysis of 42 cases of such rapid and profound change, I also present a disaggregation of this phenomenon into five distinct mechanisms of change, each one rapid and profound. This disaggregation shows that not all changes, even when outwardly similar, are the same.

Keywords Professional growth · Rapid · Teacher change · In-service education

Introduction

Teacher education literature is full of examples of teachers' changing their practice. Usually, these examples are found in research examining specific professional development models such as action research (Jasper and Taube 2004), lesson study (Stigler and Hiebert 1999), communities of practice (McClain and Cobb 2004; Wenger 1998), or more generally, collegial discourse about teaching (Lord 1994). Such research has very effectively delineated different mechanisms by which teachers change while participating in a variety of professional development setting. Conclusions show that with time and continued intervention, support, and collaboration, teachers can make significant and robust changes to their practice.

As a mathematics inservice teacher educator working in a variety of professional development settings, I have witnessed teacher change of the form exemplified in the aforementioned research. But I have also witnessed change of a different kind—rapid and profound change in practice—examples of which are not to be found in the literature. In what follows, I first introduce the unobtrusive research methodology that allowed me to identify examples of rapid and profound changes in practice. I then present the results of a

P. Liljedahl (✉)
Faculty of Education, Simon Fraser University, 8888 University Drive, Burnaby, BC V5A 1S6, Canada
e-mail: liljedahl@sfu.ca

deeper analysis of this phenomenon, which led to its stratification into five distinct mechanisms of change.

Methodology

Working as both a mathematics inservice educator and a researcher interested in the contextual and situational dynamics of the inservice setting, I find myself too embroiled in the professional development activities to adopt the removed stance of observer. At the same time, my specific role as facilitator prevents me from adopting a stance of participant observer. As such, I have chosen to adopt a stance of *noticing* (Mason 2002). This stance allows me to work within the inservice setting to achieve the professional development goals called for in the different settings while simultaneously being attuned to the experiences of the teachers involved. At the same time, this stance allows me to engage in these experiences as a researcher without the requirement of an a priori research question. In so doing, from time to time, I notice phenomena that strike me as interesting.[1] Occasionally, these are phenomena that occur in more than one setting and speak to invariance between individuals, settings, contexts, or behaviours.

Using this methodology of noticing within a diverse number of mathematics professional development settings—from workshops to learning teams to graduate programmes—I have, from time to time, noticed teachers undergoing *rapid and profound changes*[2] in their beliefs and practices. This phenomenon is rare. Most teachers engaged in inservice work follow a trajectory of change that is much more pedestrian. At first, such changes surprised me, and I was immediately suspicious of the self-reported accounts of the almost instantaneous revision of practice that some teachers spoke about. But, as more and more of these accounts accumulated, I decided to investigate further. I stepped out of my role as facilitator and assumed a stance of researcher. I began to interview teachers, to visit their classrooms, and to observe their teaching. I spoke with their colleagues, their administrators, and their students. I took field notes, and I wrote narratives (Clandinin 1992). In the end, I compiled data on 42 cases of mathematics teachers' rapid and profound changes in practice, which were analysed using a grounded theory (Charmaz 2006) approach in general and a constant comparative method in particular.

Results

From this analysis, the phenomenon of rapid and profound change fragmented and converged into five different types that I have come to call: (1) *conceptual change;* (2) *accommodating outliers;* (3) *reification;* (4) *leading belief change;* and (5) *push–pull rhythm of change.* Each of these types embodies a different mechanism for transformation

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[1] Of course, what I notice is first and foremost predicated on what I find interesting. As a person who works in professional development settings I find all things associated with teacher change interesting.

[2] In the settings in which I work my contact with teachers is discrete; constituted of a series of meetings at regular intervals. Teacher change in this setting is overwhelmingly observed to be incremental, gradual, and tentative, stretching out over several meetings and involving encouragement, planning, experimentation, and refinement. My use of the term *rapid and profound* is meant to describe changes that stand in stark contrast to this more usual form of change. So, for example, a teacher who is observed to change from no (or little) use of group work to ubiquitous use of group work wholly between two consecutive meetings without apparent trepidation is seen to have made a change that is rapid and profound.

of teaching practice that needs a unique theoretical framework in order to bring it into sharper focus. I present each of these mechanisms of change first with an abbreviated case, chosen for its ability to succinctly exemplify the category of change, and then with a brief analysis of the individual case as well as the category as a whole.

Conceptual change

Mary is an elementary school teacher with more than 20 years of experience. Although she has taught mostly every grade in the elementary curriculum, Mary has spent most of her years teaching the intermediate grades (4, 5, 6, 7). In recent years, she has become disillusioned with the teaching of mathematics. In particular, she is questioning her ability to have a meaningful impact on students within the structures that she must work. Mary is unhappy with the current mathematics curriculum as well as the dispositions of her students with respect to mathematics and the learning of mathematics. She sees the curriculum as being a collection of disjoint topics each of which needs to be mastered in turn. There is little connection across these topics and there seems to be very little thinking required of the students. She talks about the teaching of mathematics in very traditional ways—from the standard review-demonstrate-mimic-practice lesson plan to the use of homework, quizzes, and tests to leverage students' commitment to the acquisition of the taught skills. Ironically, she doesn't see herself as this type of teacher in the other subjects that she teaches.

It was at this point in her career that Mary decided to enrol in an Elementary Mathematics Education Master's program. Mary hoped that her participation in such a program would allow her to find what it was she felt was missing in her practice. She did.

As the instructor for the second course in the program, I first met Mary in November when she was still very much disillusioned with the state of affairs in mathematics education in general, and in her teaching in particular. At the beginning of January, when she returned to the classroom after the Christmas break she was a completely different mathematics teacher. Her classroom was transformed, now a place of inquiry and discovery, of meaning making and thinking. Rows of desks were replaced by tables with students working in groups of four. There was no longer a well defined 'front' of the classroom as Mary now utilized the entire classroom for teaching and students were valued as partners in learning and teaching.

Mary reported that she had participated in many professional development work-shops and district based initiatives, from constructing assessments to piloting new textbooks and resources. But each of these failed to inform her practice in any significant way. In fact, many of them simply reinforced the very paradigm she was trying to escape. Based on interviews with Mary, I determined that what eventually initiated Mary's rapid transformation was the recasting of her view of mathematics curriculum from content to context. This resulted from an in-class exercise we did in which we looked at the mathematics curriculum documents from the perspective of mathematical processes. This view places the processes (such as estimation, problem solving and communication) as the primary goal of mathematics education and relegates prescribed learning outcomes (such as addition of fractions) to the context in which these processes are actualized. This view was what Mary was missing. In

the mathematical processes she found the meaning that she felt was absent from her practice.

The transformation that Mary underwent—belief rejection followed by belief replacement—can be seen as a special form of conceptual change, which has been used to hypothesize about the teaching and learning of science. More recently, this theory of conceptual change has been applied to the learning of mathematics (Greer 2004; Tirosh and Tsamir 2004; Vosniadou 2006; Vosniadou and Verschaffel 2004).

In general, the theory of conceptual change starts with an assumption that, in some cases, people form misconceptions about phenomena based on lived experiences, that these misconceptions stand in stark contrast to the accepted theories that explain these phenomena, and that these misconceptions are robust. It is not a theory that applies to learning in general. It is highly situated, requiring four primary criteria for relevance (Vosniadou 2006) and is applicable only in those instances where (1) misconceptions are formed through lived experiences and in the absence of formal instruction; (2) there is a phenomenon of concept rejection; (3) there is a phenomenon of concept replacement; and (4) there is the possibility of the formation of synthetic models. I propose that each of these criteria is equally relevant to the instance of Mary's rapid reformation of her beliefs about mathematics and the teaching and learning of mathematics as well as to the rapid reformation of her teaching practice.

To begin with, Mary's relevant lived experiences occurred outside of a context of formal instruction (criterion 1). Through the lived experience as a learner of mathematics, she has experienced both the learning and the teaching of mathematics, and these experiences have impacted on her beliefs about the teaching and learning of mathematics (Ball 1988; Chapman 2002; Fosnot 1989; Lortie 1975; Skott 2001). Secondly, Mary had clearly rejected the paradigm in which she was working, and although this rejection did not manifest itself in her actual practice, it did exist within her belief structures around mathematics and the teaching and learning of mathematics (criterion 2). Mary then searched for something more meaningful around which she could construct her teaching practice. When she did eventually encounter the aforementioned view of 'mathematical processes as curriculum', it instantly displaced the already rejected paradigm, which she was still reluctantly using (criterion 3). Finally, a 'synthetic model' is a term reserved for the description of an incomplete or incorrect model. It is a middle ground between the initially rejected concept and the concept that is to be acquired. In many cases, it is a *synthesis* of the old and the new as the learner is making use of old resources to make sense of new ideas. In relation to Mary, a synthetic model manifested itself in the fact that although she made rapid and profound changes to her teaching practice, she still continued to make changes and improvements as time went on (criterion 4). This is not unusual. Change begets change and, as Mary settles into her new teaching practice, she saw more and more details that required attention. This is no different from the context of a student's conceptual change around a mathematical or scientific concept. The synthetic models they develop are most often temporary and tentative, often giving way to more and more refined models as conceptual understanding is achieved.

Although Mary's case exemplifies profound change through a process of conceptual change, she is only one of eight such cases that I have encountered. In general, there are many teachers who engage in mathematics inservice education opportunities because they are disillusioned with the paradigms that they are working in and they are looking for something better. Sometimes those teachers find something better, sometimes they do not.

When they do find something better, they may be able to visibly transform their practice quickly, as Mary did.

Accommodating outliers

Mitchell is a middle-school teacher with eight years of teaching experience. He has always taught mathematics and he has a very clear sense of what is important for students to learn in mathematics and what his role as a teacher is in this context. For Mitchell, mathematics is a collection of skills and facts that need to be mastered before going onto the next level.

Mitchell's mathematics classroom is a pillar of traditional teaching and he has no issues about stating so. He adheres to a standard lesson of review-demonstrate-mimic-practice and students are expected to seek his help if they are stuck or do not understand. He only uses questions that are unambiguous and lead to closed-form single solutions. He feels that mathematics needs to be taught (and learned) in this fashion and that any of the problems that face mathematics education are due to deviation from this tradition.

Ironically, Mitchell does not shun professional development activities. He attends them regularly for the purposes of getting some "good ideas", although disappointedly (and expectedly) these workshops often turn out to be "pointless". Even though I had met Mitchell as a participant in a number of single-session workshops, I did not begin to interact with him until he became a member of a district-based learning team that I was facilitating. This team was formed for the purpose of creating numeracy tasks for district-wide assessment. Mitchell came to this learning team with the expressed purpose of offering some of his expertise in creating "really comprehensive final exams".

The first task of this team was to co-construct a definition of numeracy. Initial attempts to do this resulted in definitions that were more closely associated with fluency of arithmetic. In order to get past this initial definition, I suggested that they think of students that they had taught in the past who were very good in mathematics, and to further think what qualities they possessed that allowed them to be successful. This dramatically changed the discourse about numeracy and rather quickly a more sophisticated definition emerged—"Numeracy is not only an awareness that mathematical knowledge and understandings can be used to interpret, communicate, analyze, and solve a variety of novel problem solving situations, but also a willingness and ability to do so."

The team then set out to design a task that would measure some of the capacities embodied within this definition. Over the course of four additional meetings stretched out over approximately eight weeks, the team went through three iterations of a design-test-refine process before they arrived at the final task. During this process, I saw Mitchell undergo a tremendous transition in his teaching. After pilot testing the initial version of the task, he was talking about things that needed to change in his classroom in order for his students to be successful: restructuring the way he thought about and facilitated group work; redefining his own notion of what constituted a good mathematics question and a good mathematics answer; and he was trying to find ways to change the dispositions of the students in his classroom. In a very short

period of time, Mitchell came to change most of what he held to be true about mathematics and mathematics teaching and learning.

Unlike Mary, Mitchell did not come to the learning team looking for answers. He had not rejected the teaching paradigm that he was working under, his teaching practice, or his world-view of teaching mathematics. Quite opposite to Mary, Mitchell's teaching can be seen as impenetrable. He participated in a wide variety of professional development opportunities, but nothing had any effect on his practice. Invoking the discourse of adaptation à la Piaget, we might say that Mitchell was not accommodating new ideas into his existing schema of teaching mathematics (Piaget 1968).

Mitchell tended to deal with new ideas about the teaching of mathematics in one of three ways. First, and most common, he would find within his professional development experience something specific that resonates with his current teaching. This point of commonality was then used to support his basic assumption that "I already do that". If Mitchell did not find any points of commonality, but the new ideas presented to him caught his interest, he tended to incorporate them into his practice. But he would do so without letting these ideas impact on his conception of himself as a mathematics teacher or on his general notions about mathematics and the teaching and learning of mathematics. As an example, Mitchell very easily introduced into his teaching a programme of problem-solving that he had acquired at a workshop. He had a small, but good, collection of problem-solving tasks that he gave to his students from time to time. There was no attempt to assess student performance on these tasks or to pull some of the affordances such tasks could offer into the rest of his teaching. Mitchell was not only assimilating these problem-solving experiences—he was actively not accommodating them. Finally, if Mitchell found no points of commonality and no points of interest, he would simply dismiss the new ideas presented to him as "pointless".

Ironically, it was accommodation that eventually led to the revision of Mitchell's practice. From interviews, it became clear that a real turning point for Mitchell occurred during the construction of the definition of numeracy. In particular, it was the consideration of the good students he had had in the past that led to deeper changes. Mitchell had always been aware of these students, but he had effectively not allowed their existence to impact on his aggregated vision of a mathematics student. To him, they were outliers, as were the capacities that they possessed. For Mitchell, a student was seen from a deficit perspective. They were children that lacked specific knowledge—knowledge that he possessed and would apportion out to them over the course of the school year. When he started to think of these outliers, he not only saw them as capable, but he also saw a whole spectrum of skills that he now felt all students needed in order to be successful in mathematics. At the same time, Mitchell revealed that he was beginning to make sense of why the problem-solving tasks that he had previously been using as extracurricular were so effective at developing some of these skills. Critical thinking, divergent thinking, group work, ability to articulate thinking, persistence, tolerance of ambiguity, and comfort with being stuck were now the deficiencies that he wanted to address.

In the consideration of both talented students and good problem-solving tasks, Mitchell was finally accommodating outlying information into his practice. But, this was not new information. Rather, it was information that he had kept compartmentalized away from his normal constructs of what constituted a mathematics student and a mathematical activity, respectively. In the end, the reform of his teaching happened when he began to accommodate these outliers. In my data, there are five other such cases, each one demonstrating change after they began to accommodate existing, but outlying, information.

Reification

> Danica has been teaching middle school for 13 years during which time she has always enjoyed teaching mathematics. Her teaching in general, and mathematics teaching in particular, is dynamic and progressive. She is always willing to try something new and is a keen consumer of professional development opportunities.

> Danica is unhappy with her mathematics teaching, however. She has a sense that what she is doing is correct, but her teaching "lacks cohesion". She feels that she is being pulled in so many directions by so many "new ideas" that she doesn't know who she is as a teacher any more. She is experimenting with formative assessment, pilot testing a new textbook, implementing problem solving in her classroom, and engaging students in group work. Outwardly, her teaching is a disjoint collection of innovations that she has acquired over the years. There is little sense of purpose behind her various strategies and the classroom seems chaotic at times. Danica is aware of this as she states that she lacks a holistic understanding of how it all fits together. She feels that everything she tries is sitting alongside every other thing she has tried and none of them are working in harmony with each other.

> I first began working with Danica in the same learning team as Mitchell. And like Mitchell, this experience had a transformative effect on her. Almost immediately she gained confidence as she found cohesion in her teaching. This confidence continued to grow over the course of the rest of the meetings. Danica had found a thread that bound all of her previously disjoint ideas about teaching mathematics together—a thread of communication. Danica started to build all her teaching around the idea that students needed to be able to articulate their thinking. This gave meaning to her efforts at group work, formative assessment, and problem solving. At the end of that school year, Danica came to a surprising conclusion for herself and her students— students who were better able to articulate their thinking actually thought better.

In interviewing Danica, it became clear that for her, like Mitchell, the beginning of this transformation was the co-construction of the definition of numeracy. Unlike Mitchell, however, Danica was searching for a way to bring together her discrete experiences and the construction of the definition facilitated this. For Danica, this was a reifying experience (Wenger 1998).

For the most part, teaching has no concrete form. It exists in time and space in the relationships with students and the interactions between curriculum and learners. In essence, teaching is an experience. Occasionally, this experience can be reified into some artefact that, at least for the creator, embodies the experience of teaching. For Wenger (1998), reification is "the process of giving form to our experiences by producing objects that congeal this experience into *thingness*" (p. 58).

For Danica, the definition reified teaching—ideal teaching. But, it was not her teaching—at least not yet. The process of co-constructing the definition was an act of synthesis, drawing on the teaching experiences of eight different teachers. Danica contributed to this process and, as a result, saw herself in part of the definition. However, the definition became so much more for her. This definition was the reification of the types of students she wanted to produce, and because she had contributed to its formation, she felt she had access to all that it embodied.

That is not to say that all of Danica's transformation could be attributed to the reciprocating relationship she had with the definition, but it was the start. The design and pilot

testing of the numeracy task had a similar and, I would say, more profound effect on her practice. Again, the initial task that was created was a reification of aspects of all of the teachers' aspirations. For Danica, it carried within it the capacities that she wanted to develop within her students, capacities that she knew were important, and that she knew were absent within her students. The pilot testing of the task on her own students only served to solidify her resolve as with each iteration Danica became aware of more mathematical and pedagogical affordances embedded within the task (Liljedahl et al. 2007), each of which could now be used to achieve her goals.

Danica now possessed a definition and a task, each of which embodied the essence of the teacher she wanted to be. In these, she found the thread of communication (or more specifically, the ability to articulate thinking) that bound her disjoint efforts together. Her assessment practices now had a purpose, as did her focus on group work and problem-solving.

Danica is not unique in this regard. Of the many teachers who I have worked with in the capacity of task design, 11 others have experienced a similar transformation. The co-construction of definitions and tasks reifies teaching in ways that simple lesson plans, tests, and worksheets do not. They are both connected to individual teachers and idealized amalgamations of the best of teaching. However, other experiences (not yet developed into cases) tell me that it is not unique to the goal of task design. I am seeing similar effects in a group of teachers designing performance assessments, and I have observed the same effects in teachers engaged in lesson study. The commonality in all these contexts is the co-construction of some artefact of teaching—reifying and amalgamating the best of each of their experiences and aspirations.

Leading belief change

Phil is a relatively new teacher, having taught for only 5 years. Trained as an elementary school teacher, he has been teaching grades 6 and 7 in a middle school his whole career. In many ways, Phil is a progressive teacher who makes extensive use of group work, projects, and activity based learning. When it comes to evaluation, however, Phil is quite traditional in his methods. He relies heavily on tests, quizzes, and homework to comprise the marks that he assigns his students. His only deviation from this is his frequent use of projects (usually posters) that he requires his students to make. However, these are done individually and marked using a rubric that focuses primarily on surface features such as quality of diagrams, titles, and references. He does have one portion of the project mark that measures depth of content, but this is given holistically without explication and is only worth 20% of the total mark.

I met Phil as a participant in a series of three workshops I was giving in his school district on teaching with problem solving. These sessions were staggered throughout the school year with the intention that teachers would leave each workshop with a set of activities to try in their classroom. The second of these workshops dealt exclusively with the aspect of assessment and evaluation. In this session, I challenged the participants with the idea that we, as teachers, should evaluate what we value. This is a statement that was developed reactively to the observation that students use evaluation methods to gauge what the teacher values and to appropriately adapt their learning behaviour. Proactively, this statement says much about how to communicate effectively to students what we, as teachers, value in the process of learning.

Phil was deeply affected by this statement. This was something that he immediately felt was true and he knew that he had not been practicing in his teaching. When Phil

came back to the third workshop 5 weeks later, he had chewed through each of the alternative evaluation strategies I had given for having students represent their problem solving thinking in diverse ways and was grappling with how to evaluate group work—something he valued deeply in his practice. Phil spent the remainder of the school year grappling with big ideas around evaluation. He struggled with the tension between homework as a safe place to practice mathematics and the use of homework marks as an extrinsic motivator to get students to practice mathematics, the timing of when a student must demonstrate acquisition of a prescribed learning outcome (in a lesson, on homework, or on a test), time limitations on tests and in class problem solving assignments, to name a few. Although he did not necessarily resolve each of these conflicts, he did make marked changes in his evaluation practices and came to his teaching the following year with a diversified and comprehensive evaluation plan.

Teachers' practice is not distinct from their beliefs about teaching and learning (Leatham 2006; Liljedahl 2008). At the same time, teachers' beliefs about teaching and learning do not exist in isolation from the experience of teaching (Leikin 2006; Thompson 1992) or their experience as learners (Ball 1988; Chapman 2002; Fosnot 1989; Lortie 1975; Skott 2001). Finally, and most relevant, teachers' beliefs do not exist in isolation from each other. Instead, they cluster together along lines of relevance to form robust systems of beliefs (Chapman 2002; Green 1971). For example, a teacher may believe that mathematics is nothing more than a collection of discrete facts and skills to be mastered. At the same time, they may believe that learning mathematics requires memorization of facts and practice of skills. These are two distinct beliefs, one likely stemming from the other (although which is the precursor is not always clear), that cluster together to form a belief system around what it means to learn mathematics and what the object of that learning is. This is not to say that all relevant beliefs cluster together. Beliefs can form discrete systems that may or may not be consistent with each other (Green 1971), as can be seen in the case of Phil.

Phil had, at least, two distinct belief clusters guiding his practice. The first cluster, comprised of a set of beliefs of what constituted good teaching, had been formed during his teacher education programme. It was here that he had learned about the importance of group work as well as activity-based learning. The second, and quite distinct, belief cluster guided his assessment practices. These beliefs came from his own experiences as a learner (both K-12 and post-secondary) and were comprised of a set of beliefs that evaluation needed to, above else, measure the performances of individual students in a way that was equal, controlled, and accountable. The prompt to think about evaluation as an extension of what he valued in teaching and learning caused Phil to have to reconcile these distinct belief clusters. In essence, he abandoned his former set of beliefs around evaluation and allowed a new set of beliefs to grow out of his existing beliefs about teaching.

Phil is one of six teachers whose cases exhibit such clear evidence of changes in their teaching practice coming on the heels of a shift in one clearly identifiable belief (a *leading belief change*). That is, there is a change in the configuration and composition of their system of beliefs as a result of a significant change to one specific belief.

Push–pull rhythm of change

Karen is a high-school teacher with 16 years of experience. She loves teaching and prides herself on the level of care she brings to her practice—especially the care she shows for her students and their success in the exam at the end of the year.

Karen's practice can best be described as meticulous. She is very careful to make sure that she covers every prescribed learning outcome in a timely manner so that there is plenty of time left over at the end of the year for review. This is not to say that she rushes through the curriculum—she doesn't. Her pace is deliberate and methodical with not a minute wasted. Karen builds her instruction around homework, which is carefully selected from the textbook as well as a databank of old exams. Each lesson begins with a review of any problematic questions from the previous homework. She then moves quickly into a lesson carefully designed to prepare the students for the forthcoming homework assignment. This usually involves taking the class through a carefully designed sequence of examples each one incrementally different from its predecessor.

Karen makes herself available to her students before school, after school, and at lunch and actively encourages students to come and work on their homework under her supervision and tutelage during these times. Karen's greatest pride is the level of success some 'challenged' students are able to achieve through diligent use of this service.

I first encountered Karen in a numeracy task-design team that I was facilitating. This team was similar in focus to the one Danica and Mitchell partook in although slightly different in terms of end-users. In this particular learning team, we were working on designing tasks for the team's use rather than for the whole district.

For the first two sessions of the learning team, Karen worked very happily on the activity of creating a numeracy definition and a numeracy task. Then, suddenly in the third session, she revolted against the entire enterprise espousing beliefs that the educational system as a whole did not allow for students to succeed in such activities. Surprisingly, she came to the fourth session quite excited about the shortcomings of her students in their abilities to complete the task and committed to making changes in her teaching to ensure that they would be successful. She talked about implementing group work, developing thinking and communication skills, as well as the need to engage the students in more open-ended activities as a way to nurture these capacities. Karen continued talking about changes to her teaching over the next few sessions. Then in the seventh session her discourse suddenly shifted from teaching to learning as she began sharing with us her efforts around particular students.

Karen's practice continued to change for the remainder of the year. The following fall, with the start of a new school year, her teaching practice was unrecognizable in comparison to the type of teacher she had been a short 10 months previously.

Outwardly, Karen's case can be seen as being similar to Mitchell's. In fact, their discourse on the changes they needed to make in their teaching in order to allow their students to succeed at the numeracy tasks was almost identical. Deeper analysis, however, revealed that Karen's transformation was more steeped in a natural rhythm of change than in accommodation.

The *push–pull rhythm of change*, as I have come to call it, is a phenomenon comprised of a series of two, three, or four distinct phases, always in the same sequence, each having its own associated name. The names are an amalgamation—the prefixes *exo-* and

endo- come from Greek meaning outer, outside, external and inner, inside, internal, respectively, while *-spection* comes from the Latin *specere* which means 'to look at'.

Phase 1: exo-spection (x)

The teachers work on an activity, which, at the time, occupies their focus. This could be a problem-solving exercise or the designing of a lesson, task, or assessment rubric. Whether the activity is relevant to their own teaching practice is immaterial as the teachers' focus is on the completion of the task, rather than on the potential for the task to inform their own practice. That is, the teachers are looking at the activity as lying outside of themselves. Although not a push, this phase can be characterized as a *holding* of the activity away from themselves.

Phase 2: eXo-spection (X)

The teachers realize that the problem they have solved, or the lesson or task they have built, is not commensurate with their own classroom context. They see this as a large-scale problem bemoaning the poor state of affairs of all students and the educational system in general. They look at the source of the problem as lying far outside of themselves, for example, societal expectations, the curriculum, the evils of external examinations or deterioration of standards, and speak of systemic reform as the only solution. As such, they are not only *pushing* the problem further outside of themselves, but also broadening its scope.

Phase 3: eNdo-spection (N)

Suddenly, there is a change in the teachers' disposition—the problem, regardless of where it lies, must be solved within their own practice in the scope of the classroom. Now, the conversations are about what they can do within their teaching in order to enable their students to be successful in solving a specific problem, completing a specified task, or performing well on a given assessment. The teachers are no longer pushing the problem, and any subsequent solutions, away from themselves but are rather *pulling* it back to their locus of control.

Phase 4: endo-spection (n)

Finally, there is a shift of attention to the plight of individual students. The conversations shift from the classroom to a particular student or subset of students, and with this shift comes a narrowing of focus on their influence as teachers. This final shift is also marked by a subtle shift in discourse from *teaching* to *learning* as they *pull* the problem closer to themselves.

It should be noted that I have deliberately avoided using the term *introspection*, which means to examine one's own thoughts and feelings. This is not what I am trying to capture here. *Endo-spection* is not about looking inside oneself, but about looking at something as lying inside of oneself or one's locus of control. Conversely, *exo-spection* is about looking at something as lying outside of oneself or one's locus of control.

Of the 16 teachers who exhibited this rhythm of change in their practice, three went through the XN phases of change only, while ten went through the xXN sequence. The remaining three teachers, Karen included, went through the full xXNn sequence. This is not to say that I have not also encountered teachers who have exhibited just one of these phases—I have. However, one phase does not a change make.

Conclusions

Not all change is the same. I began this article by drawing the distinction between the more common forms of teacher change that teachers experience as participants in the popular sustained and collaborative intervention settings and the phenomenon of rapid and profound change in practice that I had witnessed. But, even instances of this phenomenon, as rare as it is, are not to be seen as being the same. Within this phenomenon, I identified five distinct mechanisms of change that I came to call: (1) *conceptual change;* (2) *accommodating outliers;* (3) *reification;* (4) *leading belief change;* and (5) *push–pull rhythm of change*. Awareness of rapid and profound change in general and the nuanced characteristics of these five distinct mechanisms in particular have alerted me, as a person who works within the field of mathematics professional development, to new possibilities within both my practice and my research. Personally, I now question the assumptions upon which much of the professional growth literature has been predicated. Deeper analysis of each one of these richly diverse and complex mechanisms is certainly warranted.

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