Train spotters' paradise

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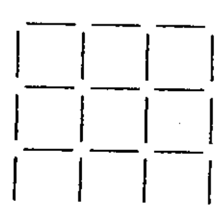
I have been in many classrooms where children have been encouraged to use their intelligence and creativity to find some mathematical properties. Children have been asked to look at particular situations and are encouraged to find connections, make conjectures and test to see if those conjectures are correct. They are encouraged to make generalisations and to express these in algebraic form. In such lessons, I am impressed by how much children are able to discover for themselves and how well they can articulate their findings. There is an atmosphere of involvement in mathematics, children are being challenged and are expressing a sense of achievement in what they are doing. Quite often they continue working on their problem at home and involve their parents. They may arrive the following day eager to share new things the family have discovered. Such times have so many ingredients of lovely mathematics lessons. Yet I feel saddened rather than joyful.

I will mention five such lessons.

In one lesson, children are asked to draw a number of networks and to see whether they can be traversed without taking the pen off the paper or going over any line twice. After a while the teacher asks them to draw up a table giving the number of nodes, how many are odd and even, the number of arcs, the number of regions and whether the network is traversable or not. The challenge for the children is to look at the table and see whether they can see any patterns.

In another lesson, children are looking at the number of matches required to make a square such as shown below.



The children draw different sized squares and collect their results in a table. A number of patterns are found and many children articulate rules.

In a third lesson, the class are asked to draw a number of circles with different radii. Using some string, they measure the circumferences of each circle and make a table of the radius, diameter and circumference. The children are asked to try to find a connection between the radius and the circumference.

The fourth lesson involves choosing a number, say 68, reversing the digits to get 86, and adding the two numbers together:

$68 + 86 = 154.$

If the answer is a palindrome, stop. Otherwise, repeat the process with the answer:

$154 + 451 = 605$
$605 + 506 = 1111.$

In the case of 68, it took three iterations to arrive at a palindrome. Thus the number 68 is called a *level 3* number. The class are divided into groups and, between them, are asked to find out what level are all the numbers from 1 to 100. They are asked to collect the results in a table and to look for patterns.

In the fifth lesson, children are listing the different outcomes that are possible if they throw 1, 2, 3, . . . coins. They are asked to collate their results in the table shown as Table 5.1. Then they are told to look for patterns and predict how the table would continue.

Table 5.1 Number of ways of getting:

	0 heads	1 head	2 heads	3 heads	4 heads	. . .
1 coin						
2 coins						
3 coins						
4 coins						
. . .						

Despite the fact that in each of these lessons children were well motivated and involved in mathematics, I am saddened because the children ended up doing a similar activity irrespective of the initial mathematical situation.

Is the diversity and richness of the mathematics curriculum being reduced to a series of spotting number patterns from tables?

Whatever the initial mathematical situation, once the numbers are collected into a table, a separate activity begins to find patterns in the numbers.

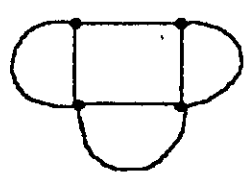
Their attention is with the numbers and is thus taken away from the original situation. After a period of time, some children have difficulty reminding themselves where all the numbers came from. I suggest that, for many children, what they find out about the numbers remains exactly that; it does not mean they have learnt anything about the original mathematical situation, only about sets of numbers in a table.

Children can find many patterns in their table, even if they have made some errors in the entries. They may find all sorts of rules, none of which apply to the original situation but then some children have long ago turned their attention away from that. Spotting patterns in the numbers becomes an activity in its own right and not a means through which insights are gained into the original mathematical situation.

Networks may come under a heading of topology; the square of matches is essentially a geometric situation; circumference of circles may come under a heading of measures or geometry; palindromic numbers within number theory; and the coins within combinatorics or probability. These initial situations span a broad cross-section of mathematical areas and yet I argue the case that each of these lessons was really under the same heading of spotting number patterns since that is what the pupils ended up attending to.

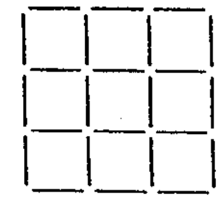
In all these lessons, the children were doing several particular examples and collecting results from these. I presume the structure of collecting results in a table offers the possibility of making general statements about these results. The trouble is that the general statements are statements about the results rather than the mathematical situation from which they came. The existence of the table places value on collecting several results rather than looking in any depth at a particular one. More might be learnt about the original mathematics if one particular situation was looked at in depth, rather than rushing through several in order to collect results.

If I consider the network



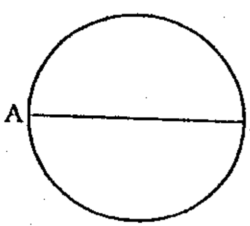
I can learn that it is traversable. However, instead of rushing on to consider another network, I could explore this one further. Are there other ways I can traverse this network? What if I keep the same starting node, where can I finish up? What if I try starting from the other nodes? How often do I visit each node? What would change if I rubbed out one of the arcs? Does it matter which one?. . .

If I take some matches and start putting them down so as to make this square:



In what order do I place them down? Do I repeat certain patterns of matches? How often? Is there a stage when I am half way through? Can I look at the final square and imagine half the matches a different colour? What about a third? A quarter? How many horizontal lines are there? How many vertical? Why? Can I see the square one size less within this square? What if I paint the extra ones blue? Can I see the square two sizes less within this one?. . .

Imagine taking the diameter line of the circle and picking up a copy of it leaving the circle with the original diameter staying where it is. Imagine I could bend the diameter although I cannot alter its length. I try to bend it so that it curves round the circumference of the circle. If this bent diameter starts at A, how far round the circumference do I think it will go? Will it go as far as B? Suppose I put a mark where it has got to and continue with another copy of the diameter, how far now? How many diameters would I need to go once round the circumference and return to A? What if I did a drawing of this and put it in a photocopier which reduces the size?. . .



If I consider adding together 68 and its reverse 86,

68 +
86

what numbers appear in each of the columns? If they are the same numbers, why do I get the same number in each column of the answer? When will I get the same? What effect does the 'carry' have? How could I change the numbers so that I do not get a carry? If I stick with the original number being two-digit, when do I get two-digit answers? Three-digit answers? Can I get four-digit answers? If the answer is three-digit, what digits could I get in the hundred column?. . .

Let me consider the situation where I have exactly one head with a number of coins. If I know how many ways there are of getting one head with two coins, do I have the same number again when I introduce a third coin which happens to be tails? If the third coin was a head, what would the other two coins have had to be? What if I consider a larger number of coins and introduce one more coin which is a tail? A head?. . .

There is so much mathematical richness that can be gained by looking at a particular situation in some depth rather than looking at it superficially in order to get a result for a table and then rushing on to the next example. By staying with the particular situation, I can learn about the mathematics inherent in it rather than learning about numbers in a table. I practise and develop different abilities rather than practising and developing the one ability of spotting number patterns. I see geometry as geometry, combinatorics as combinatorics rather than everything as spotting number patterns. I am being asked to be creative and adaptable in different situations, to see that different situations require different questions to work on them.

Train spotters go in search of trains and collect numbers. At the end of the day, they are left with numbers . . . not a train in sight.

Hewitt D. (1994), *Train Spotter's Paradise*, in Selinger, M. (Ed), Teaching Mathematics, pp47-51, London: Routledge.

Hewitt D. (1994), *Train Spotters' Paradise*, in ATM, Teachers, Learning and Mathematics, pp44-46, Derby: Association of Teachers of Mathematics.