Mathematics teaching and social media: An emergent space for resilient professional activity

by

Agnieszka Judyta Larsen

M.Sc., Simon Fraser University, 2013
B.A. (Mathematics Major), University of the Fraser Valley, 2009

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Approval

Name: Agnieszka Judyta Larsen
Degree: Doctor of Philosophy
Title: Mathematics teaching and social media: An emergent space for resilient professional activity

Examinining Committee: Chair: Sean Chorney
                                    Assistant Professor
                                    Peter Liljedahl
                                    Senior Supervisor
                                    Professor
                                    Nathalie Sinclair
                                    Supervisor
                                    Professor
                                    David Pimm
                                    Internal Examiner
                                    Senior Lecturer
                                    Elaine Simmt
                                    External Examiner
                                    Professor
                                    Faculty of Education
                                    University of Alberta

Date Defended/Approved: November 19, 2019
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Abstract

Professional activity around mathematics teaching is considered vital in the improvement of mathematics education at all levels. Research in mathematics education has identified aspects of teacher professional development that are effective, but there has been a recent push for better understanding how mathematics teacher professional development can also be sustainable. To this end, informal professional activity around mathematics teaching has become of particular interest in the field. Since many education professionals are turning to resources that are becoming increasingly available beyond the confines of institutional boundaries, such as via social media, many of the constraints of traditional forms of professional activity are being bypassed, allowing for informal professional activity to flourish. In some cases, collectives of professionals have formed in such contexts. One such collective, referred to as the Math Twitter Blogosphere (MTBoS), has remained resilient for almost ten years with ongoing activity around mathematics teaching occurring daily. Although this self-organized, bottom-up, emergent collective thrives with engagement around mathematics teaching, it has received very little empirical attention within mathematics education. As such, this study investigates the inner workings of this collective by drawing on tenets of complexity thinking to develop a more comprehensive description of its nature and how it thrives. Informed by an ethnographic journey of becoming a MTBoS participant, I select and analyze data in innovative ways to uncover both the ideational network in MTBoS and the social network that drives its existence. Analysis of these networks illuminates the influence not only of social capital, but also of ideational capital, both of which are necessary for determining ideational resilience within the collective. The results of this research indicate not only the popular topics within MTBoS, but also more importantly, features that drive ongoing and often generative activity around mathematics teaching within this online, unprompted, unfunded and unmandated professional setting.

Keywords: mathematics teaching; teacher learning; social media; complexity; professional development; network analysis; MTBoS
For Nathan, your passion for change along with your unfailing love and support helped me flourish.

For my parents, your diverse talents and resilience have supported my learning journey since day one.
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Participant

A Participant is any person who engages in MTBoS activity in any way whether it is by contributing or lurking, or whether they have engaged in the past or are currently engaging. This term is used to refer to Users particularly within Communities of Practice framings.

Reply

A Reply is an option on Twitter with which a User may Reply to any Tweet, even if the Tweet is already a Reply to another Tweet. A Reply is technically a new tweet, but Twitter organizes it in a way that users viewing it can see the content that is being replied to.

Retweet

A Retweet is a kind of Tweet made by a user that is a repost from a different user. The Retweet is published as the retweeting user’s Tweet but contains content from the original Tweet as well as indications that it is a Retweet.

Subject

A Subject is a Participant, a term used particularly within Activity Theory framings.

Tasks

Tasks refer specifically to mathematical tasks.

Tweet

A Tweet is a short microblogging message published by a User on the Twitter platform. It may be either publicly available or limited to only being viewed by those the User accepts as followers. Tweets were once limited to 140 characters until November 2017, when they were extended to include 280 characters. Tweets may also include up to 4 photos or a maximum of 2 minutes and 20 second of video, along with text as well as hyperlinks.

Twitter

Twitter is a social networking and microblogging service founded in 2006 that allows users to publish and peruse short messages known as Tweets as well as subscribe to the messages made by certain users via Following.

User

A User is an account manager of a particular Handle on Twitter, and therefore is a Twitter user. Although most Users are single people, some are also collectives, such as partnerships or brands. As such, the term User is used throughout the thesis in referring to Twitter accounts in general, often in relation to those in the data set.
Teachers are beginning to take control of their own professional development, finding new ways to learn from each other, to reflect on their own practice, and to develop learning and support networks of like-minded professionals all over the world.

(McCulloch, McIntosh, & Barrett, 2011, p. 4)

How I meet other teachers, where we discuss ideas, and how we share information has changed. Significantly. I meet them online. I learn from them online. I share with them online. (Sakamoto, 2012, para. 16)

I was boring and I was bored. Then I tried a lesson I found on a teacher’s blog, and the kids lit up. #WhyMTBoS (@KentHaines, June 14, 2014)

Love my virtual PLN #MTBoS - thanks for pushing me to do my best, try new things, and encourage me to press on (@TypeAMathLand, November 17, 2016)

Following this weird #MTBOS hashtag on twitter has changed my teaching practice in so many ways. (@MrOrr_geek, February 6, 2018)
Chapter 1 Introduction

Teaching is too hard to do alone. Too many of us do it alone. (Aion, 2015)

As a university teacher of mathematics, I have found immense value in connecting and collaborating with others who also teach mathematics and who critically examine mathematical learning situations. Listening and engaging with their musings, successes and struggles have been both inspiring and affirming; inspiring when I encounter a novel idea I have not previously thought of and affirming when they describe occurrences that are deeply familiar to me. I feel privileged to have had opportunities for such conversations, as well as the occasions to witness practices of other mathematics teachers. Noticing micro-practices such as how they deal with a struggling student, how they pose a certain problem or how they adapt their lessons to student needs have been significant sources of learning that have helped refine my practice. I cannot imagine being who I am today as an educator without the opportunities I have been granted to collaborate with other teachers in my field.

However, opportunities for such collegiality were not always available to me. Although my teaching situation allowed me the time to collaborate with others, my colleagues were often teaching when I was not. I did not have social relations with other mathematics teachers outside of my teaching context, which is at a university where I teach school-level mathematics and mathematics education courses1, with whom I could collaborate. When I contacted mathematics teachers at various public schools in my region, I was told they could not meet because they did not have time, were very busy with grading or planning, or were simply not seeking collaboration. My requests to visit classrooms of schoolteachers were frequently denied, perhaps due to a lack of social relationships with them or because of being alienated as a university professor who may have been perceived as an evaluative threat. As such, my desires towards learning with other mathematics teachers about improving our practices were unfulfillable and limited by a lack of mutual contact time and social relations. Since teacher isolation is a broader phenomenon that many teachers experience (Flinders, 1988; Little, 1990; Schlichte,

1 My primary role is teaching adults school-level mathematics prerequisite courses towards entry into university programming; however, I also occasionally teach methods courses for pre-service mathematics teachers and mathematics courses designed for future mathematics teachers.
Yssel, & Merbler, 2005), it means that sometimes, even if a teacher wants to collaborate with others, the physical and social barriers may be prohibitive.

My desire and curiosity towards peering into practices of other mathematics teachers, paired with my millennial nature for searching the Internet, led me to search online for other teachers of mathematics who may be sharing publicly about their practices. Although my initial intention was to find teachers willing to let me visit their classrooms to experience various contexts of mathematical learning, what I found in my searches was that many mathematics teachers were reflecting on and comprehensively reporting their teaching practices in online public blog pages and micro-blog posts on social media platforms such as Twitter (http://www.twitter.com). These teachers were hosting their own blog pages and posting journal-like entries relatively frequently. I quickly noticed many of them referring to Twitter and realized that Twitter was a platform that allowed them to connect with each other and provided the means through which they could find each other. What I did not realize at the outset was that this was part of a broader phenomenon of educators from various disciplines who have begun to turn to online platforms such as Twitter and blogs in recent years for collaboration due to otherwise limited professional learning opportunities (Veletsianos & Kimmons, 2012). When collaboration occurs online, the physical limitations of meeting face-to-face are removed, flexibility is added and teachers can pursue their needs and interests (Duncan-Howell, 2010). Online opportunities for professional learning seem to defy the limitations of face-to-face settings and offer an interesting alternative for teacher professional learning.

In January 2014, I embarked on my journey of looking online for mathematics teachers posting about their practices. Early on, I found myself being surprised and intrigued by a mysterious phrase being used rather frequently on Twitter by mathematics teachers, which I first noticed being written as a hashtag: #MTBoS, as in Figure 1.1.

![Figure 1.1 Noticing the MTBoS hashtag in 2014](image-url)
I followed what seemed like a rabbit-hole of links and discovered that MTBoS was an acronym standing for “Math Twitter Blog-o-Sphere”, a user-generated label that refers to an “interconnected web of math teachers from all across the world who share, reflect, and talk ‘out loud’ regarding their thoughts on what is happening in their own personal classroom” (Feller, 2017). Participating in MTBoS happens through tweets and blog pages, but blog pages are often linked within tweets for better visibility to the community. Users then subscribe to (or follow) other users to receive updates on their published content, often in the form of a newsfeed (or feed). Although the first use of the acronym MTBoS was in 2013, references to a ‘twitterblogosphere’ as “an emerging online community of math educators” were made as early as 2010 (“MTBoS”, 2015). MTBoS has grown over the years to include at least 6,000 people interested in mathematics teaching, including teachers, leaders, and coaches, more than 800 of whom publicly identify themselves as “MTBoSers” (MTBoS, 2019). Awe-inspiringly, this collective has sustained activity consistently for almost ten years.

As a whole, MTBoS is a space in which hundreds of mathematics teachers and leaders from across North America are regularly publicizing their musings about mathematics teaching in an unprompted, unfunded, and unevaluated manner that seems to be more sustainable than many other professional development initiatives. Through their individual autonomous actions, MTBoSers contribute to a decentralized, virtual professional community that is accessible around the clock and seems not only to achieve many of the goals of professional development endeavours, such as helping mathematics teachers develop and implement rich mathematical learning tasks in practice, but also to surpass the limitations of traditional professional development approaches in terms of defying constraints of time and space.

---

2 Tweets are micro-blog posts published on the Twitter platform that are limited in size, and visible to anyone unless the user lists their account as private to restrict access to only their followers.

3 Blog pages are user-created webpages on which users may publish content under a timestamp.

4 ‟Following” is another term for subscribing to the updates on the activity of the accounts one follows, which appear in one’s newsfeed (also referred to as one’s ‘feed’ for short).

5 Determined by counting the number of unique users posting or retweeting content with the MTBoS hashtag more than once between September 24, 2018 and January 2, 2019, which included 73,180 tweets made by 16,570 distinct users, 6308 of whom posted with the hashtag more than once.

Through communicating their own musings and reading the musings of others involved in teaching mathematics, participants can find lesson ideas, co-plan with others, be inspired by the stories of educators in similar contexts and challenged on their ideas by like-minded peers (see Figure 1.2).

Figure 1.2   Examples of tweets related to mathematics teaching (from 2016)

Not only does this space seem generally promising for mathematics teacher professional learning, participants frequently express their appreciation for the space and what it does to enhance their learning as teachers of mathematics. Expressing gratitude for the affordances of MTBoS is commonplace and can be encountered frequently.

#MTBOS completely changed the way I approached mathematics in the classroom. I can find a great lesson on #MTBOS, but it’s not just about finding great lessons. It’s about how to teach. My teaching pedagogy has improved solely because of #MTBOS. (Feller, 2017)

Participants claim it is the best professional development they have experienced, particularly because it is personalized and caters to their needs.

I’ve just started participating in MTBoS and am finding it to be some of the best PD I’ve had! #WhyMTBoS (@heighdeigh99, October 18, 2013)
Why do I love Twitter?? I have 4 ppl helping me plan a lesson I’ve always struggled to teach! #whymtbos #mtbos #MSMathChat (@MeganHeine, April 2, 2015)

I love being inspired by, challenged by, and collaborating with other math teachers #WhyMTBoS (@abusch38, March 20, 2015)

My best new classroom resources come from those I follow or whose blogs I read #whyMTBoS (@bobloch, October 18, 2013)

#whyMTBoS it is a huge online classroom for teachers that is accessible around the clock! A teacher's BEST asset! (@The30thvoice, October 18, 2013)

They also work to pass on benefits they have experienced to newcomers through various initiatives such as organizing initiatives to reply to people who have few followers and sending welcome messages.

We are better together. Inspiring each other and sharing what we do to help each [other] be our best teaching selves #thankful #whyMTBoS #loveyall (@TypeAMathLand, September 22, 2018)

Stuck in a rut? Need to revamp a non-engaging lesson? MTBoS will listen, question, encourage & challenge you to be your best! #WhyMTBoS (@pamjwilson, June 14, 2014)

And, many claim they feel a sense of community with those participating in the space.

These people aren't just random people on the internet. They are people who have the same interests as me, hoping to make their community love math and be the best math teachers they can be, and they do that by lifting others up. What a community. (@howie_hua, October 24, 2018)

#whymtbos? Well, where else can I find moral support and share our true love of teaching math? Best. Thing. Ever (@mathinct678, September 11, 2018)

Several also attest that MTBoS saved their careers because of how supportive and helpful this online community has been for them.

I was doing the same thing all math teachers do: Warm-up, model, guided practice, assign homework. Rinse and repeat. It's the vicious cycle for mathematics teachers in today's world. I knew something had to change because I wasn’t having as much fun as I should’ve been having. In the end, this was my first “AHA!” moment as a teacher and the part where #MTBOS and Twitter saved me from not sucking. (Feller, 2017)
Thank you to everyone in the MTBoS that has been a cheerleader for me whether it be by liking, sharing, or just sending good thoughts from time to time. You may not have realized it at the time but you were literally saving me from myself and for that I will always be thankful. (Powers, 2018)

My plan was to kick my feet up on the desk and bring in a newspaper and get out worksheets . . . and then a friend of mine who also teaches math asked me if I’m going to Twitter Math Camp. He sort of pulled me into this community, the MTBoS, the Math Twitter Blogosphere. And then summer of 2013, I went to Twitter Math Camp, and it was beyond life changing. I came back and my wife and my coworkers didn’t know who I was. When I say life changing that’s not a metaphor or hyperbole. It completely changed my view of teaching and my view of my role as an educator. (Aion, 2015)

Based on my happenstances with people who indicate MTBoS saved their careers, a typical story of those affected deeply by their participation in the MTBoS community seems appropriately represented by one teacher’s visual representation, shown in Figure 1.3, of her journey as a mathematics teacher.

![Learning curve with Twitter](image)

**Figure 1.3** Learning curve with Twitter (@GotMathHelp, December 9, 2017)

The transformative effects of participating in MTBoS touted by many of its participants is awe-inspiring and intriguing because it is unclear exactly why it has such a strong effect on their perceptions of their professional learning, and how this learning is precipitated. Empirical work on this rich phenomenon is still limited, but generally indicates evidence that MTBoS offers a promising avenue for mathematics teachers and leaders to learn about the knowledge and conceptions that support effective use of cognitively demanding tasks (Parrish, 2016b), to receive and offer advice through mentoring.
(Parrish, 2016a), and to negotiate meaning about mathematical learning situations, some of which can be generative of new materials or ideas (Larsen, 2016; Larsen & Liljedahl, 2017). There is also evidence that it creates opportunities for members to participate in a community of practice with other mathematics teachers (Risser & Bottoms, 2018; Willet & Reimer, 2018) and supports formation of social relationships that move beyond being strictly professional in nature (Risser & Waddell, 2018). However, although these studies indicate that MTBoS offers a diverse array of opportunities for professional learning, they do not explain why these opportunities are created, and how their creation is maintained. The inner workings of MTBoS and the implications it has for investigations into mathematics teacher learning are yet unclear.

As such, to understand how MTBoS works, the most natural point of entry is to take the perspective of a newcomer trying to join it. Perhaps the newcomer has heard about it from a colleague who told them to search the hashtag or encountered it through other searches for mathematics teacher related content on the Internet. Nonetheless, it can quickly become evident that although MTBoS comes across as an established and organized entity, it is not at all easy to navigate. In fact, MTBoSers have created several websites that aim to help newcomers find ways to ‘join’ (e.g., “Exploring the MathTwitterBlogosphere,” n.d.; “welcome to the mathtwitterblogosphere,” n.d.). This is because it is chaotic in nature, with disparate utterances being publicized at random times; and, these varied messages are posted as tweets in one place while being relatively independent of each other. This seeming randomness is evident when searching Twitter for a search term or a hashtag7, viewing the history of a user’s tweets, or when scrolling through a feed of tweets made by people one follows. As may be observed in the feed of tweets shown in Figure 1.4 below, tweets are independent of each other, as influenced by the design of the Twitter platform, which allows for many statements to be made at once without a need to wait to listen to others. Even though tweets may be expanded further if desired, the independence of tweets contributes to the broader phenomenon of ‘self-sharing’ as prevalent in social media settings (Naaman, Boase, & Lai, 2010). Overall, the majority of tweets on Twitter receive no replies, and only 1.5% of tweets solicit more than two replies in depth (HJH, 2014).

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7 A hashtag is a term preceded by the hash (#) sign that links all tweets that include it, and usually centres around a particular topic of interest.
Yet, MTBoS members claim they encounter meaningful conversations about teaching and learning mathematics and some tweets garner significant amounts of discussion. For instance, as Kane (n.d.) writes, “There’s always conversation happening on Twitter. It’s intimidating at first, but math folks on Twitter are incredibly welcoming” (para. 5). What is less evident, though, is that encountering meaningful conversations or impactful statements and resources in MTBoS requires participating as a member. Mere searches do not yield the same results as viewing several related posts every day over an extended period. If desired, searching with a search term can reduce the randomness of the encounters. However, viewing several random posts per day can increase the chances of seeing something unexpected that is of interest. To encounter relevant and interesting tweets, it takes finding accounts to follow, finding hashtags to follow, and scrolling through the feed regularly. Essentially, the sheer mass of posts made by people interested in mathematics teaching in MTBoS (about one tweet every two minutes\(^8\)) and their often-cryptic nature (with mysterious hashtags and abbreviations) makes it impossible to access without becoming a participant through ‘lurking’ at minimum.

Hence, my inquiry into the inner workings of this space has inherently involved me becoming a participant\(^9\). Over five years, I worked at navigating the space both as a teacher of mathematics and a mathematics education researcher with an effort to make sense of the structure within the chaos of MTBoS. As my insider view grew wider, so did

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\(^8\) Tweet frequency calculated based on tweets, including retweets, collected between September 24, 2018 and January 2, 2019, that included 73180 tweets over 2569 hours.

\(^9\) A more detailed account of my journey in becoming a participant is included in §4.2.
my interest in the phenomenon. And, in becoming an insider, I innately discovered what it takes to feel connected to the community, which took more than merely lurking through other members’ posts. For me, it was not until I began physically meeting other MTBoSers that I felt more inclined to share musings online. Knowing who I was talking to helped me direct my comments towards an ‘imagined audience’ (Marwick & boyd, 2010) and, as I posted and received responses, I discovered more about engaging in this space than I would have been privy to solely by lurking. What we experience is what we can attend to, and for me, I had to experience MTBoS as a participant to uncover the hidden processes that drive this intriguing mathematics teacher professional activity.

As such, this thesis comprises not only a research journey into the inner workings of MTBoS, an online professional community around mathematics teaching, but one that is innately interwoven with a personal journey of becoming a teacher of mathematics and a mathematics educator who feels integrally connected with a community.

If we are to facilitate the professional development of teachers, we must understand the process by which teachers grow professionally and the conditions that support and promote that growth. (Clarke & Hollingsworth, 2002, p. 947)

MTBoS seems to offer sought-after opportunities for mathematics teacher professional learning and is a vibrant and continuously evolving phenomenon of professional activity around mathematics teaching that has sustained operation for almost ten years. Although there is some evidence that MTBoS provides opportunities for encountering and implementing cognitively demanding tasks, making meaning and generating new ideas with peers, as well as being part of a community of practice of mathematics educators, it is a phenomenon worthy of deeper pursuit towards understanding its inner workings and implications. As such, my inquiry in this thesis is driven by a curiosity towards what MTBoS is, how and why it proves to be a sustainable form of professional activity around mathematics teaching and what it can reveal about the nature of professional learning related to teaching mathematics.
Chapter 2　Relevant Literature

Given the novel nature of this phenomenon and the sparse amount of existing empirical work pertaining directly to it, I grappled with identifying pertinent literature to guide and inform my inquiry. Fundamentally, MTBoS is a case of professional activity around mathematics teaching that occurs without prompting in a public online space and presumably offers teachers of mathematics a source of professional learning. Therefore, it is nested within both literature pertaining to mathematics teacher professional learning and social media activity, both of which have rich empirical track-records. Although engagement in MTBoS could be considered a form of professional digital scholarship, the short nature of Twitter posts\textsuperscript{10} that happen quickly and often in-the-moment makes it more of a form of informal communication rather than a premeditated form of publication. Therefore, my focus in reviewing the literature is to explore findings and conceptualizations around mathematics teacher professional learning with a particular focus on informal settings and around how social media, specifically through micro-publishing, provides a means to extending such professional activity. As such, in what follows, I review the landscape of mathematics teacher professional learning, key findings from social media research and results from investigations that have recently begun exploring MTBoS. In each of these areas of literature, I identify gaps and point to the findings and conceptualizations that are most pertinent to investigating MTBoS as a source of professional activity around mathematics teaching.

2.1 Mathematics teacher professional learning

Mathematics teachers are entrusted with the noteworthy and complex responsibility of helping students learn and appreciate mathematics. Choices made by mathematics teachers in the classroom greatly affect students’ experiences of mathematics and their visions about what it means to do mathematics (Boaler, 2002; Towers, Takeuchi, Hall, & Martin, 2017). Yet, facilitating mathematical learning is a complex endeavour and has been attributed to qualities mathematics teachers are expected to develop, such as mathematical content knowledge (Ball, 2003; Davis & Simmt, 2006; Hill, Rowan, & Ball, 

\textsuperscript{10} Tweets are limited to 280 characters (which used to be 140 characters) aside from hyperlinks, with a maximum of four attached photos or a video under 2 minutes and 20 seconds in length.
2005), mathematical knowledge for teaching (Ball & Bass, 2002; Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008) and beliefs about both mathematics and what it means to learn mathematics (Liljedahl & Oesterle, 2014; Rolka, Rösken, & Liljedahl, 2006). Amidst this complex network of expectations and sites for dilemmas and tensions, teachers are often left in isolated situations due to factors such as geographical location, pedagogical incompatibility with local peers, lack of funding, or time constraints. As such, I explore literature about why mathematics teachers may want to engage in professional learning, what barriers exist that prevent engagement in professional learning opportunities and what attributes of professional learning initiatives for mathematics teachers have made them successful.

At the heart of mathematics teacher professional learning lies a desire. The landscape of competing expectations teachers face can be rooted in disparities between what they are required to do, what they want to do, and what they believe they should do. These competing expectations can result in feelings of inner turmoil, referred to as teacher tensions (Adler, 2001; Berry, 2007), which may remain unresolved. However, such tensions may also create a desire for working towards resolution (Rouleau & Liljedahl, 2017). One source of tension, for instance, may arise from the incongruence among teachers’ experiences as learners, their interactions with other teachers who use traditional approaches to teaching and the curricular mandates that push for achieving reform-oriented pedagogies and practices (e.g., BC’s New Curriculum, 2018; NCTM, 2000). Reform-oriented practices typically involve supporting productive struggle, designing and using rich mathematical tasks, and creating environments of inquiry and critical thinking, which are generally much unlike the persistently dominant teacher-led modes of mathematics instruction (Silver, Mesa, Morris, Star, & Benken, 2009; Stigler & Hiebert, 2000). When teachers have not had opportunities to experience the practices they are expected to carry out, implementing recommended changes may be difficult to achieve, even if there is a desire to do so. Other tensions may also emerge from opposing pressures in assessment structures imposed on teachers (Reay & Wiliam, 1999), public opinions about how mathematics should be taught that influence parental concerns (Rodney, Rouleau, & Sinclair, 2016) and students capitalizing on institutional norms to subversively avoid learning mathematics (Liljedahl & Allan, 2013a). Therefore, teachers encounter a wide variety of pressures that can contribute to their desire for
professional learning, and to a more global demand for creating effective and sustainable professional learning opportunities. However, barriers to this exist.

Although some teachers naturally find ways to collaborate with colleagues (e.g., Bryk, Camburn, & Louis, 1999), the isolated nature of the teaching profession can create barriers for teachers to connect with each other resulting in limited opportunities for collegial interaction (e.g., Flinders, 1988; Hargreaves, 1993; Little, 1990; Schlichte et al., 2005). Professional development initiatives are commonly limited to sparse one-time events facilitated in face-to-face synchronous settings which, due to their temporal nature, are not typically conducive to building communities that engender ongoing professional growth (Ball, 2002). They are also generally driven by facilitator perceptions of what teachers need rather than by what teachers want (Liljedahl, 2018). In turn, various approaches to professional development, such as lesson study (Stigler & Hiebert, 1999) and communities of practice (Wenger, 1998) to name two, have been explored. What is known from this vein of research, is that the robustness of a professional development initiative is dependent on ensuring both teachers and facilitators adopt a stance of inquiry, activity reflects and is driven by teacher needs and interests, and community building and networking are at the core (Lerman & Zehetmeier, 2008). This implies that ongoing teacher collaboration is indispensable. However, the amount of collaboration may require consideration towards effectiveness.

The intensity of collaboration becomes an important determinant – too much collaboration and learning are stifling, too little collaboration and teacher isolation inhibit growth, just enough collaboration and teachers receive the stimulation and support from colleagues necessary for change. (Opfer & Pedder, 2011, p. 386)

Overall, there have been a wide variety of ways in which teacher collaboration has been stimulated within professional learning settings around mathematics teaching. For example, Chapman (2008) uses narratives to have teachers explain teaching cases to each other. Similarly, Sherin and Han (2004) consider teacher learning within a video club context where teachers discuss videotaped teaching episodes. In a more self-organized context, Arbaugh (2003) identifies the values and organizational structure of a school-based study group of seven secondary geometry teachers. And, in a more informal context, Horn (2005) studies the opportunities for learning that occur within informal interactions in a high school mathematics department. All these studies involve growing awareness around mathematics teaching through inquiry and reflection, and
networking and community building through collaboration. In fact, Mason and Davis (2013) elucidate that developing greater awareness is essential for teachers to develop abilities for making in-the-moment choices within their teaching towards guiding students appropriately in their own awareness-making within mathematics. In particular, the work of Schön (1983) around reflection-on-action and reflection-in-action, and of Mason (2002) around noticing through heightening one’s awareness either in action, in discipline, or in counsel, are helpful in teaching. However, collaboration is also found to enhance these forms of inquiry. To this end, Lerman (2001) emphasizes that reflection needs to be situated in context with others because “for reflection to say something about how people learn, [it] involves others in one way or another” (p. 41). As such, reflective practice should be considered as part of the social context in which teachers participate, and such contexts can serve as structures that support teacher learning.

There is also a draw towards empowering teachers to pursue their own lines of questioning, as for example within instances of action research, lesson study, and teacher study group initiatives. In action research, teachers undertake systematic inquiry in their own classrooms, pursue research questions, and in turn, develop of their teaching (Altricher, Posch, & Somekh, 1993; Carr & Kemmis, 2003; Elliot, 1991; Kemmis, 1985; McNiff, 1988; Stringer, 2013). This teacher-as-researcher movement has received a fair amount of attention (e.g., Burnaford, Fischer, & Hobson, 1996; Cochran-Smith & Lytle, 2012; Flake, Kuhs, Donnelly, & Ebert, 1995; Franke et al., 2001; Richardson, 1990) as also indicated by the formation of various action research journals (e.g., Action Research, Educational Action Research, The Canadian Journal of Action Research). Notably, many instances of action research lie within an academic programme, or are supported by some sort of academic association or group (e.g., Brookhart, Andolina, Zuza, & Furman, 2004; Price, 2001).

One particular example within action research reveals a case in which teachers are provided with as naturalistic of a setting as possible. Jaworski (1998) studied six teacher-researchers in the Mathematics Teacher Enquiry project, where mathematics teachers engage in action research within their classrooms and collaborate with each other and with the researchers during regular meetings. The supportive collaborative structures paired with an action research perspective contributed to the teacher-researchers as being “seen as reflective practitioners, developing knowledge and awareness through enhanced metacognitive activity” (p. 3). Jaworski finds that the
teachers’ classroom-based inquiries, guided by collaboration with fellow teacher-researchers and academic researchers, were evolutionary in nature. The researchers organized supportive structures to sustain teachers’ research projects but intended to preserve their autonomy as much as possible. She also aimed not to impose a research agenda on the teachers, but rather, teachers were “identifying and pursuing their own research questions independently of an academic programme” and the focus of the research was “to explore in what ways teachers themselves might motivate, direct and sustain such research” (p. 5, her emphasis). As a result, they became more independent as the study evolved and were able to tackle important issues with less support from researchers by the end of the two-year project. Jaworski underscores “the importance of collaborative structures . . . to provide support through collegial interaction” (p. 5). She suggests teachers found the project valuable since they volunteered to continue the project through the second year even though it was initially planned only for one year. Most importantly, “teachers’ independent activity was subjected to social scrutiny in an environment which encouraged and stimulated thought and action” (p. 28). Notably, however, teachers in this study spent a considerable amount of time both engaging in their projects and attending regular meetings over the two years of the project, and research funds were made available to cover their costs. The presence of experts in the field was also available. And, Jaworski notes that “the teachers would have been unlikely to engage with and sustain research without the motivation of the project” (p. 28).

Another strand of research where teachers are treated as active agents in studying their practice is in lesson study (Stigler & Hiebert, 1999, 2000), which can be considered a form of action research (Lewis, Perry, & Friedkin, 2009). Lesson study originates from a process for teacher collaboration around lessons in Japan. The Japanese version of lesson study is very structured and consists of “small groups of teachers [who] meet regularly, once a week for several hours, to collaboratively plan, implement, evaluate, and revise lessons” (Stigler & Hiebert, 2000, p. 10). These groups of teachers are supported in their endeavors with time that is specifically allotted for such work. It allows them to discuss lessons they are dissatisfied with by reading about what other teachers do for those lessons and what educational recommendations are on teaching those particular topics. They then revisit their lessons and try them in different ways, taking turns observing each other’s implementations. This sort of immersive and iterative practice is often considered to be unsustainable in the North American schooling model,
but nonetheless, several researchers have attempted to alter it so it can become a feasible form of teacher professional development (Fernandez & Yoshida, 2004; Hart, Alston, & Murata, 2011; Lewis, Perry, & Hurd, 2009). Many of these attempts have been successful, but they all depend on an availability of creating time for regular meetings and providing groups with training on how to approach collaboration according to the lesson study model. Lewis et al. (2009) provide an example of a North American lesson study case that proved effective for mathematics teachers’ knowledge and beliefs, their professional community, and their teaching-learning resources. Teachers were guided by mathematics teacher-coaches during a 2-week summer workshop in using four key lesson study features: investigating a lesson, planning a lesson, researching a lesson, and reflecting on a lesson. The lesson study groups consisted of a variety of schoolteachers, one of whom was a part-time mathematics coach. This mathematics coach attended 15-20 foundation-sponsored professional learning days per year. Lewis et al. note that although this is a successful example of lesson study, it is difficult to generalize their results more globally due to the unique nature of their study site. As with Jaworski’s (1998) *Mathematics Teacher Enquiry* project, Lewis et al.’s instantiation was also supported by collaborative structures that promoted reflective practice as well as the presence of academic coaches or researchers who initiated and organized the project. Such supports inevitably contributed to the projects’ successes.

Effective efforts towards occasioning mathematics teacher collaboration have also centered on initiating teacher study groups (Jones, 1997; Meyer et al., 1998; Short, Glorgis, & Pritchard, 1993), which are related to action research and lesson study models, but are not nearly as structured. Arbaugh (2003) defines a teacher study group as “a group of educators who come together on a regular basis to support each other as they work collaboratively to both develop professionally and to change their practice” (p. 141). In her case, she took part as a participant, facilitator, and researcher in a study group of seven secondary geometry teachers. Their school had received a *Toyota Time Grant*, which was sponsored by the NCTM (National Council of Teachers of Mathematics), in order to support the school in developing and implementing a more inquiry-based geometry curriculum. This means teachers were supported with release time for using school hours and stipends for participating in the project during out-of-school hours. They met ten times over the course of a year for between 1.5 and 6.5 hours each time depending on how much school time they could use with the funds
provided. Arbaugh was contacted by the school’s math department chair to help design and implement the professional development experience. Their main activity consisted of “discussing task selection for their classrooms, and then reflecting on implementation of those tasks” (p. 151). She asked probing questions, challenged teachers’ ideas verbally, and found articles for teachers to read between meetings to stimulate discussion. She found teachers had mixed feelings about being in the group and were drawn towards quitting because of all the other activities they were expected to involve in. Regardless of the hesitation of several teachers, they pursued engagement in the group. Overall, Arbaugh claims that teachers valued participation in the group because it “provided an opportunity and support for them to build community and relationships, make connections across theory and practice, engage in curriculum reform, and increase their sense of professionalism” (p. 154). In addition, they claimed the release time provided was very important, and many would likely not participate without it. It is clear that teachers lead busy lives, and the tension between professional space and personal space comes into play when aiming to occasion teacher learning.

The scenarios of teacher interaction presented above are unique instantiations of successful teacher learning initiatives that supported teachers’ reflective practice and collaboration in which they could develop new meanings and ideas around mathematics teaching. Without such well-designed immersive initiatives, the teachers in these cases would likely continue working in isolated conditions with rare opportunities for collegial interaction, as they do in the majority of schools. Unfortunately, mathematics teachers’ contact with other mathematics teachers is typically limited to that of department meetings and teacher lunchrooms, where teachers usually plan lessons and grade papers individually (Arbaugh, 2003). One evident reason for the lack of a more widespread implementation of such inquiry-oriented collaborative in-service teacher support systems is a lack of resources such as time, funding, and facilitation. Teachers lead busy lives and are often struggling to achieve a work-life balance. Also, school districts generally cannot afford to offer course releases and stipends, and researchers and teacher coaches are low in numbers compared to the number of practicing teachers. The supports that these successful initiatives therefore involved, were high. In Jaworski’s (1998) study, “research funds paid for teachers to come out of school to attend meetings” and researchers “organised regular meetings – two in each of the three terms of the school year – at the university for all researchers” (p. 11). In Lewis et al.’s (2009)
study, a foundation-sponsored mathematics teacher coach co-founded the lesson study initiative in collaboration with Japanese lesson study teachers, and the North American teachers participated in a 2-week summer workshop that was partially subsidized. In Arbaugh’s (2003) study, teachers were provided with release time and overtime stipends due to a unique one-time funding opportunity. As such, time, funds, and facilitation were necessary to provide support for successful collaborative and reflective teacher practice. This means that professional learning models such as those described above, albeit successful, are not necessarily sustainable over time and cannot be maintained unequivocally in all schools. As such, the question of how to support continuous teacher learning opportunities sustainably proves attractive, which has two possible responses. One is to look at what teachers are doing in schools informally to support their learning, and the other is to consider the affordances of technology in this endeavour.

In terms of informal learning around mathematics teaching, limited research exists. However, to address this issue, Horn (2005) takes on a situated perspective to pursue the question of what sorts of opportunities for teacher learning occur in schools that do not have access to organized professional development initiatives. She looks at the situated and informal teacher learning that occurs within two different reforming high schools through the framework of communities of practice (Wenger, 1998). She emphasizes that Wenger warns against using the term ‘community’ as being associated with a positive valence, but rather to take community as a more neutral notion where the focus is on what interactions are taking place. She finds that although ongoing teacher learning was prevalent in both schools, the schools differed in how teachers interacted, and therefore, in the nature of the learning that occurred. She argues that the school with processes that supported teacher reflection, exchange of materials, and collaboration, evidenced transformative learning for teachers (Mezirow, 1991). This means they were more prone to challenge existing assumptions and change their practices more significantly. In contrast, the other school limited teacher interaction to surface level processes. Instead of deeper reflective activity, their interactions involved the use of simple slogans for pedagogical support, such as “less is more” and “group-worthy”, and meanings were not typically challenged or negotiated. In turn, the nature of teacher learning was different at both of the schools. Consequently, Horn (2005) stresses a need for further exploration of informal teacher learning settings and the processes that occur in these settings since they can be disparate. She claims that “by
recognizing the common resources for everyday learning within groups of teachers, we can begin to leverage these for more effective and ecologically valid forms of professional development" (Horn, 2005, p. 230). Ultimately, school culture plays a large role in the nature of learning that teachers may be drawn to. Since school cultures can differ greatly, it is possible for teachers who feel isolated in their schools to be marginalized from collaborative learning opportunities.

An alluring solution to bypassing teacher isolation is often found in technology. In recent years, virtual opportunities for teacher learning around mathematics teaching have been investigated, such as those found in online courses in mathematics education master’s programs (Lee, Chauvot, Plankis, Vowell, & Culpepper, 2011; Makri & Kynigos, 2007), online courses in mathematics teacher education programs (Silverman & Clay, 2010), and online collaborative environments designed for practicing teachers (Chieu, Herbst, & Weiss, 2011; Chinnappan, 2006; Stein, 2009). Most of these have remained within closed online networks with minimal numbers of teachers and that hinge on either teacher educator or researcher guidance and support. Nonetheless, such environments have shown some success with fostering teacher collaboration and reflective practice. For example, Silverman and Clay (2010) find that the highly structured online asynchronous environment they designed for a mathematics teacher education course accomplished a safe collaborative space for pre-service teachers to think and question mathematical ideas. To achieve this, they alternated between periods of individual pre-service teacher activity and collaborative pre-service teacher activity within the online environment. This was done in the hopes of giving the pre-service teachers space to think individually on assigned problems in a reflective manner, and then prompting collaboration in relation to problems and problem-solving experiences encountered.

Open online communities have also been fostered around certain objectives, such as developing materials or tasks. For instance, the EdUmatics Project (Aldon, Barzel, Clark-Wilson, & Robutti, 2013; “EdUmatics,” n.d.) offered a platform that supported secondary mathematics teachers in the use and integration of technology within their classrooms. Another long-standing effort at establishing an open online learning community around mathematics teaching was that of The Math Forum, which has a long and complicated history (Shumar, 2017), but that eventually involved both mathematics education researchers and teachers in developing tasks and resources (Renninger & Shumar, 2002). In particular, their EnCoMPASS project (Fricchione, Silverman, Roberts, Shumar,
& Klein, 2013; Klein, Shumar, & Silverman, 2014; Shumar, n.d.), funded by the National Science Foundation, aimed to develop an online community of mathematics teachers and to support them in honing skills around analyzing student thinking. While such initiatives have revealed a variety of benefits around building community and developing resources for teaching, they ultimately depend on funding sources to exist.

The key benefit that such virtual environments offer is that of asynchronous communication, which mitigates the problem of teachers lacking time for professional learning. However, learning still takes time whether or not it is asynchronous, and therefore, the technology only partially addresses the problems related with teachers’ busy teaching schedules. The problems of funding and facilitation also still exist. Because the virtual learning environments mentioned above require teacher educator or researcher facilitation, initiatives come with a financial cost. This cost is either covered by teachers through masters or teacher education program tuition fees, or by districts or grants. Therefore, it remains unclear how to best support a generative and sustainable model of teacher professional learning that maintains opportunities of teacher collaboration and reflective practice while mitigating the obstacles of time, funds, and facilitation. As such, there is a gap in the opportunities available for supporting sustainable and meaningful teacher collaboration around mathematics teaching that help teachers manage the competing demands found within practice.

2.2 Social media activity

Given the challenges of creating contexts that support all the features necessary for successful professional learning activity, social media venues have become globally recognized as attractive and plausible sites for professional learning in a variety of fields. While research on social media and the natural activity that occurs with it are vast, studies that focus on social media used specifically for discussing and connecting about mathematics teaching remain sparse. Within education literature, most social media studies tend to treat social media and blogs as tools for enhancing structured settings either where students learn subject-specific topics, such as mathematics (e.g., Andrà, Brunetto, & Repossi, 2018; Engelbrecht & Harding, 2005; Jones, Geraniou, & Tiropanis, 2013; MacBride & Luehmann, 2008), or where preservice teachers learn about teaching through reflecting on practice experiences (e.g., Chan & Ridgway, 2005; Hramiak, Boulton, & Irwin, 2009; Makri & Kynigos, 2007; Wopereis, Sloep, & Poortman, 2010;
Yang, 2009). In these cases, attention is often placed on the effectiveness of the technological tools for fostering mandated activity within closed settings with a limited number of participants, and many of these cases continue to require time, funding, and facilitation. Their closed nature also cannot be considered representative of the more natural and unprompted features found in MTBoS, which situates in an open, unconstrained, informal and autonomously driven social media environment. Instead, MTBoS is more aligned with an open and informal professional learning network (PLN) structure that allows for teacher interaction. As Rehm and Notten (2016) note, “informal learning networks can contribute to capacity building of teachers by providing a platform to engage into a collaborative exchange of insights and experiences” (p. 215). And, social media allow for such “teacher driven activities [that] tend to be more effective than top-down interventions” (p. 216). While there is growing interest in the affordances of social media use by teachers towards professional development, it is worth bearing in mind the findings from studies that explore the capacities of social media more generally to account for natural activity it can support. Therefore, my review of scholarly work around social media focuses first on informal social media settings more generally, then on informal social media use within professional educator circles, and finally, on the few studies that have begun to explore MTBoS specifically. Although there are a variety of forms of informal self-publication and micro-publishing through various social media technologies, I focus on social networking sites with specific attention to Twitter and blog pages, since these are the tools most prevalently used for communication in MTBoS.

Very broadly, social media sites offer broadcasting access to anyone with an Internet connection, with limited barriers. No longer do individuals need to go through publishing companies to share their musings, they simply publish them directly online, where they can hypothetically be read by anyone with an Internet connection. This has resulted in a personalization of politics (Bennett, 2012), which allows for “large-scale individualized collective action that is often coordinated through digital media technologies” (p. 20). That is, micro-publishing has allowed for the mobilization of power among individuals who congregate around certain issues, often producing strong political effects, which sometimes result in and provoke organized activity, such as protests. Although bottom-up social movements existed before such technological advancements, easy access to social networking sites has greatly increased the capacity for such movements to occur, which can spread like wildfire. Content that 'goes viral' in online settings is not only
produced by media companies, but also by individuals themselves. Given the influential power some of these movements have had on the body politic and directing individual activity, marketing researchers have worked on determining factors involved in viral publications (e.g., Berger, 2013; Berger & Milkman, 2012; Kaplan & Haenlein, 2011), which may give insight into influential ideas within educator networks.

To this end, Kaplan and Haenlein (2011) identify that in order for a viral epidemic to occur, “three basic criteria must be met: the right people need to get the right message under the right circumstances” (p. 256). They further outline constraints for each of these factors. First, the message needs to be memorable and interesting. For this, they use a medical analogy to illustrate the difference between infections that affect a small number of people and die out quickly, versus those that create longer lasting epidemics. They specify that for a message to be more significantly infectious, it needs to create an emotional response in readers, which typically involves an element of surprise combined with either positive or negative emotional triggers (also found by Dobele, Lindgreen, Beverland, Vanhamme, and van Wijk (2007)). Along this vein, Berger (2013) stipulates that it is not only the emotion that needs to be triggered for transmission to increase, that emotion needs to be a high-arousal emotion (detailed more empirically in Berger (2011), and Berger and Milkman (2012)). For instance, awe, excitement and amusement are more highly arousing than contentment, and anger and anxiety are more highly arousing than sadness. Although positive emotions are typically more share-worthy than negative ones, Berger (2013) finds it is more about the intensity of the emotion evoked than the specific emotion itself. Further, he includes five other aspects to qualities in a message that make it contagious. Aside from evoking high-arousal emotions, he claims the message needs to involve social currency in that it makes people look good if they share it, and practical value in that it both fills an information-gap (Loewenstein, 1994) and stimulates curiosity. Berger outlines that the message should also incorporate stories that are integral with the message or product, triggers that link to peoples’ daily lives, and be publicly visible enough so others can imitate it.

Although Berger argues that the qualities in the message are incredibly important for transmission, Kaplan and Haenlein (2011), as well as Gladwell (2002) more seminally, highlight the importance of the messengers who carry the messages in a network. They both refer to the Pareto principle (Koch, 2001), which states that roughly 80% of the effects come from 20% of the causes, therefore implying that about 80% of the infectious
messages come from 20% of the messengers. Therefore, who receives the initial message matters greatly. Kaplan and Haenlein (2011), using Gladwell’s (2002) terminology, identify three groups of messengers: market mavens who are the first to access messages, social hubs that are tightly knit with hundreds of connections, and salesmen who have more convincing power. Each of these groups are important because market mavens actively diffuse ideas they find to their short-range connections, social hubs transmit ideas among themselves efficiently, and salesmen carry the messages and amplify them with their social status. In this way, messages are found, passed on, and advocated for. When this sort of synergy occurs, the idea can go viral.

However, the qualities in the message and the kinds of messengers it gets passed on to are not necessarily sufficient for an idea to go viral, the right environment is also needed. Kaplan and Haenlein indicate that the environment must occasion appropriate timing, which essentially involves luck, but also, an appropriate initial seeding population. They suggest the initial message needs to be sent to a diverse array of people in various subcultures or social hubs so that it has greater reach and that no more than 150 of these messages are initially sent. This is because most typically, “messengers will only pass on the message when they think it’s not already something everyone knows about” (p. 257). And 150, referred to as Dunbar’s (1992) number, is found to be the rough threshold for the number of people a person may feel socially connected to. As such, Kaplan and Haenlein suggest that “instead of concentrating on having as many seeds as possible, firms should instead focus on having an infectious message . . . and seeding it to many disconnected subcultures” (p. 257). Therefore, they indicate that social media platforms, which offer space for various subcultures to co-habit in it simultaneously, are “particularly well-suited to kick off viral marketing phenomena” (p. 257).

More specifically to the Twitter platform, Rieder (2012) reveals how such factors can prevail naturally, contributing to not only high diffusion of information in the network, but also refraction of the information. To this end, he studies a particular sphere of French and French-language Twitter users who take interest and post about political topics and current events. Taking a very large sample of data produced over a three-month period, with over 300K users and over five million tweets, he investigates the topics that were tweeted about by this group of people over time. Topics included significant time-sensitive events such as an earthquake off the coast of Japan, as well as more drawn-out issues such as negotiations around Internet privacy policies. One of the more time-
sensitive topics was a response to breaking news of “Dior’s chief designer John Galliano [who was] arrested by police after an anti-Semitic rant in a Paris bar [and who was] subsequently first suspended and then fired by his employer” (para. 12). Although the case at first involved tweets typical of responding to such situations, with the most popular tweets invoking some sort of humour or jest around the issue, the swift dismissal of Galliano caused what Rieder initially calls, a subject drift. That is, this smaller event triggered public attention to more global issues. In this case, these more global issues pertained to racism and the accountability of public figures towards racist actions. Since other public figures did not get dealt with as Galliano did for making racist remarks, the public sphere ‘shifted’ to the more global issues this represented.

From this, Rieder suggests that not only does information become diffused in spaces such as Twitter by replication of content, it also gets refracted, as information passes on heterogeneously rather than homogeneously. When information is shared on Twitter, it is often presented through a personal perspective, and as he reveals, the more popular tweets are often those that are phrased more personally and that add to the conversation in some way rather than reiterating the facts.

We find that neutral “reporting”, in the form of merely relaying or linking factual accounts without commentary, is the exception rather than the norm. The most successful tweets are most often those that add a “twist” to the topic and “spin” it in a certain way, i.e., that “refract” it. (para. 36)

Therefore, topics and ideas can become skewed over time as they are presented by various users in different ways, which challenges dominant views of Twitter as a place for information diffusion directly along network ties (as rooted in social network analysis). Rieder also distinguishes the nature of Twitter messages from being considered as a diffraction of information. He argues that while diffusion implies linear movement and diffraction implies outward movement, refraction implies a shift in direction of movement, and that therefore, refraction is a more appropriate metaphor to describe the nature of Twitter messages over time. This is because he finds that messages around a particular topic on Twitter slowly shift in meaning and are passed on in a way that is somewhere “between identical reproduction and total heterogeneity” (para. 34) due to the personal nature of messages. As such, he nuances the ideas of Hermida (2010) around Twitter’s tendency to create a fragmented news experience, in which disparate information is diffused through the network in chaotic ways. Rather, by highlighting how global
snapshots of tweeted content by a group of users shifts over time, he reveals that topics not only replicate in a network, but also transform in meaning and relevance over time.

In this way, Rieder also nuances the commonly discussed notion of Twitter as an echo chamber, as proposed by Sunstein (2001), who posits that users only follow those who they take interest in, and therefore, only see a small range of ideas that remain rooted in their own belief systems, which results in ideological self-segregation. In an echo chamber, there is great homophily among members, and diversities among them are limited (Mcpherson, Smith-lovin, & Cook, 2001). In turn, this reduces opportunity for discussion and confrontation since members share views and generally agree with each other. For instance, in examining political networks on Twitter, Colleoni, Rozza, and Arvidsson (2014) find that there tends to be a significant amount of homophily among members who are identified as either Democrats or Republicans, with the homophily being especially strong among Democrats. However, Colleoni et al. argue that there should be more diversity available within Twitter networks to maintain the integrity of the public sphere, which should allow for “public dialog and reasoning through the advancement of claims and information that lead to deliberation” (p. 318).

This is similar to Kilpatrick's (2013) call for more critical ‘foxes’ in the field of mathematics education, alluding to how many mathematics education researchers tend to remain within their ‘hedgehog’ views of dominant theories and worldviews rather than weaving between various views, like a fox. In this way, Kilpatrick calls for an increase in engagement not only in producing research, but also for more friendly critique among researchers towards fostering critical self-awareness within the field of mathematics education itself. Public dialogue that not only supports, but also critiques can move ideas to evolve. As such, even though networks on Twitter tend towards political homophily, topics still emerge and transform over time as members respond to each other in critical ways. Therefore, Rieder suggests that rather than referring to Twitter as an echo chamber, it can be considered a refraction chamber, which emphasizes that ideas can change over time even within a tight network of users given they have enough diversity. He accentuates this nuanced shift in view, claiming, “instead of merely being exposed to like-mindedness, we consider that the users are the driving force behind the production of shared values and understandings” (para. 46). To this end, activity on Twitter does not merely involve diffusion of information, it creates possibilities for pockets of ideational growth, where ideas can percolate and transform.
As Rieder underscores, Twitter can be conceived of as offering a space for ambient journalism through its nature as an “asynchronous, lightweight and always-on system [that enables] citizens to maintain a mental model of news and events around them, giving rise to awareness systems” (Hermida, 2010, p. 297). That is, even when users do not actively interact through publishing, and only participate through reading (or lurking), they develop a piecemeal awareness of currently prevalent content in the spheres they attend to; the ambient journalism becomes part of their local context. As Hermida highlights, “the value [in tweets] does not lie in each individual fragment of news and information, but rather in the mental portrait created by a number of messages over a period of time” (p. 301). Over time, this sort of ambient journalism can contribute to a sense of not only which content matters in our current sphere, but also who is listening when we consider engaging publicly. This notion of a “mental conceptualization of the people with whom we are communicating” (Litt, 2012, p. 331) has been referred to and conceptualized as an imagined audience (e.g., boyd, 2010; Brake, 2012; Litt, 2012; Marwick & boyd, 2010), and has been used within a variety of publishing contexts. As users engage in reading and publishing content in a particular sphere, they begin to build an awareness of who they are writing to, which in turn, may direct their activity.

While this is the case in a variety of different publishing contexts, an imagined audience within the Twitter context is also determined through not only viewing the published content of others in one’s sphere, but also through the immediate responses and interactivity one can receive. Over time, an awareness of not only others in the sphere develops, but also of those who are likely to respond. However, although an imagined audience is emergent through interacting in the sphere, one’s imagined audience may be misaligned with an actual audience, which can lead to unintended consequences (Litt, 2012). Since much of social media sharing involves public persona development that involves managing impressions, users tend to imagine their audience by their most sensitive members such as parents, partners, or bosses, which may limit personal discourse on Twitter and stifle vulnerable ideas (Marwick & boyd, 2010). This has to do with the context collapse within social media settings where multiple identities are traversed, and a user mitigates the diversities among the contexts in which they live.

Further, an imagined audience may move beyond that of merely a networked public that one perceives as their audience, to that of an imagined community, which involves the perceived community one feels connected to. The notion of an imagined community is
originally introduced by Anderson (1983) who formulates it to describe our imagination of the existence of a nation. However, more recently, it has been extended by Gruzd, Wellman, and Takhteyev (2011) to that of imagined communities particularly within Twitter. Gruzd et al. indicate that the asymmetric nature of the Twitter microblogging service contributes to opportunities for feeling connected to a community without any physical aspects or even interactions. Although such a connection can be available in other settings such as within writer circles or topic-specific print media publications, Twitter offers specific features that afford new kinds of interactions. For instance, the unprecedented efficiency of communication on Twitter and the uniquely brief nature of publication creates possibilities for novel community structures. Nonetheless, Gruzd et al. focus on highlighting the similarities between other contexts in which imagined communities may exist. To this end, they involve not only Anderson's imagined communities, but also concepts from Jones' (1997) virtual settlement and McMillan and Chavis' (1986) sense of community. According to Anderson's writing, an imagined community requires users to engage in using common language, that their activity has temporality and happens over time, and that there are certain high centres of influence in the social network. As Jones (1997) writes, virtual settlement requires interactivity, more than two communicators, a common-public-place for users to meet and interact, and sustained membership over time. As McMillan and Chavis writes, a sense of community can occur when users have membership in the community, influence on the community, an integration and fulfillment of needs, and shared emotional connections.

Towards a synthesis of these ideas as relevant to Twitter activity, Gruzd et al. use them to consider how “Twitter can sustain and provide grounds for development of an online community that is not simply imagined by each user but that is built on the shared sense of community” (p. 1298). By examining the network data of one user and messages of that user in light of the aforementioned concepts, they find the user’s network to be both real and imagined in that many reciprocated relationships formed the basis of ongoing interactions, which were real, but that many messages were also sent on the basis of an imagined sense of community. They conclude by highlighting that a ‘community’ on Twitter is both collective in a broad sense, and necessarily personal in the local sense. Although each user has an individual view on activity in the space, the activity seems to imply various “norms, language, techniques, and governing structures” (p. 1312).
As such, it is interesting to consider not only the collective layers formed through Twitter activity, but also how and why individuals choose to participate and what affordances they can experience, particularly for educators who use Twitter for professional activity as is most pertinent to the study presented in this thesis. In general, Twitter is prone to a sort of sharing economy (Hamari & Ukkonen, 2016), a give and take exchange of ideas and resources through the opportunity for collaborative consumption. For educators, this means there can be collaborative consumption around ideas and resources for teaching practice. Some have begun to explore this sort of informal activity that educators are pursuing through either Twitter and blogs (e.g., Carpenter & Krutka, 2014; Forte, Humphreys, & Park, 2012; Luehmann, 2008a, 2008b; MacBride & Luehmann, 2008; Rehm & Notten, 2016; Trust, Krutka, & Carpenter, 2016) or other informal online forums (e.g., Duncan-Howell, 2009, 2010; Hur & Brush, 2009; Macia & Garcia, 2016). This work typically tends to focus on how and why educators use these sites and sometimes, what it can afford them in developing professionally and in their practices. What is clear in this work is that educators use Twitter and blogs in vastly different ways.

Almost every study that explores informal educator engagement in online social media or forum contexts indicates the range of ways available for participation. This includes that of lurking, as well as more active forms of participation, such as commenting and creating new content. Maci and Garcia (2016) point to how although lurking is
considered a form of peripheral participation, that this is an incredibly common and important form of participation in informal online networks. They refer to Seo and Han (2013), who find that “only 1% of the community under study uploaded the materials that were used by the rest of the teachers [and that] these teachers tried to justify their lack of participation through referring to busy schedules, time constraints or lack of expertise” (Maci & García, 2016, p. 301). This sort of pattern is also seen in other cases. For instance, in a six-week action research study with eight teachers in the same elementary school who learned to use Twitter to support their professional learning networks, Deyamport (2013) indicates that only three of these teachers posted 96% of the group’s tweets. He uses interview data to identify common characteristics among these active contributors and identifies that they were generally open to new experiences, had social personalities, seemed willing to share personal material, and their teaching subjects were well established within Twitter. Interestingly, while most lurked, all participants indicated Twitter was an effective platform for developing their professional learning networks, 88% of them indicated they would continue using Twitter for professional learning, and 63% noted it had a positive effect on their classroom practice since they found resources or practices they could implement.

Hur and Brush (2009) further explain that the possibility for anonymity in engaging in self-generated online teacher communities allows for an increase in teachers’ self-esteem and confidence, particularly when this anonymity allows them to share emotions and be exposed to the emotions of other teachers. And, Buechel and Berger (2018) find, microblogging, through offering the possibility of undirected communication, is particularly valued by those who may feel socially apprehensive and who may not want to impose communication on anyone specific. Therefore, social apprehension is one reason why social media sites may be attractive over other in-person settings, and to some extent can contribute to participation through lurking, which is often a predominant mode of participation in online social media settings. Although lurking is sometimes viewed negatively since it does not advance the production of materials in the shared space (e.g., Smith & Kollock, 1999), it has recently become more accepted as a valid form of participation. For instance, Preece, Nonnecke, and Andrews (2004) find that 30% of their survey respondents indicated they were lurking because they were still learning about the group and 54% of them indicated that browsing gives them what they are looking for. And, lurking is often a form of legitimate peripheral participation that can
allow entry into more active participation. In one study, Lai and Chen (2014) compare knowledge-sharing intentions between posters and lurkers. They find factors that motivate behaviour of posters as opposed to lurkers, which implicate ways in which lurkers can be lured into posting. The related factors in differences in engagement they find include reciprocity, enjoyment of helping others, knowledge self-efficacy, perceived moderator’s enthusiasm, offline activity, and overall enjoyability of knowledge-sharing. Therefore, by feeling the need to reciprocate contributions that have helped them, being aroused by a moderator’s enthusiasm, gaining self-efficacy in their knowledge, or engaging in offline activity that gives them a better sense of who they are posting for, lurkers may become attracted to posting. While informal online knowledge-sharing communities need posters, lurkers are important and are growingly being accepted.

Beyond lurking, however, the posters who contribute to an informal online knowledge-sharing space can still engage in vastly different ways and can be motivated by different needs. According to Carpenter and Krutka (2014), 96% of their respondents (out of 755 K-16 educators using Twitter) indicated they both shared and acquired educational resources via Twitter. Their respondents expressed great enthusiasm about professional development they experienced through Twitter and indicated that they felt it helped them combat teacher isolation through finding a community to glean ideas from, and to contribute to. Forte et al. (2012) find similar results through triangulating survey data with in-depth interviews and content analysis from data produced by educators involved in the Twitter hashtags #edchat and #edtech. Through this study, they add further nuance to the self-reports of educators who claim Twitter is an incredible source of professional development. Namely, they find that engaging in such hashtags allows educators to import new ideas into their local contexts of practice, giving them social capital in their local communities, and strengthening their self-efficacy in advocating for and sometimes even implementing reform-oriented practices they encounter on Twitter. Carpenter and Krutka (2014) indicate, though, that it is yet unclear whether their participants’ claims of Twitter engagement affecting their practice indeed influenced changes in their classrooms or not. Although it may not be the case that teachers adopt all strategies encountered on Twitter or change practice significantly, at minimum, they develop a social capacity to act as a bridge between global efforts at educational reform and local educational contexts. As Forte et al. argue, this sort of bridging can be fundamental in creating collective action that is more powerful than fragmented activity. They use the
notion of social capital to emphasize the effects that participating in Twitter networks (such as those within #edchat and #edtech) have on educators beyond that of practice-related engagement. They note that teachers view Twitter as “a source of new ideas and a way of keeping abreast of educational technologies in particular” (p. 110). While the effects of their engagements may not necessarily directly affect practice, their interest in participation involves developing an awareness of the tools that others are using to enhance practice. This, in turn, can contribute to a sort of enriched ‘example space’ of teaching ideas encountered, making it more likely for them to creep into practice.

Interestingly, 64% of posts tagged with #edchat in Forte et al.’s study were also tagged with #mathchat, indicating the strong tendency of education related discourse on Twitter to be centered around mathematics teaching. And, resource sharing in their study remained at the forefront of all activity, with 54% of posts analyzed from the hashtag #edchat involving resource sharing. The evident tendency towards resource sharing in education related activity on Twitter is supported by other studies as well (e.g., boyd, Golder, & Lotan, 2010; Carpenter, 2015). Therefore, resource sharing is clearly a central form of activity in education-related professional social media activity.

Although Twitter is often used as a venue for sharing details of one’s personal thoughts and daily activities, teachers are using Twitter as a place to share resources and to make and respond to others’ requests for information. (Forte et al., 2012, p. 110)

Beyond resource-sharing, however, the activity of educators participating on Twitter have also been identified as involving responding to others and requesting information from others (Forte et al., 2012), as well as more specifically to MTBoS, mentoring (Parrish, 2016a). While Naaman et al. (2010) indicate that 80% of general Twitter users engage in meforming, which refers to the practice of self-sharing in a self-indulgent and self-promotive manner, and only 20% engage in informing, which refers to information sharing, these proportions are different for education-related activity. As Forte et al. reveal, only 2.5% of the educator accounts they studied involved meforming, and instead, most instead focused on informing. This exposes the knowledge-oriented nature of educators using Twitter as opposed to the more general uses of Twitter. Considering Naaman et al.’s finding that those who engage in information sharing on Twitter are generally more prone to being embedded in social activity and interaction, this indicates that knowledge-oriented activity on Twitter moves beyond the original intentions of the
Twitter platform. That is, not only are ideas being shared as updates, they are also being discussed, negotiated, and built on. While Twitter originally designed their platform for simple personal updates, users have shifted the use of the platform towards creative and collaborative efforts, and the platform has more recently been finding ways to further accommodate a focus on content rather than personal popularity (Dorsey, 2019). As is becoming evident, Twitter is supporting professional participation in informal online collectives, where agents can expose themselves to opportunities for resource exchange, developing social capital, and collaboration with like-minded colleagues. Not only can it allow teachers to share ideas they may not feel comfortable sharing in their respective local in-person contexts due to social apprehension (Buechel & Berger, 2018; Hur & Brush, 2009), they may also encounter opportunities for affirmation given the overall 'like-culture' (Burrow & Rainone, 2017) that prevails in social media contexts.

More broadly, education-related professional activity in online forums (Duncan-Howell, 2009, 2010; Macià & García, 2016) and blogs (Luehmann, 2008a, 2008b) have yielded indications of how participation in informal online communities can have an effect on classroom pedagogy (Duncan-Howell, 2009, 2010) and critical inquiry into practice (Macià & García, 2016). For instance, in Luehmann's (2008a, 2008b) investigation of one reform-oriented science teacher blogger over the course of one year, several affordances of blogging became evident. Blogging helped shape the professional identity development of this reform-oriented science teacher blogger as she wrote about her teaching-related experiences and growth over time. Through this work, various kinds of topics the teacher wrote about, the types of work they engaged in through blogging, and aspects of practice blogging helped them with are revealed. Luehmann's results indicate that the reform-oriented science teacher wrote about a variety of issues including those pertaining directly to the classroom, to personal matters as both an individual and a teacher, and more globally, to life as a professional both inside and outside the classroom. The types of work engaged in included community building activity such as sharing resources, mentoring, dialoguing, connecting, and encouraging, integrating personal matters such as through displaying competence, setting goals, sharing emotions, and critiquing oneself, as well as exploring new ideas and promoting perspectives. New ideas and perspectives can be brought forth through documenting as well as advocating for certain positions. Finally, her work also illuminates how blogging can support a teacher in more deeply exploring the richness in curriculum,
understanding and valuing student culture in her classroom, and increasing confidence as a teacher. Although blogs allow for a much more in-depth engagement than Twitter does due to the limitations within Twitter, these results are interesting to consider since they reveal the possibilities for teacher growth over time. MTBoS activity includes blogging as many blogs are shared via Twitter, bridging the media spaces together. Therefore, such affordances may be available as well to MTBoS participants.

Another study that also examined the affordances found among educators within social media settings in which they engaged deeply is a study conducted by Stephansen and Couldry (2014). Unlike Luehmann’s focus on a single teacher and her identity development through deep engagement in a blog, Stephansen and Couldry pursue an action research study in which they design a knowledge-producing community using Twitter by creating a departmental Twitter account for a humanities department within a college setting. In this way, they solicited engagement from a few hundred teachers and students who interacted in the same space. In their design of directing activity through the departmental Twitter account, they use a digital story circle approach of narrative exchange and knowledge production. Although in their interview data, some participants found it awkward to share personally in a public space particularly because both students and teachers were involved, others commented on how it helped the department form cohesion. Stephansen and Couldry indicate that Twitter platform helped break down conventional hierarchies and enabled agency among participants. More specifically, a ‘levelling of the playing field’ occurred where students and teachers were no longer bound to their respective roles as students or teachers, and the focus was instead, on knowledge production. This may be in part because the Twitter platform naturally emphasizes content over user details in how information displays. While it was directed by researchers, this study points to the possibilities available within a Twitter context for knowledge-production, agency-development, and community-engagement.

Although a variety of studies point to the significant opportunities available for educators in using informal digital media tools, little research focuses on the very prominent activity occurring specifically around mathematics teaching. Most studies involve either an interest in marketing implications, political influences, or broad educational outcomes, but few studies focus on the implications of such activity for the field of mathematics education, which is important given the high volume of activity around mathematics teaching evidently occurring in such spaces. Mathematics education researchers are
clearly interested in professional development opportunities for mathematics teachers, as is evident in the high volume of publications around mathematics teacher professional learning with particular interests in teacher collaboration (e.g., Brodie, 2014). But attention to informal activity around mathematics teaching is still sparse. Given the well-touted benefits of Twitter as a source of professional development for educators, and the desires of mathematics education researchers to uncover ways to more effectively deliver and address professional development initiatives, it is therefore promising to consider MTBoS and how it sustains professional activity around mathematics teaching. Amidst this current research landscape, MTBoS is evidently worthy of further attention.

2.3 MTBoS

In recent years, aside from informal references to MTBoS in teaching publications (e.g., Bourassa, 2017; Newel & Orton, 2018; Soto & Hargis, 2017), some researchers have taken interest in studying MTBoS empirically (Fesmire, 2016; Larsen, 2016; Larsen & Liljedahl, 2017; Larsen & Parrish, in press; Parrish, 2016b; Risser & Bottoms, 2018; Risser & Waddell, 2018; Waddell, 2019; Willet & Reimer, 2018). Overall, various approaches and perspectives have been taken. Some have focused only on activity accessible within content posted online within MTBoS, while others have looked at specific initiatives that bridge physical professional learning spaces with those available through MTBoS, aiming to identify what it offers as an extension. While this work is still sparse and often preliminary in nature, taken together, it showcases possibilities for what MTBoS offers in terms of professional activity around mathematics teaching.

In terms of content analysis on Twitter, the most comprehensive examination of the specific mathematics-oriented professional learning possibilities in MTBoS is conducted by Parrish (2016b), who explores how MTBoS supports the development of teacher attributes towards selecting and implementing cognitively demanding tasks. To this end, he takes tweets that include the MTBoS hashtag and all linked blog post material from two randomly selected weeks in the Fall of 2015. Out of this data, he randomly selects 75 tweets from each day to form a data set of 1048 tweets and 95 blog posts. He then qualitatively codes this data using an a priori framework based on Wilhelm’s (2014) claim that teachers who can select and implement cognitively demanding tasks must have sufficient mathematical knowledge for teaching (Hill et al., 2008), visions of high-quality math instruction (Munter, 2014), and views for supporting struggling students. In
addition, he also uses inductive coding to emerge further themes. Overall, Parrish (2016b) identifies that 40% of his data set evidenced mathematical knowledge for teaching in content, with just over a quarter of it being purely mathematical in nature. He indicates that sharing and co-planning resources for teaching were also prevalent in the data as well as sharing about how students may think about a given topic, which indicates the use of pedagogical content knowledge in engagements. Further, 46% of his data set involved visions for high quality mathematics instruction, particularly within classroom related discourse focused on teacher questions and student thinking, where the teacher’s role was that of a facilitator. Many of the engagements involved acknowledgement of students justifying their thinking. In addition, Parrish (2016b) also finds that 85% of mathematical tasks in his data set were observed as requiring high levels of cognitive demand. However, only 3% of his data revealed ideas around supporting struggling students. Finally, 12% of his data was community-related and did not involve any mathematics. As such, Parrish (2016b) claims the content produced within MTBoS that pertains to mathematics teaching is promising in terms of supporting effective implementation of cognitively demanding tasks.

To support this, Parrish (2016b) also conducts interviews with five randomly selected participants who engage in MTBoS with varying degrees of activity to explore how they perceive the possibilities for developing attributes towards selecting and implementing cognitively demanding tasks in MTBoS. From these interviews, it became evident that although the participants claimed MTBoS helped them develop mathematical knowledge for teaching, they tended to focus on classroom practices. When pressed to discuss the mathematical tasks they found within MTBoS, they emphasized that the tasks were “classroom ready, peer-recommended, crowd-sources, teacher-created, and teacher-adapted” (p. 194). One of the interviewees indicated they felt no change from participation in MTBoS other than gaining a ‘sense of vindication’ since his educational philosophies aligned with those in MTBoS. Most interviewees also emphasized the fine-tuning of classroom practices they learned from participation in MTBoS such as wait time, questioning strategies, and listening to student thinking, but did not indicate that it helped them support struggling students. However, they stressed the personal aspects of MTBoS community, alluding to the personal nature of interactions as incredibly important in their professional growth. That is, they generally pointed to the value of interacting with others who came across to them as like-minded. Taken together, Parrish
concludes that MTBoS offers possibilities for mathematics teachers to improve at selecting and implementing cognitively demanding tasks, with almost 85% of tasks discussed in his data set having the potential to elicit high levels of cognitive demand.

However, he indicates that more research is necessary to further explore the nature of interactions and other aspects of the community that participants referred to as being supportive of their professional growth. He claims that since he only considered individual tweets with the MTBoS hashtag, his study did not account for interactions within threads. He notes, “viewing all related tweets or comments within a community conversation would have been useful in better understanding how the community responded to or discussed topics within mathematics education” (p. 188). He also mentions he did not consider which content was responded to, how, and why. In his suggestions for further study, he asks, “what content of MTBoS community receives the greatest community response?” “Around what topics do interactions and discussions tend to occur?”, and “are some members of the community particularly influential?” (p. 188). It is thus evident that MTBoS is a rich phenomenon of interest and more inquiry is needed to examine why it is so sustainable and conducive to conversations about mathematical teaching practices which aim to support high cognitive demand and emphasize flexibility in student thinking.

To this end, some of my preliminary research has shed light on a few of these questions. Of highlight, in Larsen (2016), I explore one particular thread of dialogue between MTBoS members that began on Twitter and extended into the blog space. I use the construct of the duality between participation and reification from Wenger's (1998) communities of practice framework to explore how the social production of meaning within blogging and micro-blogging can be carried out. In the examined episode, three prominent MTBoS participants discussed a ‘pyramid of abstraction’ metaphor for thinking about designing and sequencing mathematical tasks within Twitter. Through the production of public reifications, further participation was prompted, and participants became accountable for their musings. Although exemplified through one particular case, I reveal in this paper that the blogging and micro-blogging mediums allow for participation and reification to be so closely intertwined that it drives the negotiation of meaning in a co-evolving and co-implicated way. Further, in this study, I note that the opportunities of asynchronicity, permanency, and publicity afforded by the blogging space allow for participation to continue over days or even months. And, it also allows
participation to be visible by lurkers, whose presence may make participants accountable for their reifications and may contribute to attracting more participants to converse. Through this work, I conclude that a more nuanced treatment of participation and reification is needed because they are so intertwined within the MTBoS context given that every participation forms a reification. While participation and reification are so simultaneous that they may be unnecessary as a framework, it is evident that this nature of the medium may be a factor in the continuity and sustainability of negotiation in it.

Subsequently, in Larsen and Liljedahl (2017), we explore a particularly well-responded tweet that asked about how others would respond to a particular student error in solving a quadratic equation. This initiating tweet resulted in 254 replies made by a total of 87 users, 52 of whom were identify as mathematics teachers. Within these replies, some threads were determined as continuing while others were non-continuing based on the depth of the threads. Given our interest in possibilities of emergent ideas within longer threads, we examined only the continuing threads. To this end, we use the notions of diversity and redundancy among agents, from Davis and Simmt (2003) and Davis and Sumara's (2006) complexity thinking framework to aim at understanding how complex emergence can occur within self-organized online communication between teachers of mathematics. By comparing and contrasting the redundancy and diversity found in various continuous threads, we conclude that the threads that precipitated complex emergence through the generation of new task materials designed to help students understand the error required redundancy between members to carry the conversation, but also diversity to produce novel material. That is, when there was not enough redundancy between conversing members, threads did not continue much. With much redundancy, members reached agreement quickly without producing novel ideas. However, when members had enough redundancy in terminology use and associated meanings, and also found differences in their interpretations, they began communicating their differences through the production of novel content. Although limited in data size, the analysis presented in this study gives a list of aspects that were redundant and diverse among members within conversations, which contributes to the more global effort of understanding what MTBoS is.

The consensual domain of the MTBoS includes patterns of interaction such as thinking like a learner, generating examples, invoking shared language, and using instructional routines, as well as being guided by pedagogical values related to teaching without telling and guiding students towards
However, it is also essential for there to be diversity around approaches and representations of mathematical ideas to allow for emergence of novel ideas for teaching and learning mathematics. (p. 136)

Therefore, our inquiry reveals the possibilities for complex emergence in MTBoS through various patterns of interaction and by the serendipitous encounters of members, who share redundancies and feel comfortable enough revealing their diversities to each other. The diversities then prompted further negotiation, and in turn, novel content. However, it is still unclear how much of this sort of activity actually happens within typical MTBoS activity, and how members achieve such redundancies and diversities. That is, how does the MTBoS form and how does it sustain activity on a larger scale? Although this remains unclear, our findings indicate the emergent potential within conversations.

Along this vein, Parrish (2016a) also identifies the possibilities for informal mentorship within MTBoS. By using the same data set as in Parrish (2016b), in Parrish (2016a), he examines the data for evidence of mentorship, in which participants offer details or advice about specific aspects of teaching mathematics. To this end, he identifies 22 out of 46 blog posts that evidence mentorship. This mentorship occurred either through giving advice for teaching specific students, making suggestions for teaching or resources, suggesting advice for goals or issues in teaching, revealing transparency in planning mathematical instruction, and making direct requests for mentoring. His work implies that mentorship in MTBoS can be either solicited or unsolicited and that it quite often involves vulnerability in sharing ideas. More recently, upon re-examining the interview data in this same data set, in Larsen and Parrish (in press), we further identify that members in MTBoS engage in “identifying and selecting resources frequently, that they value resources that are inspiring, relevant, and reliable, and that they establish values through identifying resources with attributes of specificity, like-mindedness, credibility, and through repeated exposure over time” (para. 1).

As such, MTBoS offers a rich space for engagement around mathematics teaching. However, the abovementioned studies only point to the most opportune settings. Such cases of productivity and quality in engagement around mathematics teaching may not always be prevalent in activity in MTBoS overall. That is, by looking at specifically successful instances, the findings only indicate possibilities for the affordances without adequately addressing limitations. To this end, some related studies have investigated
the nature of questions in MTBoS, how they get answered, and how frequently this happens. In Romkema (2017), several mathematics-related hashtags are explored to identify how mathematics-related questions are answered and how the friendship network supported response to these questions within tweets between March 2011 and June 2017. The findings suggest that the proportion of mathematics-related questions answered is very small, and that it is equally as likely to receive a response from a friend or a stranger. About 10% of their data set involved mathematics-related questions, and only 4% of these were answered. This points to the issue of not only what is posted, but also which questions received response and why.

Along this vein, in Larsen (2017b), I examine the 180 questions posted with the MTBoS hashtag within a two-week period in August 2017. Although my primary aim in this work is to qualitatively analyze the kinds of questions that were asked, a cursory examination of this data shows that 65% of these questions received very little response, and only 7% of them resulted in 10 or more replies, which aligns with Romkema's (2017) findings. However, I dig further into the nature of these questions towards identifying what teachers seek when approaching MTBoS. To this end, I coded tweets for topics that were asked about as well as for elements of Liljedahl's (2014) framework of teacher wants (originally developed within physical professional development settings). Through such analysis, I reveal that teachers in MTBoS were either “seeking affirmation, seeking connection, seeking pragmatism” or were “willing to entertain, willing to reorganize, [or] willing to rethink” ideas that they were asking about (Larsen, 2017b, p. 73). And, the topics these participants asked about included questions about “tools, class structure, curriculum, and class culture” (p. 73). Interestingly, 40% of the posts pertained to the mathematics curriculum and the other topics were around 20% of the data each. This indicates a strong interest in asking questions about the mathematics curriculum, either around the mathematics itself, or about how to sequence mathematical ideas. Further, I also pursue a cross-analysis between the overarching wants and the topics of interests. From this, I conclude that although teachers in this data were willing to rethinking classroom structure and culture, the most common way of engaging in using social media for professional growth in this case involved seeking “pragmatic resource exchanges pertaining to virtual or physical tools, ideas for class structure, and approaches to curriculum” (p. 73). These findings are consistent with other findings
about the prominence of resource exchange in educator social media settings, but they emphasize an overall pragmatist orientation.

While the abovementioned studies have focused on activity found strictly within the online site of MTBoS, other studies have also investigated the possibilities of MTBoS as a supplement to in-person interventions or events. In one such case, Fesmire (2016) designs and implements a professional development module aimed at supporting participants in using technology to connect with virtual colleagues towards helping them reflect on teaching practices and improving instruction. She grounds her analysis in the theoretical constructs of Lee's (2014) *virtual communities of practice*. To this end, she first surveys a random sample of about 100 MTBoS participants to examine the predictive factors in their social media use intentions around improving instruction. She finds that “performance expectancy, effort expectancy, and intrinsic motivation – specifically experienced competence – [were] predictive factors of math teachers' social media use intention” (p. 92), but that no significant predictors of social media use behaviour were identifiable. Based on these findings, she designs a three-day professional development program in which a small group of teachers were guided in learning how to engage in MTBoS. The first session focused on reading and commenting on math blogs in MTBoS, the second session focused on using MTBoS search engine to find resources efficiently, and the third focused on creating blogs and Twitter posts to connect with the MTBoS community towards improving instruction. Through this module, some observations were made indicating an increased sense of community and overall improvement in the teachers’ tendencies to be reflective and curious. However, Fesmire (2016) indicates the limitations of her study and calls for further investigation of affordances available in blending traditional professional development structures with the virtual community of the MTBoS.

In a different setting, Risser and Waddell (2018) and Waddell (2019) have explored the more natural unprompted backchannel activity occurring in relation to in-person conference experiences of participants. Namely, they explore the online activity related to the conference organized by MTBoS members, referred to as Twitter Math Camp¹¹ (TMC), which attracts a significant amount of backchannel tweeting. Unlike in Fesmire's (2016) study, their interest is not in directing activity, but rather, observing it as it occurs.

¹¹ Twitter Math Camp (TMC) and its origins are further outlined on pg. 93.
naturally. In Risser and Waddell’s (2018) work, they examine the conference
backchannel tweeting patterns of both #TMC15 (Twitter Math Camp 2015) and
#NCTMBoston (The NCTM conference in 2015), and include data from the six-month
periods that immediately followed the conference times. It is noteworthy that they take
interest in the backchannel after-effects resulting from the in-person conferences rather
than activity during the conference since this reveals the longevity and continuity that
Twitter offers. By coding the 176 tweets made by 110 users following #NCTMBoston and
the 455 tweets made by 144 users following #TMC15 for purposes, they aim to uncover
the differences in content as well as among conversation patterns between backchannel
activity that followed online after the two conferences took place.

Through this, they reveal a window into the kinds of relationships that can form after an
in-person conference that continue into the online space. By comparing the backchannel
tweeting patterns between the two conferences, they reveal nuanced differences, which
they attribute to how more explicit attention is given to forming relationships within TMC
functions than in those hosted by NCTM. The participant numbers are also different,
resulting in different opportunities for interaction, with NCTM generally attracting more
participants and TMC having a maximum number of 200 participants. This reflected in
their data as they were able to indicate more evidence of a sense of community
(McMillan & Chavis, 1986) among the TMC backchannel participants than among those
from NCTM. More specifically, TMC tweets tended to involve more replies and mentions
than those from NCTM, and more of the TMC tweets included linking to the work of
others, which indicates stronger opportunities for relationship formation. This finding
around the possibility for users to experience a sense of community is noteworthy and
aligns with research conducted by Risser and Bottoms (2018) on data from MTBoS in
2011, when it was first forming. In this work, they also indicate the possibility for users to
experience a sense of community in MTBoS. Further, Risser and Waddell find that
participants in TMC backchannel conversations often engaged in interactions that
moved beyond that of being strictly professional in nature, alluding to the importance of
not only professional interactions, but also personal ones. Although TMC revealed a
stronger tendency towards engendering a sense of community among participants, the
overall types of tweets remained similar between the two conferences, with about 30%
of tweets involving meforming, 60% involving informing, 7% involving info-requests, and
1% involving questioning in both cases. It is interesting to additionally note that Risser
and Waddell found only four tweets out of their 631 tweet data set that directly addressed mathematical content. This may be somewhat surprising given the conferences were designed for focusing on mathematics teaching, and seems to contrast the findings of Parrish (2016b) and Larsen (2017b).

In Waddell’s (2019) more recent work, he takes a similar approach at examining backchannel conference related activity, but instead investigates only the backchannel activity pertaining to #TMC17 (Twitter Math Camp 2017). In his data set, he includes content posted through 17 days before the conference, four days of the conference, and a month following the conference. He uses a mixed-methods approach in which he uses social network analysis to examine the hashtags and users included in his data set, and the theoretical framework of cultural-historical activity theory (CHAT) (Engeström, 2001) to analyze contents of tweets qualitatively. Interestingly, not only did participants of the physical conference engage in posting to the #TMC17 hashtag, but so did users who were ‘tuning’ in remotely to the activity at the conference through tweets. Both attendees and remote participants’ contributions were included in the data. Through both social network analysis and qualitative inquiry, Waddell identifies that the network of participants (through mentions, or replies directed at each other) formed a tight crowd in social network analysis terms, and parsed into 26 tight crowd clusters with clusters connecting around similar ideas. Remote participants were found to be just as well welcomed as attending participants. Further, he investigated what the human activity was of conference participation in #TMC17 using CHAT analysis, finding that individuals “worked together to form a community which taught the rules to new attendees by modeling behaviors” (p. 133). The rules he identifies include “gratitude for other individuals, being actively engaged in the community, and being willing to share time and knowledge with others” (p. 133). Further, tools such as the wiki site for sharing links, hashtags, and Tweetdeck helped make communication more efficient so participants could find information. And, in the examination of division of labor towards objects through community, he identifies “how the multiple types of sessions contributed to the forming of a sense of community and the objects of learning” (p. 134). However, since learning can only occur in an activity system with the presence of tensions (Engeström, 2001), Waddell indicates the tensions he identified within his data set as being conducive for creating opportunities for learning.
The first tension found was the concept of vulnerability. The participants were asked to engage actively in activities that created a sense of vulnerability through tweeting, reflecting, and responding to others. The second tension found in the data was the stress between practice and learned activity. The participants were learning processes, activities, and ideas that they could implement in their classroom, but there was stress between knowing and implementing. This stress was a tension that created opportunity for participants to learn. (p. 134)

This highlights the norms in the MTBoS community around learning. It also aligns with and extends some of the more preliminary findings presented in Larsen and Liljedahl (2017) around redundancies among patterns of interaction of MTBoS participants, and with notions I presented in my Fields MathEd Forum talk (Larsen, 2017a) around social network analysis approaches and the importance of a sense of belonging in MTBoS.

Taken together, it is evident that MTBoS offers a plethora of advantages that not only mimic, but also extend current possibilities for professional learning around mathematics teaching. It is clear that MTBoS serves as an example of an incredibly sustainable form of professional activity around mathematics teaching that seems to present not only professional activity, but also personal activity and a place for belonging to a community if mathematics practitioners. However, it remains a mystery how this sustainable activity forms and what processes are involved in sustaining it. As such, my aim in this study is to consider the natural activities found in MTBoS engagements and to ground my inquiry in concepts that pertain most closely to explaining what MTBoS is, how MTBoS becomes what it is, and what this implies for professional development initiatives. While MTBoS activity has been blended with in-person activity, I aim to focus only on the digital activity and what it reveals about the nature of professional learning about mathematics teaching itself. This sort of broader perspective of the online MTBoS activity as a whole is still evidently lacking and may be revealing of novel insights.

### 2.4 Gleaning from the literature

In considering the literature pertaining to mathematics teacher professional learning, social media activity, and more specifically, MTBoS, various findings influence my approach in this study. As such, in what follows, I identify the most significant sensitizing concepts gleaned from the literature presented in this chapter and identify how they guide me in my inquiry into the inner workings of MTBoS.
From literature around mathematics teacher professional learning, the most influential concept on my approach in this study is around the importance of teacher *agency*. That is, the autonomy teachers can take on in bridging the worlds between professional learning and practice through reflection-on-action and reflection-in-action. This sort of activity that is focused on teacher needs and attending to reflective practice is evident within the various approaches to professional development design that have proven effective (i.e., lesson study, communities of practice, and narrative inquiry study groups). From this research, it is evident that effective professional development is focused on activities relevant for teachers and that involves participation in a community in which they can *collaborate*. However, many of these physical settings depend on significant inputs in terms of time, funding and facilitation. Meanwhile, MTBoS seems to offer many of these attributes found to make professional learning settings successful, and it requires no such inputs. Therefore, the literature on mathematics teacher professional learning serves as a backdrop against which the results of this study may be compared to in efforts to identifying new approaches to professional development.

From the social media activity literature, particularly from that which studies social media implications for marketing and political movements, it is clear that social media offers an alluring solution for connecting mathematics teachers with each other across regional boundaries in ways that can be influential on them. The notions of information *diffusion*, and how ideas are not only diffused through Twitter, but also refracted in a sort of *refraction* chamber can be useful in viewing the ongoing and developing nature of MTBoS. Further, the findings around how high-arousal *emotions* impact take-up of messages are revealing because not only are publications made, some forms attract different forms of response than others. As such, it is interesting to consider not only the content of what is included in MTBoS posts, but also how it is presented and whether this affects how it is responded to. Finally, the notion of *ambient affiliation* in conceiving the collective as experienced by users in a partially imagined way points to how participating in a collective such as MTBoS may create a sense of community through its very presence and how users imagine the networked public in which they immerse.

Although several studies have explored MTBoS specifically, many of these remain either preliminary in nature, focused on a very specific aspect in MTBoS, or attend to the use of the MTBoS space in combination with an in-person setting. While these findings highlight the various beneficial attributes that MTBoS may offer for teachers of
mathematics, they do not yet explain what MTBoS is as a whole, how it has formed, and what processes make it so sustainable. Also, a common approach across all of the studies of MTBoS discussed earlier is to use single tweets found through hashtag searches to analyze contents either qualitatively or quantitatively. Only one preliminary study explores the conversational threads that can occur in this space (Larsen & Liljedahl, 2017), and none of the studies examine how the initial tweets are responded to in terms of being liked, retweeted, or replied to. Given the fact that in marketing research, various attributes of content entail different forms of response to content, and that in political movement research, it is found that certain topics get taken on and refracted more significantly than others, it is therefore interesting to consider how such results prevail within MTBoS data. That is, not only what is being posted about, but also how it is taken up, and what that means for the growth of the system.

As such, through examining this literature, I refine the aims of my study of MTBoS to that of focusing in on the formation, sustainability, and the implications of MTBoS as an ongoing self-organized collective oriented around communicating about mathematics teaching. To this end, in this study, I aim to more deeply examine how MTBoS acts, what the driving factors that direct activity in it are, and what this may reveal about professional activity around mathematics teaching. This is particularly appropriate given the unique nature of the MTBoS space as offering a source of raw data that is largely uninfluenced by any sort of researcher presence or intervention. However, it does live in a space that has certain constraints and possibilities that allow for not only content to be published asynchronously, but for it to be interacted with long after it has come to exist in the public sphere. This context therefore offers insights into professional activity around mathematics teaching that are worthy of deeper pursuit. To this end, in what follows, I explore the various theoretical framings and influences that position my theoretical perspective, with an aim of further refining my focus and identifying research questions.
Chapter 3  Theoretical Views

Deciding on a single theoretical view in this research has been arduous because theories robust enough to attend to the complexity and nuance involved in MTBoS as a site for professional activity around mathematics teaching most often rely on assumptions rooted in physical contexts in which they have been developed. However, the social media context in which MTBoS situates involves features that defy laws of time and space. This is because published musings in MTBoS are ultimately traceable and not limited by time or location in garnering a response. As well, the communication in the context is autonomous, asynchronous, publicly accessible, and media enhanced. While theories that address collectives such as MTBoS within digital settings have developed (e.g., Siemens, 2005; Wenger, White, & Smith, 2009), these tend to place focus on the digital tools themselves more so than the nature of the collectives they uphold, and often, are reframings of earlier theoretical work that is itself more robust. As such, my intent was to ground my theoretical views in frameworks that do not necessarily address technological aspects commonly found in social media settings, but that instead, offer a robust set of tools with which to approach the study of MTBoS as a phenomenon of self-organized professional activity around mathematics teaching. Thus, in this research, technology simply offers an opportune portal for observing naturally occurring activity among professionals interested in mathematics teaching without researcher influence. While the effects of technological tools on communication between MTBoSers are not ignored, and their role is acknowledged and embraced in analysis, the dominant focus in this study is on examining how and why MTBoS sustains self-organized professional activity around mathematics teaching as precipitated by the technologically saturated context. Therefore, I gravitated to frameworks that conceptualize professional learning as situated in context and where individuals are considered social participants in a collective. To this end, I was most drawn to the theoretical framings offered in activity theory (Engeström, 1987, 2001; Leont’ev, 1978; Vygotsky, 1981), communities of practice (Lave & Wenger, 1991; Wenger, 1998), and complexity thinking (Davis & Simmt, 2003; Davis & Sumara, 2006; Davis, Sumara, & Simmt, 2003). Inevitably, by considering each of these theories deeply, they consequently contributed to my views on MTBoS, offering me various sensitizing frameworks (Patton, 2002) from which I could draw on in my observations.
However, within the process of designing this study, I encountered a split tension in deciding whether to integrate aspects from each of these theories, or to dispose of certain theories to more comprehensively utilize a single theory. This tension arose because of the remarkable nature of MTBoS, in which no theory seemed to completely address all its inner workings. With the recent interest in mathematics education around the networking of theories towards refining and expanding on theoretical frameworks in the field (e.g., Prediger & Bikner-Ahsbahs, 2014; Rolka & Roesken-Winter, 2015), integrating theories seemed attractive. Given the philosophical compatibility of the three theories in consideration, albeit with differences in epistemological foci of interest, integrating them posed a relatively feasible possibility. However, in ‘trying on’ each theory with preliminary MTBoS data, I discovered that while activity theory (i.e., Engeström, 1987) and communities of practice (i.e., Wenger, 1998) offered very specific tools for analysis, their guiding assumptions and intentions consistently clashed with the self-organized nature of activity in MTBoS. In contrast, complexity thinking (i.e., Davis & Sumara, 2006) suggests an overall worldview that presupposes, and requires, self-organized activity as a foundational aspect to the formation of emergent collectives. As such, given its strong compatibility with the nature of MTBoS, I decided to use complexity thinking as my primary overall guiding framework, but chose to still draw on ideas from activity theory and communities of practice whenever theoretical tools from complexity thinking did not suffice in addressing certain occurrences in the data. In what follows, I draw out the key sensitizing concepts from each of these theories and point to how they shaped and influenced my overall approaches and attunements in this research. I then use these views, along with information from literature and the MTBoS context itself, to specify the research questions that guide my inquiries in this thesis.

3.1 Drawing from activity theory

The first theory I considered, was activity theory, which is a descriptive meta-theory on learning and change where activity is tool-mediated, object directed, and situated in context. In contrast with traditional psychology, where an outside force drives change, activity theory posits that participation in activity produces change in participation. That is, when we participate in a task, this participation changes us. As such, there is an inseparability of consciousness and human activity, and activity as a whole is a unit of analysis, rather than any of its components. Although there have been several
‘generations’ of activity theory, they stem from the founding concept of object-directed tool-mediated action, which first included psychological tools when written about by Vygotsky (1981). His initial mediation triangle of activity has been commonly reformulated into what is now considered first-generation activity theory (see Figure 3.1 below). As Bakhurst (2009) emphasizes, in Vygotskian cultural-historical psychology, “the idea is that human behaviour is not simply called forth by stimuli, but is mediated by artifacts that are created to prompt or modulate action” (p. 199). This notion was pivotal in forming my sensitivity to conceptualizing MTBoS activity as object-directed, mediated by artifacts, and a whole unit of analysis. It drew my attention to the role of tools, such as those that support communication in MTBoS (e.g., hashtags, media, mentions, etc.), and that through activity, the subjects, objects, and mediating artifacts change each other.

Figure 3.1  First-generation activity theory (Bakhurst, 2009, p. 200)

This initial formulation was subsequently extended by others, such as Leont’ev (1978) and Engeström (1987), with relatively significant differences. Some of the key variances between these extensions lie in a difference in interpretation of the term object, which in Russian, can translate to either predmet or ob’ekt. The first refers to a space of intention or purpose whereas the latter refers to a concrete topic or an object being worked on. Leont’ev (1978), who used the interpretation of predmet, developed a strand of activity theory, now considered second-generation activity theory, that lends itself to inferring the unconscious motives lying behind goals and operations carried out by subjects in activity. In his interpretation, he distinguished individual ‘actions’ from collective ‘activity’ to account for the individual-collective relation, indicating that although an action is made by an individual, the activity is the purpose of the collective. To illustrate this, he suggested an analogy of a ‘beater’ in a hunter-gatherer society where the action of the ‘beater’ is to beat a bush in order to startle the animals, but who does this towards the collective activity of hunting. In this way, the predmet “is the moving force that governs the processes of activity at the collective level” (Roth, 2014, p. 10). This notion provided
me with a sensitivity towards how individuals act within the activity of a collective towards a *predmet*, and that in activity, these aspects co-implicate each other. It also illuminated for me that I could not assume that MTBoSers were working towards a collective *predmet*. Although some users seem to work at organizing and defining the space, many go on without evidencing such advocacy or awareness of the structures in the collective beyond their individual experiences and intents. While Leont’ev’s formulations provide a robust set of tools to infer motives of individuals in a collective, my interests in this study were not centered around individual actions, but rather in how they contribute to collective activity without presuming an overarching aim. Therefore, this concept remained merely as a sensitizing concept and did not play a role in my analysis. However, the framing of the individual-collective relation contributed to refining my scope of inquiry, shifting away from individual experiences towards collective activity.

Another significant development of Vygotsky’s (1981) formulations stems from the *ob’ekt* interpretation of *object*, as proposed by Engeström (1987), who takes the original notion of object-oriented tool-mediated views on human activity and extends it to the case of collective activity, treating the collective as an activity system working as a whole. Based on this difference in interpretation, for Engeström, the object of activity is the topic or item being worked on rather than the motivating purpose towards which activity is directed. As such, his model is aimed at understanding the organizational structure of activity rather than individual motives lying within activity. This focal stance informed the level at which I aimed to investigate MTBoS. The key feature of Engeström’s extension of activity theory to the collective level is his addition of *community* as a third element in the subject-object relationship. His model therefore includes a three-way interaction between subject, object, and community (as in Figure 3.2 below).

![Figure 3.2 Engeström’s (1987) three-way model (Kaptelinin & Nardi, 2012, p. 34)](image)

Engeström also preserves the notion of tool-mediation, but because there is now a three-way interaction, each pair of these three elements is assigned a more specifically
defined means. Just as Leont'ev, Engeström defines the subject-object interaction as being mediated by tools, which may also be real or ideal. However, since Engeström now posits a three way interaction, he further specifies the means that mediate the other community related interactions. Namely, the subject-community interaction is defined to have mediating means of \textit{rules} and the community-object interaction is defined to have a mediating means of \textit{division of labour}. In this manner, his activity system triangle is born (as shown in Figure 3.3 below) and is often referred to as \textit{third-generation activity theory}.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{activity_system.png}
\caption{Engeström's (1987) activity system (Bakhurst, 2009, p. 201)}
\end{figure}

He also adds that an activity system as a whole produces an \textit{outcome}. Thus, the system works as a unit on the object to produce an outcome. The activity system is a single unit of analysis, and although there are multiple components in the system, the key premise of this strain of activity theory is that the focus remains on the dynamics and continuous development of this system rather than any particular component. For example, an outcome-oriented system could be the Desmos\textsuperscript{12} (2019) team. This team is comprised of several paid members who work at designing new functions within the Desmos program. A Desmos team member works with their community (or team), under certain rules. These rules may be implicit norms or explicit policies found within their working environment. The team may also have an implicit or explicit manner in which they designate tasks (their division of labour). A member of the team works on the current version of the software (the object) with the aid of various instruments (tools) and their community. In this process of working on the object, the object transforms into a newly defined object, which may be an outcome and may be either real or ideal.

Engeström (2001) further develops the theory to illustrate how \textit{continuous development}, or learning, occurs in such a system, and posits that such development is driven by

\textsuperscript{12} Desmos is a technologically advanced cloud-based graphing calculator application that allows for smooth rendering of various complicated equations as well as simple programming of function animation. It also allows for users to work together on one graph virtually in real-time.
contradictions in the activity system. These are parsed into four types: a first level contradiction has tension within a particular component, a second-level contradiction has tension between components, a third-level contradiction has tension between the system and a desirable outcome, and a fourth-level contradiction has tension between activity systems (Kaptelinin & Nardi, 2012). The fourth is often referred to as fourth-generation activity theory since it offers a view on multiple activity systems working towards a shared object (as illustrated in Figure 3.4 below).

![Engeström's extended activity system](image)

**Figure 3.4  Engeström's extended activity system (Bakhurst, 2009, p. 201)**

Examining these sorts of contradictions can illuminate driving forces in an activity system. For instance, perhaps in the Desmos team, there is tension between the cost of a particular technological development and the desired object of the team, which triggers a reorganization of how labour is divided towards developing tools at lower costs. This notion of contradictions within a system as driving forces of change in the collective was ultimately influential in my theoretical developments within this thesis. However, the presumption of a shared object continued to clash with the nature of MTBoS. This is in part due to the labour-oriented origins of activity theory.

Activity theory has been further extended and drawn on in many other cases, and has been used to study a variety of settings, including work environments (e.g., Engeström, 1993, 1996, 2000), educational settings (e.g., Wells, 2004; Yamagata-Lynch, 2003), teacher professional development (e.g., Abdullah, 2014; Roth & Tobin, 2002; Yamagata-Lynch & Haudenschild, 2009), and human-computer interaction systems (e.g., Gay & Hembrooke, 2004; Kaptelinin & Nardi, 2006; Nardi, 1996). The aims of these studies typically involve examining processes and systemic contradictions that drive participants’ co-evolution with their contexts towards outcomes. For instance, in teacher professional development, questions around how professional activity contributes to changes in practice are effectively explored through activity theory. As such, it provides a useful
framework for bridging the individual and the collective within contexts that aim towards an outcome and highlights processes mediated by tools, which include the community, and that collectively transform the activity participants engage in. Earlier formulations also attend to the motives and intentions that drive individuals in collective activity.

Therefore, the various formulations of activity theory sensitized my overall theoretical perspective by casting MTBoS as a collective formed through individual subjects acting towards a collective object, which through tool-mediated activity and systemic contradictions, continues to change and drive its ongoing development. This system is treated as a unit of analysis that aims to unify individuals in a collective through activity. Such a view highly influenced my interests around attending to the layer between the individual and the collective, and how these intertwine. Other notions from activity theory, such as tool-mediation, the community as a tool, and motives that drive individual actions in activity also sensitized my views and informed my analytical attention when approaching MTBoS data. Most prominently, the cyclical and iterative nature presented in activity theory offered me a perspective on MTBoS that illuminated the collective production of ideas over time and how social interactions develop the culture of a collective, which in turn, further determines social interactions. And in particular, that contradictions (or tensions) in the activity system can drive change and development.

However, I did not ultimately use an activity theory perspective as a primary theoretical worldview because of its founding presuppositions around outcomes that collectives define and work towards. Activity theory explains systems that have been defined by certain (often physical) constraints and objectives. But MTBoS reveals a new aspect to this in that it is a loosely defined system that identifies itself as it forms. In the case of MTBoS, there is no overarching authority that determines a purpose to activity, and it is questionable whether the activity is aimed at any given individual or collective objective. Rather, MTBoS is an informally defined space to post about mathematics teaching that is fundamentally self-organized and may not have yet formed an activity system. That is, the driving interest of this research about MTBoS pertains to how it forms and sustains as a collective, which may be a precursor to the views offered by activity theory.
3.2 Drawing from communities of practice

The next theory I considered was that of communities of practice (Lave & Wenger, 1991; Wenger, 1998), a social theory of learning where learning is considered as socially situated and existent through increasing participation in the pursuit of collectively valued enterprises that are meaningful in a given context. It is grounded in a robust span of social learning theories, including those that address social structure (e.g., Foucault, 1970; Lévi-Strauss, 1963), situated experience (e.g., Heidegger, 1962; Maturana & Varela, 1980), social practice (e.g., Engeström, 1987; Habermas, 1984; Vygotsky, 1978), and social identity (e.g., Eckert, 1989; Giddens, 1984). It is therefore one of many subsequent developments out of activity theory, but additionally integrates other theoretical traditions into its scope. The result is an incredibly robust framework that offers both broad-level conceptualizations of collectives as well as very specific constructs that describe how such collectives form and develop over time. Although in its philosophical grounding it continues with a flavour of labour-oriented perspectives found in activity theory, it maintains interest in conceptualizing individual actions as situated in a social collective and offers more specificity around certain aspects of interactivity between participants, their social locations in their community, and ways in which they can belong in it. This specificity is perhaps related to a nuanced shift in focus: activity theory is concerned with learning as object-directed action, whereas communities of practice is concerned with learning as increasing participation in negotiating meaning and identity formation. The unit of analysis is therefore repositioned from the activity system as a whole, to the mutual engagement between participants, which is a more localized view on how individuals form a collective. Although my interests were to include a broader view on MTBoS activity\textsuperscript{13}, the specificity afforded by considering localized interactions was also useful in my inquiries.

The key notion in communities of practice is that learning occurs within social participation, where participation is defined as the “process of being active participants in the practices of social communities and constructing identities in relation to these communities” (Wenger, 1998, p. 4). Therefore, it centres on participants engaging in

\textsuperscript{13} Although the term ‘communities of practice’ refers to engagements in the collective as ‘participation’, I continue use of the term ‘activity’ (or ‘activities’ when plural) to refer to the ongoing processes and engagements that occur within MTBoS. It is not used here to refer to mathematical activity, but more generally, any activity that naturally occurs within MTBoS.
social negotiation of meaning, which defines the community and shapes it over time. As Wenger notes, “participation shapes not only what we do, but also who we are and how we interpret what we do” (p. 4). Because of the ongoing nature of participation, Lave and Wenger (1991) emphasize that “communities of practice are engaged in the generative process of producing their own future” (p. 57). This cyclical, iterative, and interconnected view on collectives that develop over time continued to establish my perspective on MTBoS as a collective that undergoes ongoing development. However, since it presumes a reciprocity in participation through mutual engagement, it was difficult to use as my primary theory as this would limit my scope to only considering instances of reciprocated participation. While some MTBoSers engage in reciprocated participation, many do not. Nonetheless, the constructs offered by communities of practice were particularly useful in guiding my analytical pursuits of instances where reciprocity was occurring, and where mutual interaction was evident. The level of specificity offered within constructs in communities of practice overall allowed for refinements in my observations. As such, I reveal the key guiding tenets from communities of practice that most strongly influenced and refined my views.

In general, the primary components of communities of practice, which serve to characterize learning as social participation and act as the founding tenets of the theory, include meaning, practice, community, and identity. For Wenger (1998), meaning is “about our (changing) ability – individually and collectively – to experience our life and the world as meaningful” (p. 5). Practice is “about the shared historical and social resources, frameworks, and perspectives that can sustain mutual engagement in action” (p. 5). Community is “about the social configurations in which our enterprises are defined as worth pursuing and our participation is recognizable as competence” (p. 5). And, identity is “about how learning changes who we are and creates personal histories of becoming in the context of our communities” (p. 5). In Wenger’s theorizations, “practice and identity constitute forms of social and historical continuity” and social practice is considered the “production and reproduction of specific ways of engaging with the world” (p. 13), a necessarily foundational aspect of collective activity. Through practice, social identity, which is considered the “social formation of the person, the cultural interpretation of the body, and the creation and use of markers of membership such as rites of passage and social categories” (p. 13), is formed. As he indicates, “it is not easy to become a radically new person in the same community of practice” (p. 89) because of
the overall stability of one’s identity within the community. Practice and identity therefore work together and are necessarily interwoven. Wenger further explains that learning is caught in the middle between practice and identity because “it is the vehicle for the evolution of practice and the inclusion of newcomers while also (and through the same process) the vehicle for the development and transformation of identities” (p. 13). While he defines and describes identity in detail, it is treated loosely in this thesis as a concept that broadly refers to how participants are socially recognized based on the ways in which they participate in the collective and the roles they develop and maintain in the community. That is, my attention is directed more prominently on the notions developed around practice, and participation in practice, towards ongoing collective development.

One particularly influential founding concept within communities of practice is around the degrees of involvement with which subjects participate within the community. This pertains to the apprenticeship metaphor in which the theory is rooted, and is referred to as legitimate peripheral participation, which “provides a way to speak about the relations between newcomers, old-timers, and about activities, identities, artifacts, and communities of knowledge and practice” (Lave & Wenger, 1991, p. 29). By building on this notion of legitimate peripheral participation, Wenger (1998) further distinguishes trajectories for participation through which members build their identities. These are illustrated in Figure 3.5 below and include peripheral trajectories, inbound trajectories, insider trajectories, boundary trajectories, and outbound trajectories.

![Figure 3.5](image)

**Figure 3.5**  Trajectories of participation (Wenger, 1998, p. 167)

The various trajectories members take can shape their experience and competence in the community and contribute to their identity. Considering these various trajectories of participation, it is evident there are various ways in which one may learn how to participate in the community, and this learning ultimately involves traversing from peripherality to a presumed ‘core’ in the community. While I am challenged by the notion of there being one core and one periphery, the ideas around trajectories within the social
collective overall informed my view on the existence of various social positions and roles that participants can take when engaging in activity in the collective. This notion was also influential in my decisions around becoming a participant in the MTBoS since I held an awareness that I traversed various social positions and roles throughout my time participating in MTBoS activity. These in turn offered me various perspectives that I may not have been privy to without such prolonged participation. While the overall aim of this thesis is not around how users learn to participate, but rather, what is afforded by participation, this conceptualization was influential on informing how I approached data collection and how I chose to examine the social landscape found within MTBoS.

Another influential concept gleaned from communities of practice is around the nature of practice itself, and what it entails. A practice involves both explicit and tacit elements. Explicit elements can include “language, tools, documents, images, symbols, well-defined roles, specified criteria, codified procedures, regulations, and contracts” (Wenger, 1998, p. 47). Tacit elements can include “implicit relations, tacit conventions, subtle cues, untold rules of thumb, recognizable intuitions, specific perceptions, well-tuned sensitivities, embodied understandings, underlying assumptions, and shared world views” (p. 47). Wenger indicates that many of these tacit elements “may never be articulated, yet are unmistakable signs of membership in communities of practice and are crucial to the success of their enterprises” (p. 47). This distinction between explicit and tacit elements of practice attuned my views in MTBoS towards considering both those aspects of activity I could observe explicitly, and those that occurred more implicitly. This ultimately crept into the analytical design of my study as well as throughout my analysis. The awareness of there being tacit elements, however, was only made possible by choosing to become a participant myself. As such, communities of practice fundamentally informed my stance and entry into MTBoS activity.

Further, another significantly influential conceptualization from communities of practice on my views of MTBoS activity stems from Wenger’s foundational concept of negotiation of meaning, which drives ongoing development in a community of practice. Although it presumes mutual engagement around a joint enterprise through a shared repertoire, it also is defined as an ongoing duality between participation and reification that breeds continuous evolution and represents a give-an-take dynamic nature to the negotiation of meaning. This duality is illustrated in Figure 3.6 below.
The give-and-take nature between participation and reification shapes not only the participant, but also the community in which they participate. In some sense, this relates to what Engeström (1987) suggests when including community as a tool that co-evolves with the subject towards objectives of the collective. Wenger (1998) nuances this concept further and explains that participation is essentially about taking part. Whether it is conflicting or harmonious, intimate or political, competitive or cooperative, or isolated or integrated, our actions in the world are always necessarily social in this sense since others are always implicitly involved in participation. And, as we participate, we produce traces of activity. These traces are referred to by Wenger as *reifications*, since reification is “the process of giving form to our experience by producing objects that congeal this experience into ‘thingness’” (p. 58). Just like participation, reification can also shape our experience of participating in a practice. For example, “a word processor, for instance, reifies a view of the activity or writing, but also changes how one goes about writing” (p. 59). As such, reification can refer to both a process and its product because they imply each other. And, reifications necessarily prompt subsequent participation since they provide an opportunity for reinterpretation and renegotiation of meaning. This is because they inherently carry a history in how they were produced, even if this history is obfuscated. The interplay between participation and reification is therefore foundational to the negotiation of meaning and is the driving duality in communities of practice.

Interestingly, participation and reification require and enable each other, and interact with each other, which allows one to question how the production of meaning in various situations is distributed. Various combinations of the two will produce different experiences of meaning, and together they can create dynamism and richness in meaning. This happens when a particular balance between them is struck. Wenger
notes that “when too much reliance is placed on one at the expense of the other, the continuity of meaning is likely to become problematic in practice” (p. 65). If participation dominates, and “most of what matters is left unreified, then there may not be enough material to anchor the specificities of coordination and to uncover diverging assumptions” (p. 65). Perhaps two members speak to each other, but do not write anything down. They then have no reason to question each other’s statements. However, if reification dominates, and “everything is reified, but with little opportunity for shared experience and interactive negotiation, then there may not be enough overlap in participation to recover a coordinated, relevant, or generative meaning” (p. 65). In such a case, meaning is not made relevant to each member because they do not participate in interpreting the reification. In essence, participation allows for re-negotiation of meaning, and reification creates the conditions for new meanings. In this interwoven relationship, the two aspects work together to drive the process of negotiation of meaning. In MTBoS, given the nature of communication, everything that is visible between members, is reified, because all communication is published. As such, I decided to treat all reifications as traces of participations but specify reification further for instances when ideas revealed more ‘thingness’ than other ideas within the traces of participations. Therefore, this duality was incredibly useful to consider in the MTBoS space given its power to nuance the ongoing give-and-take of communication within a community.

Further, Wenger also highlights how the duality between participation and reification emphasizes the interplay between the personal and the social. By participating, we make our actions observable to others through reification, but participating is also innately personal. Therefore, in participating, we contribute to the public in which we participate. Such a reciprocity between our personal experience and the collective social world is illustrated by Wenger through a metaphor of a mountain with a river:

The world as we shape it, and our experience as the world shapes it, are like the mountain and the river. They shape each other, but they have their own shape. They are reflections of each other, but they have their own existence, in their own realms. They fit around each other, but they remain distinct from each other. They cannot be transformed into each other, yet they transform each other. The river only carves and the mountain only guides, yet in their interaction, the carving becomes the guiding and the guiding becomes the carving. (p. 71)

This sort of co-activity between individual actions and collective activity was enlightening in my process of inquiry into the workings of MTBoS since all of the participants in the
space are working individually, but somehow, they form a collective. Thus, it further reinforced my choice of perspective towards empirically studying MTBoS activity.

Finally, some of the other influential concepts on my theoretical sensitivities in approaching MTBoS activity included conceptualizations around mutual engagement, joint enterprise, and shared repertoire, as well as boundaries. While these concepts were not specifically used in my analysis, they informed my overall attunements. As Wenger indicates, through practice, a community coherence may arise, which involves a joint enterprise and shared repertoire developed through give-and-take engagements of participation and reification. Together, these define the community and drive its learning. However, their presence can be considered a strength or a weakness depending on the nature of the community. Some communities require more coherence than others, and coherence keeps the focus of the community attuned to established reifications and the joint enterprise that prevails. However, coherence to some extent is necessary. To this end, in order for a community of practice to be coherent, mutual engagement must involve processes around community maintenance with participants taking on various roles to help in maintaining the community and ensuring accessibility. This is seen in MTBoS activity when participants organize newcomer orientation initiatives.

Wenger also emphasizes that mutual engagement does not imply homogeneity, and that “what makes engagement in practice possible and productive is as much a matter of diversity as it is a matter of homogeneity” (p. 75). As such, “mutual engagement involves complementary contributions as well as overlapping forms of competence” (p. 74). In essence it allows for relationships to be made and maintained, but requires diversities that lead to disagreements, tensions, and conflicts to drive learning in the community. The result of mutual engagement is then a joint enterprise. While it is not necessarily a stated goal, it orients participants’ relations of mutual accountability that are integral to practice. Out of these, a shared repertoire develops through the joint pursuit of an enterprise. These can include words, tools, routines, ways of doing things, ideas produced by the community, or stories living in the collective space. The repertoire is therefore both reificative and participative and includes not only processes and histories of participation, but also styles by which they are expressed. Taken together, these aspects drive ongoing learning and development in a community of practice, and overall informed my view on the specific nature of collective activity that is simultaneously both individually and collectively emergent.
Also, through these processes, continuities and discontinuities in membership and participation develop, determining the boundaries of practice. The notion of boundaries was also influential on my perspective because in MTBoS, the boundary is exceptionally elusive and obscurely defined. That is, it is difficult to tell who or what lies at the boundary. To this end, Wenger emphasizes the difference between boundaries and peripheries. These both refer to edges but emphasize slightly different aspects. Boundaries, “no matter how negotiable or unspoked - refer to discontinuities, to lines of distinction between inside and outside, membership and non-membership, inclusion and exclusion” (p. 119). However, peripheries, “no matter how narrow - refer to continuities, to areas of overlap and connections, to windows and meeting places, and to organized and casual possibilities for participation offered to outsiders or newcomers” (p. 120). The nature of boundaries and peripheries in communities of practice can illuminate the type of accessibility that a practice makes available both to insiders and outsiders. Since peripheries and boundaries weave through one another in various ways, it is possible to see multiple practices as a social landscape of practices. And, various manners of boundary-crossing and multi-membership can produce new communities of practice. Boundaries are therefore interesting because various artefacts from other communities can cross these boundaries, contributing to the overall diversity within the community. It is therefore intriguing to consider the various artefacts that exist, where they originate, and whether they are accompanied by brokers or are renegotiated into novel ideas. This notion further specified my view on not only how members participate in the social landscape of a collective, but also what artefacts they communicate about, where those artefacts originated, and how much renegotiation occurred.

As such, communities of practice offers a robust theory that illuminates specific elements within a small-scale community that exhibits sustainability through mutual engagement in a joint enterprise through shared repertoire with acknowledgement of a shifting periphery for participation and the opportunity for artefacts to be reified and negotiated. While the interest is around sustained relationships, these do not need to be harmonious, and may precipitate conflicts. Attention is placed not on individual participants, nor the collective as a whole, but on the in-betweenness in the relationships between individuals who form the collective. It also embraces the naturally emergent nature of such communities without necessitating pre-scripted organizational structures that direct participation. As Wenger indicates, a community of practice cannot merely be called one, it needs to have
characteristics of one. And, the more of the features presented as characteristic of a community of practice that a social configuration exhibits, the more likely it is one. Therefore, while many of the specific conceptualizations offered by communities of practice served as sensitizing concepts that guided the perspectives, methods, and analytical approaches in this study of MTBoS activity, I decided not to use it as a primary framework because I did not want to presume MTBoS is a community of practice.

This tension is highlighted by Wenger, Trayner, and de Laat (2011), who indicate a need for blending the theoretical approaches of communities of practice with that of personal learning networks or social network analysis more broadly. In their work, it is worth noting how they extended Wenger’s (1998) earlier conceptualizations of communities of practice by integrating it with that of personal learning networks (e.g., Tour, 2017; Trust, Krutka, & Carpenter, 2016). In particular, they claim that “community and network [are] two aspects of social structures in which learning takes place” (p. 9). Personal learning networks are fundamentally individual in nature and do not necessarily comprise a community that is oriented around any specific focus of interest. However, networks afford “information flows, helpful linkages, joint problem solving, and knowledge creation” (p. 9) even though they remain distributed in structure and do not tend to offer a sense of shared identity. They note that the balance between networks and communities can be used to conceive of social structures that involve both. This can be seen also in the findings revealed by Rost (2011) who uncovers that tie strength between agents in a network is more important in precipitating innovation in a network than the number of connections it has. Although this provides a social network analysis view, it supports the notion proposed by Wenger et al. (2011) about synthesizing personal learning networks with communities of practice to account for the need for both networks that support information flow and communities that support relationships and negotiation of meaning.

If compared to Wenger’s earlier notions about peripherality in a community of practice, an interesting synthesis may be considered. That is, if the periphery of a community of practice is structured as a personal learning network, which provides access into the core community, this means balancing the ideas between a personal learning network and a community of practice creates a new structure, one that reaps the benefits of both. More specifically, it further bridges the individual and the collective perspectives because individuals use their agency to develop their own personal relationships and connections through interactions with others, and a subset of the collective may begin to act as a
community through development of a shared identity around a mutually agreed upon domain of interest. Such a synthesis further refined my views towards conceptualizing the trajectories of participation in a space such as the MTBoS, where individuals participate in local interactions, which seem to at times contribute to global patterns of activity. It also further illuminates the possibility of MTBoS not necessarily exhibiting a community of practice, but that perhaps a subset of it does. As such, in this thesis, I refer to MTBoS primarily as a collective rather than a community, unless there are moments in which it reveals aspects of operating as a community.

Further, in my search for theoretical frameworks, I also considered some of the adaptations of communities of practice made for digital contexts. However, none of them seemed particularly robust enough to attend to the singular nature of MTBoS as a case of self-organized professional activity around mathematics teaching. For instance, one such theory is that of connectivism (Downes, 2005; Siemens, 2005), which claims to offer a theory of learning for the digital age and emphasizes that knowledge is distributed through a network, and develops through the use of technology and connection. To this end, Siemens identifies several principles of connectivism that indicate learning in a digital age requires a diversity of opinions, is a process of connecting information, may reside in non-human appliances, and involves remaining current with up-to-date ideas. However, many of its central tenets root heavily in robust theories such as activity theory (Engeström, 1987; Leont’ev, 1978), communities of practice (Lave & Wenger, 1991; Wenger, 1998), social learning theory (Bandura, 1977), and even collective intelligence (Jenkins, 2004). While I considered the principles of connectivism as a theoretical framework with which to approach the MTBoS, I quickly found the tenets too apparent in that the principles were observable, but not explainable.

In another case, Wenger, White, and Smith (2009) extend Wenger’s (1998) earlier work around communities of practice, adapting it to digital habitats. They maintain many of the same concepts but redefine them to include technological affordances. Taken together, they provide a refreshed take on what a community of practice can be within digital spaces. However, their theoretical focus remains similar to that found in original communities of practice formulations except that they are enhanced with consideration of technological implications. Their treatment of technology also remains broad since there are many technological tools and platforms available, resulting in constructs that come across as relatively simplistic. For instance, they offer an orientation chart to be
used as a spider diagram that can help profile a digital community along aspects such as how meetings occur, how expertise is accessed, the degree to which conversations remain open-ended, etc. While these are worthy aspects to consider, they do not attend to the inner processes behind the formation of such features, nor how these features contribute to ongoing professional activity around mathematics teaching.

Overall, communities of practice offers a theoretical perspective that highlights individuals working within a social landscape under a shared domain of interest in which they mutually engage. It illuminates very specific processes involved in the production of meaning in the community through the negotiation of meaning that cycles through participation and reification. And, it gives terminology to notions such as a periphery, boundary encounters, artefacts, and most prominently, negotiation of meaning. The ideas around practice are intertwined with identity as a bridge between the personal and social, implying individual actions contribute to a collective, or even, a community.

However, the primary orientation of communities of practice as a theory is on learning to participate, and in turn, on membership. Consequently, its framings were helpful in guiding me to embark on an ethnographic journey of becoming a member. The specificity in its theoretical tools were also helpful in analyzing local interactions of negotiation between participants. However, it failed to serve as my dominant theoretical framework in this study for several reasons. First, its level of inquiry being on local interactions between individuals illuminated for me that my interest was more at the level of the collective as it forms from individual activity rather than within interactivity between individual participants. While this is a very nuanced distinction, much of the communication in MTBoS is unidirectional, implying that individuals participate with an ‘imagined community’ much of the time. Although there may be a subset in MTBoS that acts as a community of practice, I wanted to remain attentive to other aspects that may be occurring that drive ongoing activity.

Second, I sought a theoretical framework that would explain and account for the self-organization in MTBoS, since this is one of the defining aspects in MTBoS. While communities of practice allows for self-organized behaviour, it is not built on an assumption of self-organization. At its core, it developed from study of an in-person work setting in an insurance claim processing agency with a centrally organized structure and overarching mandate. Although Wenger indicates it is difficult to synthetically produce a
community of practice, many studies have indeed attempted and succeeded at designing communities of practice in professional settings (e.g., Goos & Bennison, 2008; Graven, 2004; Horn, 2010; Zaslavsky & Leikin, 2004). However, MTBoS has no central organization body, and while it may involve some of the processes of a community of practice, there may be processes in it that move beyond the confines of more centrally organized settings. Finally, while learning how to participate is important in MTBoS, my curiosity in this research moved beyond learning how to participate, towards attending to how MTBoS develops and what it affords through its existence. As such, while I was highly influenced by concepts from communities of practice, and I consequently drew on some of its specific constructs and terminologies, it did not ultimately serve as my primary theoretical influence.

3.3 Drawing from complexity thinking

While activity theory and communities of practice both offer robust theoretical foundations for approaching phenomena that involve individuals working within a collective, they both in some sense presuppose an aim for collective activity and are built fundamentally within physical production-oriented environments. Since MTBoS seems to defy physical constraints and does not orient around a clear production-oriented\textsuperscript{14} practice, such theories may not treat the phenomenon with fidelity due to its emergent and chaotic nature. Bearing this in mind, I was consequently drawn towards considering the theoretical influences of complexity science/theory/thinking (Davis and Sumara, 2006) given its roots in biological metaphors of evolution, which embrace chaotic self-organized emergent behaviour. Complexity theories in general take interest in the emergent collective behaviour arising out of self-organized individual activity and do not presuppose activity as object-oriented. For instance, one popularized depiction of complexity science is presented by Johnson (2001), who bridges anthills, emergent cities, and adaptive computer games such as Sim City to exemplify cases of complexity, and in turn, emergence. Another depiction is presented by Waldrop (1992), who provides an introduction to complexity science, revealing it through various perspectives.

There are a variety of branches within complexity theories, arising from various epistemological perspectives. Richardson and Cilliers (2001) parses them into three

\textsuperscript{14} By production-oriented, I refer to not only physical productivity, but also ideational productivity.
broad stems: reductionist complexity science, soft complexity science, and complexity thinking. Reductionist complexity science remains close to tenets of analytic science, aiming to reduce the richness of reality to simpler ‘truths’ or equations with a presumption that reality may be determined. Soft complexity science adapts these to allow for interpretive nuance as necessary for social systems while still using metaphors developed within reductionist approaches. More recently, however, complexity thinking, as initially suggested by Richardson and Cilliers, involves focusing on “the epistemological consequences of assuming the ubiquity of complexity” (p. 7). That is, complexity thinking presumes the universe is necessarily complex and that all understanding is provisional. Therefore, embracing a stance of complexity thinking can illuminate the processes involved in emergence without presuming it can be controlled.

To this end, while definitions of ‘complexity’ remain diverse, the most influential theoretical contributions towards my study of MTBoS include those initially presented in Davis (2005), Davis and Simmt (2003), and Davis, Sumara, and Simmt (2003); and later, more comprehensively in Davis and Sumara’s (2006) seminal book: Complexity and Education: Inquiries into Learning, Teaching, and Research. In this work, complexity thinking is brought into educational settings, and approaches and conceptualizations of learning are reconsidered. Namely, in defining learning, Davis and Sumara reject analytical and mechanical models of learning and push away from seeing learning as acquisition. Instead, they utilize biological metaphors to introduce learning as maintaining fitness in context, much in line with enactivist framings (Maturana & Varela, 1980; Reid & Mgombelo, 2014; Varela, Thompson, & Rosch, 1991). They note that “in contrast to demands for validity, reliability, rigor, and generalizability, complexity thinking is more oriented toward truths that are viable, reasonable, relevant, and contingent” (p. 26). As such, “learning is not understood in terms of a directed movement of information or knowledge, but in terms of continuous adaptation or coping as these co-implicated phenomena maintain their respective coherences” (p. 30). Further, they do not differentiate between individual and societal loci of knowledge, but rather, claim that complexity thinking “supports the common sense usages of the word knowledge to refer to what humans, nonhumans, and human collectives know” (p. 30, their emphasis). In this way, learning can be seen at various levels, including the individual and the collective levels. This was exactly the view I found most desirable in my study of MTBoS.
Moreover, Davis and Sumara outline that complexity thinking moves beyond correspondence theories, which statically parse the world into categories, and even coherence theories, which embrace evolutionary dynamics. Instead, they focus on the coherence and relationships between agents. Although complexity thinking shares many attributes with coherence theories, it also includes the notion of self-organization and emergent transcendent forms. Most importantly, every element in a complex system is intricately tied to every other element, and these elements cannot be simplified into separate entities. To this end, Davis and Sumara use notions from chaos theory (e.g., Lorenz, 1993) to explain how although initial conditions are determined, the complexity of a dynamic system will produce varying results every time. Notably, predicting how complex systems will evolve over time is deemed impossible. However, Davis and Sumara emphasize that it is not about getting these systems to reach predetermined or predicted goals, but rather, to become fascinated with the process of constant elaboration that such systems undergo. This stance is influenced by the post-structuralist emphases on dynamism, emergence, and inter-objectivity (Maturana, 1987), where the observer is always implicated. It also grounds in phenomenology (Husserl, 1954/1970), pragmatist notions of fitness (Darwinian evolution), and psychoanalysis (Freud, 1920). The result of using these influences is a theory that treats learning as adaptation to dynamic circumstances in which the observer is a member of the system, and where knowing is being able to select a well-suited action in response to an immediate context from a range of possible actions. Additionally, there is also a focus on the lining between structures as a place of unification, or simultaneity (Davis, 2005), with a focus on continuity across boundaries rather than discontinuity.

Based on these key notions and theoretical influences, Davis and Sumara define nine characteristics of complex adaptive learning systems. They then use these to develop six conditions necessary for complex emergence, which they arrange into three dyads (or tensions). These formulations heavily refined my views and theoretical sensitivities towards MTBoS because taken together, they describe the characteristics and processes involved in collectives of agents that gain a capacity to act in response to changing conditions, or more simply, to learn as a collective. This stance is unique from other formulations about collectives that learn because it illuminates a focus not only on individuals in a collective, but the collective itself, which seems to emerge as greater than the sum of its parts. In so doing, complexity foregrounds various emergent
phenomena that occur in unprecedented ways such as stock market crashes, city formations, or human evolution. As Waldrop (1992) writes, “complexity research is trying to grapple with questions that defy all conventional categories” (p. 9). By importing ideas from complexity science/theory/thinking into educational settings, Davis and Sumara (2006) aim to challenge existing norms and perspectives on education and within its very structures through highlighting vital simultaneities to consider within education and educational research, such as transphenomenality, transdisciplinary, and interdiscursivity (Davis & Sumara, 2008; Mason, 2008). As such, given the novel nature that MTBoS offers, the characteristics and conditions for complex emergence developed by Davis and Sumara are worthy of consideration towards efforts in further understanding its emergent qualities and what it affords for professional activity, or even learning, around mathematics teaching. Although complex learning systems can evidence a wide variety of characteristics, the nine that they identify are self-organization, bottom-up emergence, scale-free networks, short-range relationships, nested structures, ambiguously boundedness, organizationally closedness, structure determinedness, and existence that is far-from equilibrium. These qualities are profoundly inter-woven, inter-connected, and difficult to pry apart.

The first of these, self-organization, is by far the most foundational as a quality found in complex systems. Self-organization means there is no authoritative force imposing any sort of organizational structure in the system. Rather, autonomous agents join into a collective in which they become interlinked and co-dependent naturally and spontaneously. They may not have much in common and may not be working towards the same goal, but they somehow begin working as a collective. This relates with bottom-up emergence, but additionally highlights that “complex unities manifest properties that exceed the summed traits and capacities of individual agents” (p. 5). This means that groups can outperform their best member, and all of this can happen without a central organizer, which ties in with Surowiecki’s (2004) research on the wisdom of crowds. Surowiecki (2004) argues for decentralization because people are more engaged when they take responsibility for their actions and act independently. Also, people with local knowledge are best equipped to make efficient and workable local solutions that may not emerge within a top-down organization. To this end, Davis and Sumara emphasize that “intelligent collective action arises out of the bottom-up, independent (but co-specified) actions of individual agents who act out of self-interest
and who may even be motivated by profound selfishness” (p. 85). They note that intelligence stems from agents selecting the most well-suited actions out of a range of possible actions they identify in the moment. When each agent works towards their own needs, a collective can emerge that is more powerful than if each agent were governed by a central authority. This description clearly aligns with the organization within MTBoS.

Another foundational aspect to complex systems pertains to structure, which for complex emergence to occur, needs to involve not only decentralized, but scale-free networks that allow for short-range relationships. The scale-free aspect builds on a fractal metaphor (Mandelbrot, 1982) in which a smaller structure is the same formation as the larger structure. To hone this, Davis and Sumara describe three types of networks based on findings of network theorists (e.g., Barabasi, 2000; Buchanan, 2003): centralized, distributed, and decentralized (or scale-free) networks (see Figure 3.7 below).

![Figure 3.7](image)

Figure 3.7 Three types of network structures (Davis & Sumara, 2006, p. 52)

A centralized network will develop if a decentralized network is stressed by certain conditions that mandate more efficient communication. For instance, when a teacher is pressured by a governing authority to implement certain curricula or assessments, the information flows efficiently to them but does not allow their ideas to be heard or to flourish in novel ways. Davis and Sumara note that a centralized organization structure “militates against an intelligent collective, as it prevents agents from pursuing their own self-interests and obsessions, which in turn prevents the representation and juxtaposition of diverse interpretations and actions” (p. 89). Centralized organization also tumbles when one node in the network fails because communication depends on nodes to transfer information efficiently. For instance, if a certain task is only given to one individual, if they cannot complete the task, the whole network suffers as no one has been fostered to replace their contributions. Therefore, a centralized network offers efficiency, but the drawback is endangered viability and limited generative capacity. In
contrast, a distributed network develops when resources are abundant, and stresses are low. An example of this would be in a self-paced learning setting where communication is not very efficient, and it is difficult for members to act jointly or cohesively since members are isolated. Although the system can remain viable, it will not be very efficient. In between these two extremes, however, is a decentralized network (or a scale-free network). This is where each agent has opportunities to specialize, and there are enough connections made by agents to maintain efficient lines of communication while preserving individual responsibility and accountability. It is this sort of decentralized network structure that is associated with complexity, and a decentralized structure is necessary for the system to be intelligent because it strikes a balance that maintains both viability and efficiency of the system. Interestingly, this kind of network structure may also be found at different layers (i.e., cells, people, collectives, societies).

Further, the notion of short-range relationships highlights the individual agency that is granted to agents in a system to maintain relationships with close neighbours. It emphasizes the need for nurturing local interactions as opposed to top-down commands. These sorts of short-range local interactions hold the capacity to contribute to an effect of global behaviour, which in turn, stimulates bottom-up emergence. As such, the decentralized scale-free network architecture of a complex system implies that any particular node may be a component of a node in a different level of analysis. In this sense, there is a nested structure. To this end, “complex unities can be (and usually are) simultaneously autonomous unities, collectives of autonomous unities, and subsystems within grander unities” (p. 92). That is, they are nested within each other and are inherently related to each other even though they are considered to be different levels of organization. Their biggest differentiating feature is the time it takes for them to change. Davis and Sumara further note that it is up to the analyst to choose which layer they wish to look at, but they need to be cognizant of such inter-relations. The notion of co-dependent nested layers of scale-free decentralized complex networks implies that boundaries between these layers are permeable and co-evolving, existing with vital simultaneities. As such, my interest was drawn towards the layer between the individual and the collective. Also, the ambiguously bound nature of MTBoS, with an open boundary where matter and energy can be exchanged with surrounding contexts, allows for agents to congeal within the boundary, but to draw from their own respective contexts. In a sense, this allows for the sort of multi-membership as alluded to within
communities of practice, but attention is placed within complexity thinking on the co-evolving, intertwined, and ongoing permeability of such boundaries.

Although constantly shifting in nature at their boundaries, complex systems have the property that they are inherently stable, or organizationally closed. They can maintain their general form even when they interact with their outer contexts through their ambiguously bound boundaries. Interestingly, Davis and Sumara indicate that “judgements about [complex systems’] edges are usually based on perceptible and sufficiently stable coherences” (p. 6). These coherences are what make the complex system organizationally closed. However, these seemingly stable coherencies can also change since “a complex unity can change its own structure as it adapts to maintain its viability within dynamic contexts” (p. 6). In MTBoS, this can be seen in the shifting nature of participants who come and go, but somehow activity maintains and evolves as a grander whole. Therefore, while boundaries are permeable, there is a prominent coherence in the overall structure, the combination of which can contribute to the system’s adaptability to changing conditions. Complex systems are also never in balance. In fact, “a stable equilibrium implies death for a complex system” (p. 5). Davis and Sumara explain that complex systems seek an interplay between positive and negative feedback. Positive feedback amplifies, like mob mentality, and negative feedback regulates, like a thermostat. Complex systems operate somewhere in between these two because positive feedback is essential for learning to be possible, but negative feedback allows for the system to be kept in check. Therefore, there is an inherent unpredictability of responses such a system will produce given a particular perturbation. In this sense, while complex systems are organizationally closed, they are structure determined, meaning their internal structures determine the response they will have to emergent conditions. And, it is in this manner of adaptation to emergent conditions that a complex system learns while embodying its histories. As such, I was drawn to considering how positive or negative feedback structures prevail and affect MTBoS activity, and what opportunities it has to adapt to changing conditions.

While the abovementioned features are necessary for complex learning systems to form and exist, this list is not exhaustive and serves only to highlight the baseline features of complex unities. However, in the study of various complex phenomena such as those of anthills, distributed networks in brains, and non-equilibrium thermodynamics, some complex unities do not simply exist, they seem to thrive beyond mere survival.
Not only did the unities seem to self-organize, but they seemed to learn. Indeed, some of them appeared over time to get more intelligent – that is, capable of more flexible, more effective responses to previously unmet circumstances. (Davis & Sumara, 2006, p. 74; italics in original)

By considering the implications of this for learning environments, Davis and Sumara studied learning settings within schools and with cohorts of teachers to develop a further set of conditions that not only allow a complex unity to emerge, but also to learn, that is, to become more adaptive to changing conditions. To this end, they conceive education “in terms of expanding the space of the possible rather than perpetuating entrenched habits of interpretation,” and therefore claim that the study of education “must be principally concerned with ensuring the conditions for the emergence of the as-yet unimagined” (p. 135). By taking this perspective, they develop six necessary conditions for complex systems to have a capacity to learn in this way and to emerge as-yet unimagined possibilities through their inner workings and adaptations to changing conditions. These six features are placed into three dyads (or tensions): specialization, trans-level learning, and enabling constraints. Specialization involves a tension between internal diversity and internal redundancy among agents. Trans-level learning involves a tension between neighbour interactions and distributed control in the system’s organization. And, enabling constraints involve tension between randomness and coherence in the collective. Together these provide a framework for approaching various learning-oriented contexts to identify if they have the conditions for complex emergence. While these conditions may be seen as relatively broad, attending to them and forming new specifications as pertinent to contexts such as MTBoS, ultimately illuminated the aspects that drive activity in MTBoS, and reasons behind why it generates novel ideas.

The first of these dyads is that of specialization, which is the living tension between internal diversity and internal redundancy and exists at all levels of organization even though it is most prominently observed among components (or agents) in the system. Internal diversity refers to there being enough diversity among members to allow the system to have a breadth of possible responses to emergent circumstances, and therefore ensures adaptability of the system. Further, “diversity cannot be assigned or legislated; it must be assumed to be present” (p. 138). That is, assigning roles in a top-down manner does not necessitate internal diversity and a mandate for diversity cannot control or enhance diversity. Also, diversity can remain dormant and may not surface if conditions are not conducive to requiring the diversities. For instance, “it is unlikely that
diversity, even if expressed, will be recognized and valued if the task set for a collective is trivial" (p. 138). So, not only does the collective require diversities among its members, conditions that evoke these diversities must be available. On the other hand, the collective must also have enough internal redundancy among members since having duplications and excess of necessary aspects in the system strengthens it, allowing for complex activity to initiate. This has to do with providing support for structural coupling to occur between agents so that they can interact (Reid & Mgombelo, 2014). Further, as Davis and Sumara (2006) posit, internal redundancy has two roles for a complex system: it "enables interaction among agents, [and] when necessary, makes it possible for agents to compensate for others' failings" (p. 139). They also note that such redundancy may be constructed by providing members with some sort of common ground like an artefact or an experience. Interestingly, in a mechanical system that is complicated but not complex, redundancy is often seen as unnecessary because duplicates are treated as excess. However, in a complex system, redundancies among members are necessary for systemic cohesion and viability. This change in perspective stems from a difference in goals between complicated and complex systems. The former aims for efficiency, whereas the latter aims for adequacy and maintaining fitness.

Consequently, the dyad of specialization allows for both stability and novelty, the balance of which has also been referred to as the zone of creative adaptability (Regine & Lewin, 2000). Davis and Sumara explain that a complex system must have a dynamic mix of both redundancy and diversity.

Whereas internal diversity is outward-oriented, in that it enables novel actions and possibilities in response to contextual dynamics, internal redundancy is more inward-oriented, enabling the habituated, moment-to-moment interactivity of the agents that constitute a system. (p. 139)

Both are necessary and their desired balance can depend largely on the context in which the complex system lives. In a stable setting, it is not necessary to have much redundancy because there are no potential threats to the system. Minimizing redundancy can reduce the robustness to the system and also decrease adaptability. In a volatile setting, maximizing redundancy can promote the viability of the system to keep it intact. However, with too much redundancy, the system loses its ability to adapt quickly to changing conditions. Therefore, a balance is necessary for the system to maintain adaptability and viability. Most importantly, internal redundancy and diversity cannot be
controlled. Rather, it is necessary that they are both existent in the composition of the collective, and that they are allowed to surface within the context as necessary.

The next dyad (or tension) is that of trans-level learning, which is an organizational dyad that works to enable neighbour interactions through decentralized control. Davis and Sumara use this dyad to offer insight into how conditions can be manipulated to ensure that individual and collective interests are not in competition. Rather than seeing these levels as being in competition, they account for individual agency while also addressing collective potential. Learning within the complexivist frame is about expanding the space of possibility in order to be able to adapt to new conditions. As such, learning is by nature a trans-level phenomenon because it involves the system and the context that it is situated in, as well as the contexts that are situated within it. If looking at an individual learner at the person level, learning involves both the collective context in which they are situated and their interior cell biology. In the case of the person being the collective of their cells, the cells experience decentralized control and billions of close neighbour interactions that allow for the collective system (i.e., the person) to learn. If this notion is scaled “up” to a collective of people, with people as nodes, the same principles should apply. The decentralization of control and close neighbour interactions can trigger collective learning where the collective ‘knows’ more than any of its constituents. An example of this may be a loose social circle where nothing is inherently organized, but members interact closely and frequently enough to produce various pockets of rich discussion that guide and determine subsequent activity.

Davis and Sumara emphasize that in order for a complex system to learn, “not only must there be neighbour interactions, there must be a sufficient density of neighbours to interact” (p. 143). Density allows for more frequent ‘bumping’ of ideas that spontaneously collide and interact, which can trigger emergence of new ideas. In this way, a complex system can become generative. They add that neighbours “in knowledge-oriented communities are not [necessarily] physical bodies or social groupings, [but] rather, the neighbours that must interact with one another are ideas, hunches, queries, and other manners of representation” (p. 142). They claim that “the ideational network rides atop the social network” (p. 143) and that these cannot merely be confounded as they exist at different levels, albeit closely related ones. However, as levels are porous, learning happens across levels. This allows for collective knowledge to emerge that is difficult to attribute to any individual member even though members act out of their own self-
interests without centralized direction. As they note, “there should be ample room for self-interest in a complex collective – but the global project should never be reducible to those self-interests” (p. 140). As such, individual agents may act out of their own desires, but the collective becomes more than any of these desires, developing a life of its own.

Just as neighbour interactions in a knowledge-oriented system involve the bumping and colliding of ideas, decentralized control that allows for neighbour interactions in turn, pertains to emergent conceptual possibilities. That is, control emerges from ground-level localized activity that does not necessarily intend on creating common interpretations, but rather, creates space for various possibilities of interpretation that arise in a shared space through the local neighbour interactions. Therefore, structures and outcomes of the collective are not controlled by any single member, but rather, “shared/decentralized control prompts our attentions away from matters of a leader’s actions towards consensual domains of authority” (p. 145). In this sense, authority in a complex system does not involve imposition, but rather, can be conceived of as the capacity to involve prevailing discourses, or to act within a consensual domain. That is, individuals carry an authority in the system, and the authority is therefore distributed by those who can maintain fit in the collective. Such decentralized control may also be considered as decentralized authorship, where what is created by participants is indistinguishable from products of the collective. To this end, for complex emergence to occur, “external authorities cannot impose, but merely condition or occasion possibilities” (p. 145). This is not to say that educational leaders should abandon their responsibilities of organizing learning spaces if that is their role, but that if they are given this role, they may find ways to decentralize control and encourage neighbour interactions so that authority may be distributed among agents in the system. If a system has no identified leaders and is truly self-organized, such capacities for collective learning through decentralized control and neighbour interactions, assuming they are dense enough, can naturally form emergence.

Further, in addition to specialization and trans-level learning, Davis and Sumara also introduce the dyad (or tension) of enabling constraints, also referred to as liberating constraints (Davis & Simmt, 2003), in which a balance between randomness and coherence is sought after to stimulate adaptability and to ultimately generate collective knowing. Enabling constraints are “the structural conditions that help to determine the balance between sources of coherence that allow a collective to maintain a focus of purpose/identity and sources of disruption and randomness that compel the collective to
constantly adjust and adapt” (Davis & Sumara, 2006, p. 147). Coherence allows for a sufficiently constrained domain of possible action, and randomness allows for unanticipated possibilities. In the context of a collective, coherence can orient agents’ actions, but randomness can allow for a more flexible and varied response. Sometimes when conditions are left too open, it is difficult to produce anything valuable. However, if conditions are slightly constrained, it can lead to more creative production. An example of this would be asking someone to draw something on a piece of paper versus asking that someone to draw something that does not have straight lines or intersecting curves. The results would be quite different based on these initial conditions. The first case may not even produce much because the task may be too open to prompt immediate action. They also point out that constraints should be proscriptive rather than prescriptive. That is, constraints should “not [be] imposed rules that one must obey in order to survive, but [rather,] conditions that one must avoid in order to remain viable” (p. 148). Further, “by proscribing unacceptable action rather than prescribing acceptable action, not only are the conditions for group identification (i.e., coherence) set, but an unexplored space of possibility (i.e., randomness) is opened” (p. 148). Therefore, in order to promote complex emergence, a delicate balance should be struck between coherence and randomness in the collective, with coherence allowing it to have identity and randomness providing it richness. Interestingly, this defies any expectation of a system attaining predetermined goals. Rather, complex systems are unpredictable. So instead of viewing this as a pitfall, the possibilities of complex emergence should be embraced because what a complex system produces may exceed the quality of expected outcomes.

Overall, these three dyads (or tensions) reveal the possible inter-related conditions necessary for complex emergence that is generative of the as-yet unimagined, as has been found through empirical study of emergent activity within informal in-service teacher groups as well as in a seventh-grade classroom (Davis & Simmt, 2003; Davis & Sumara, 2006). It is acknowledged by Davis and Sumara that these are not the only conditions that may be necessary. Rather, these are the conditions they found most important within an educational setting. To communicate this incompleteness and offer space for further emergent possibilities for developing other conditions necessary for emergence, they include a list of other conditions that may be useful to consider more deeply based largely on their readings of Johnson (2001). These include “the possibility of dying,” “means to preserve information,” “stability under perturbations,” and negative
and positive “feedback loops” (p. 151). Most importantly, Davis and Sumara make it clear that complexity cannot be scripted or predicted, it can only be occasioned.

As such, complexity thinking offers a cohesive set of inter-related features that can contribute to complex emergence. Together, they embrace the unpredictable and chaotic behaviour of complex systems, avoiding assumptions commonly rooted in centralized contexts. Given the fundamentally self-organized nature of MTBoS, complexity thinking is therefore innately representative of what is described by Davis and Sumara as characteristic of a complex adaptive system. Not only is it self-organized, it is also bottom-up emergent, ambiguously bound, organizationally closed, and consists primarily of short-range relationships between agents. Complexity thinking also takes a primary interest in attending to the space of possibility in a complex system rather than presuming it works towards an objective. Without presuming there is an objective, this frees a researcher to attend to emergent possibilities in the system, which may be more illuminating than if such objectives were presumed, or even expected.

Given the novel nature of MTBoS as a space that may be telling of insights into professional activity around mathematics teaching in an ‘un-touched’ manner that is naturally emergent, my most all-encompassing theoretical influence is therefore that of complexity thinking. This is because it gives the most space to attune to what MTBoS naturally offers without overly pre-determining what it should be. It also brings my attention closer onto the collective layer, to consider how the collective as whole ‘learns’ in response to changing conditions. Although individual interactions contribute to this whole and remain important, neither of the other two theories explored attend to the collective itself as an organism that has a living quality beyond that of its individual components. The reason why this perspective is so attractive in my conceptualization of MTBoS is that it seems to provide a view that holds the potential to illuminate not only individual actions around discussing issues in mathematics teaching, but also the nature of mathematics teaching in itself as emergent from unprompted activity by individuals who have no mandate to engage in discussing it. That is, it appears to offer an attentive perspective on the nature of what professional development around mathematics teaching naturally wants to be rather than what it has been scripted to be by those leading professional development initiatives. By taking a complexity thinking perspective, I found myself more equipped to focus on the emergent qualities in MTBoS. Namely, the notion of attending to the internal diversity and redundancy in the system at all levels of
analysis became central to my observations and analytical views. And, the ideas around
the collective’s sources of coherence that identify it as an ambiguously bound entity as
well as the sources of randomness that allow it to fluctuate and maintain viability also
became incredibly influential in my analyses.

By guiding me to attend to what makes the collective operate and maintain viability,
complexity thinking offered the most explanatory power towards understanding why
MTBoS has sustained activity over 10 years and why it continues to grow. Amidst all the
attempts at designing professional learning settings through socio-cultural perspectives
such as those rooted in activity theory or communities of practice, none seem to have
persisted to evolve without incoming inputs of resources over this much time. As such,
complexity thinking offers an alluring perspective that seems to attend not only to the
aspects that have been observed in professional learning settings, but also to new ones
we may not have yet encountered. Therefore, the conditions offered by Davis and
Sumara served as my primary theoretical framework, which guided my research
questions, approaches to structuring methods of analysis, analysis, and interpretation of
results. While notions from communities of practice have helped me enter the space
more deeply and to label certain aspects of participation, and ideas from activity theory
have guided my initial considerations around tool-mediation in the space, complexity
thinking informed my dominant stance and theoretical worldview in this study.

This choice was by no means easy. To reach this decision, I arduously entertained each
of the aforementioned theoretical frameworks towards interpreting the MTBoS context.
To some extent, they have all influenced my thinking and it is difficult to remove their
influences from my theoretical sensitivities. Each theory offers certain insights and could
well-have been used in studying MTBoS since they each address individuals in a
collective. Therefore, when notions from complexity thinking fail to provide a certain level
of specificity, I still draw on language from the other theories to support and enhance my
analysis. This is possible because of an underlying compatibility of the theories given
their acknowledgement of the possibility of self-organization, their attention to individuals
in activity with a collective, and some overlap in theoretical roots from which they build
on. However, complexity thinking ultimately serves as a dominant framework because it
offers a more suitable aim in its overall philosophical intentions and assumptions.
3.4 Specifying research questions

This research is fundamentally driven by a global interest in what MTBoS activity can indicate about the nature of professional activity around mathematics teaching, as this is what initially drew me into this research. However, before such broader implications can be made, a closer look at what MTBoS is and why it is sustainable is needed. In taking the dominant view of complexity thinking, as well as being informed by relevant literature and the nature of MTBoS itself, I further refine these interests into more specific research questions. As Davis and Sumara (2006) indicate, complex learning systems have an “ideational network [that] rides a top the social network” (p. 143). This notion was particularly influential on my sensitivities towards observing and structuring my inquiries of MTBoS. It also resonates with how MTBoS is referred to by its participants. At times, it is referred to as “the MTBoS“ or simply, “MTBoS“, while at other times, it is referred to as “the people of the MTBoS” or more simply, “the people of MTBoS”. This fundamentally highlights the interwoven nature between people and ideas in this collective.

I think the #mtbos is my happy place =D (@TypeAMathLand, October 8, 2015)

The #mtbos is a welcoming bunch! (@sumunderwood, August 3, 2015)

The #MTBoS is a deep well of resources. (@MrVaudrey, April 26, 2019)

Okay, #MTBoS, what do you make of these results? (@MathProjects, July 16, 2019)

I think about all of the people of the #MTBoS that have impacted me. Too many to name & afraid I’d forget some :( (@druinok, August 3, 2017)

People of the #MTBoS: I find your Ss thinking fascinating and your reflection on it inspiring. Thank you for sharing. (@DavidKButlerUoA, April 26, 2017)

Many people of the #MTBoS have made some awesome activities too. (@Desmos, December 16, 2016)

That is, MTBoS is simultaneously a collective and a collection of parts, but the parts seem to come together as a whole that is greater than the sum of its parts. Therefore, my central perspective in this study resides in the interwoven relation between individuals and the collective they form. In particular, the relationships that form a
connective tissue between the ‘people of MTBoS’ and the ideas that sustain professional activity around mathematics teaching in ‘MTBoS’. This sort of bridging perspective is informed by complexity thinking as well as the ways in which MTBoS is referred to. A visual representation of this view on the phenomenon is shown in Figure 3.8 below.

**Figure 3.8  Views on the collective as composed by individuals**

As such, with this theoretical backdrop in mind, as well as my aims of investigating what MTBoS is and why it is sustainable, the following research questions guide my inquiry:

**RQ1:** What is the nature of ‘MTBoS’ and how does it emerge from ‘the people of MTBoS’?

**RQ2:** How and why does ‘MTBoS’ invoke a sustainable form of professional activity around mathematics teaching?

Taken together, these questions aim to unpack the phenomenon of interest, which is that of the formation and sustainability of MTBoS as an ongoing self-organized collective oriented around communicating about mathematics teaching.

More specifically, RQ1 aims to examine what the collective layer in MTBoS is and how it emerges from bottom-up, self-organized activity. In this question, my focus is on identifying both the topics that are centrally communicated about and the processes around how they come to formation. That is, interest in both the ideational space and the social space is taken. This naturally leads into RQ2, which aims to unpack how and why the processes identified as contributing to the formation of the broader collective remain sustainable. Although it is beyond the scope of this study to examine a longitudinal approach towards the ongoing sustainability of MTBoS over time, the processes identified within the data collected are considered in terms of their implications for sustainability. Again, these aspects of sustainability also include both ideational and social aspects as highlighted within tenets of complexity thinking.
Taken together, RQ1 and RQ2 aim to illuminate the global inquiry that initially drew me into examining MTBoS, which is that of contemplating what can be learned from this context that is applicable to the larger body of work pertaining to professional activity around mathematics teaching. That is, how do the overall topics of interest in MTBoS inform what we know about the natural interests around mathematics teaching? What do the processes of communication and structures in activity imply about the nature of professional activity that center around mathematics teaching? And finally, how can these findings inform future professional development initiatives for mathematics teachers and the field of mathematics education overall? These implications will be explored based on the findings arising from investigations into RQ1 and RQ2.
Chapter 4  MTBoS and me

As noted in the Introduction, the chaotic, dense and cryptic nature of MTBoS required me to become a participant so that I could access the phenomenon, identify relevant data sources, and develop the capacity to interpret data\textsuperscript{15}. Accessing MTBoS generally involves finding, sifting through and decoding tweets and blog posts related to mathematics teaching. Using search terms or hashtags to find content in MTBoS only scratches the surface of what happens in this space, because participants typically engage through following a set of users who post tweets regularly, and whose tweets are collated in the participant’s newsfeed (also referred to as a feed, for short). Typical participation involves checking one’s feed frequently, exploring content and hyperlinks, and if one so chooses, to respond to tweets of others or to create original tweets. Being able to observe this ongoing nature of participation affords the possibility of attending to how the collective emerges and what holds meaning and value in the space. Mere searches do not yield a holistic perspective, nor do they allow for feeling the effects of participation, which is essential in developing the capacity to interpret how others have chosen to participate. As such, my orientation and desire towards understanding emergence in MTBoS made it essential for me to become a participant, as informed by communities of practice, so my window on the space could be more all-encompassing.

My entry into MTBoS occurred through my stepping in as a participant-observer; that is, participating through my role as a teacher of mathematics and observing through my role as a mathematics education researcher. It is noteworthy, though, that becoming a participant was not as easy an endeavour as I had initially imagined. Although participation in MTBoS is often considered to be defined, at minimum, by reading content produced by MTBoS participants and merely ‘showing up’ (see below), it takes time to feel the effects of even this degree of participation, to know how to navigate the self-organized space, and to have a sense of what things mean and what matters.

[In response to: “How do I officially join #MTBoS?” from @LeslieVFL] No dues! No mailing lists! No spam! No registration! No meetings! Just show up when and if you feel like it. (@Trianglemancsd, July 14, 2014)

\textsuperscript{15} Not only is MTBoS tweet data hard to access due to quantity, there are also formats and nuanced forms of writing that are challenging to navigate and understand without becoming an insider.
In what follows, I outline the ethnographic approaches that framed my stance in participation, the pivotal moments of my journey in becoming a participant as I moved from legitimate peripheral participation to core member participation, and how this insider perspective allowed me to engage with and identify pertinent data for research.

4.1 Participant ethnography

Research into education, and mathematics education in particular, is necessarily an anthropological endeavour. (Rowland, 2000, p. 14)

With the aim of accessing and understanding the underlying socio-cultural processes that drive collective activity in MTBoS, of which I was only able to see mere traces of when I first discovered its existence, an ethnographical stance of participant observation became apparent early in the study as the most suitable and simultaneously necessary approach to take. An ethnographic approach is also ontologically compatible with complexity thinking since an observer is necessarily part of the system in which they act. That is, there is no observation without an observer. An observer is necessarily co-acting, co-evolving, and co-implicated with all related components they engage with in a given context. As Davis and Sumara (2006) indicate, “complexity thinking might be understood as an acknowledgement of one’s complicity – not just complicity with/in one’s research interests, but with/in the grander systems that contribute to the shape of and that are shaped by those research interests” (p. 25). Therefore, my aim with choosing an ethnographic approach with which to enter the phenomenon was fundamentally intertwined with my overarching worldview of complexity thinking. More pragmatically, it allowed me to observe what I would otherwise not be privy to. Through engagement and contribution, my view, my context, and my focus of inquiry evolved. As Patton (2002) writes, “the participant observer can discover things no one else has ever really paid attention to” (p. 263). Although this research is not exactly ‘an ethnography’ since it does not satisfy ethnographical aims of uncovering views of participants and their relations with their environment (Reeves, Kuper, & Hodges, 2008), ethnographical processes of participant observation allowed me to enter the phenomenon, to learn how to identify pertinent data, and to engage in a co-evolution with the phenomenon of interest. In what follows, I outline the key aspects of ethnography that informed my stance, my choice of perspective within an ethnographic approach, and the affordances this stance has offered in my inquiry.
While there is debate about what constitutes and doesn’t constitute a ‘real’ ethnography (Agar, 1996; Hammersley & Atkinson, 1983; O’Reilly, 2004), ethnographic approaches primarily aim to uncover and interpret the culture of a certain group of people (Patton, 2002). Ethnographic processes involve using “some model of cultural or social process in both the gathering and interpretation of data” (Spindler & Spindler, 1987, p. 3). They also require the researcher to immerse themselves in the cultural context of study until new activity in the context is no longer novel or surprising (Wolcott, 1978). However, after becoming deeply involved in an environment, detachment is also necessary (Wolcott, 1978) as it allows for observation that carries with it a layer of reflexivity. Since purely objective observation is impossible, an ethnographer involving in participant observation ultimately takes a perspective that lies between the etic and the emic (Patton, 2002). Etic-emic theory, introduced by Pike (1967), introduces the tension between an insider’s view, referred to as the emic, and an outsider’s view, referred to as the etic. Darling (2016) conceptualizes this tension between these two perspectives as that of a ‘tightrope’ that a researcher ‘dances’ between. That is, no one can solely confine themselves to either of these perspectives, but rather, observation involves both. Some methods involve one more than the other; and, methods that embrace both offer opportunities for traversing between the two perspectives in an iterative way (e.g., Allan, 2005). In contexts where information about the socio-cultural environment are inaccessible from an outsider perspective, an insider perspective is innately necessary since it affords a rich and well-informed viewpoint from which the phenomenon of interest may be considered (Wolcott, 1978). However, being more of an insider means that attention must be taken to step back and to question anything that happens that may otherwise be taken for granted (Erikson, 1976; Whitehead, 2005).

As such, participant observation involves more than merely participation and more than merely observation. Rather, both of these processes are involved in building a description of the context of study in a manner that insiders would agree with but would not themselves have necessarily identified, which is the aim of an ethnography (Hammersley & Atkinson, 1983). Achieving such a description most classically involves immersion into the socio-cultural context, consistent collection of rich fieldnotes, and ongoing interviews to varying degrees of formality (Hammersley & Atkinson, 1983; O’Reilly, 2004; Whitehead, 2005). In the case of contexts that are digital, the processes are similar, but are at times referred to as a netnography (Kozinets, 2010), which
similarly aims to “arrive at the ethnographic understanding and representation of a cultural or communal phenomenon” (p. 60), but uses data from computer-mediated communications. In either case, within data interpretation and analysis, the key aim in ethnographic research is to describe and interpret socio-cultural behaviour in the context of study. This includes developing an understanding of “the socio-cultural contexts in which these behaviours occur, the socio-cultural processes of behavioural contexts, and the socio-cultural meaning that these contexts and processes have for those who practice them” (Whitehead, 2005, p. 9). And, there are a multitude of ways to achieve this. Ultimately, “ethnography names an epistemology – a way of knowing and a kind of knowledge that results – rather than a recipe or particular focus” (Agar, 2006, para. 57).

Consequently, an ethnographic approach of participant observation affords building a ‘sensitivity’ to what matters most in a given socio-cultural context. Through immersion, fieldnote taking, and informal interviews with participants, the researcher can develop such a sensitivity to the meanings that participants typically ascribe to certain discourse and occurrences in the space. It builds a sort of attunement to the nuanced interpretations taken by insiders. However, as Patton (2002) notes, “observers do not enter the field with a completely blank slate” and are guided in their observations by their own ‘sensitizing framework’ built from the perspectives that influence their views (p. 279). As such, in my approach of participant observation, I was guided in my observations by ‘sensitizing’ concepts from various aspects of my lived experience. Namely, through the theoretical underpinnings identified in Chapter 3, concepts I am guided by in my own teaching practices as emergent from my experiences as a teacher of mathematics and a graduate student in mathematics education, as well as the context-based knowing I developed through participation in the collective itself over time.

Notwithstanding, the aims of ethnography typically revolve around making meaning of how subjects of the space interpret their context rather than what the context is itself and is thus a primarily egocentric research approach. Although, more recently in some educational research, there has a turn in ethnographic approaches being used to study contexts, involving subjects only inasmuch as they are entangled with context (Gordon, Holland, & Lahelma, 2001). However, the focus of my research is not egocentric, nor is it context centric, but rather, attunes to the collective as a complete system involving the collection of subjects and their context. Therefore, this study cannot be considered an ethnography in its most complete sense, since my interest is not focused on individuals.
nor only on their context, but rather on how the collective emerges from the individuals. Nonetheless, **ethnographic approaches** were fundamental in my process of identifying and interpreting the data as well as in my stances towards participation in the collective as a participant observer. Therefore, ethnography informs the stance with which I approach data collection and analysis. This is not to say that I ignored meanings and interpretations of participants, but rather, that my focus moved away from participants’ interpretations of the context, towards the context itself. Their interpretations of the context, through my happenstance encounters with them, merely informed my entry into the phenomenon and my stance in the study overall. In other words, ethnographic approaches afforded me entry and access into the phenomenon and guided by interpretations of the data, but more global research approaches were used for the primary components of the study.

Since an aspect of ethnography is its fundamentally reflexive nature, my journey of becoming a participant within the development of my position as a participant observer cannot be ignored and is pivotal in my methodological choices in this research. Therefore, before outlining my methods of data collection and analysis, I first reveal my journey in becoming a participant, which involved finding ways to move from peripheral participation towards legitimate core participation (Lave & Wenger, 1991). Being a participant observer involved observational attunement to key ‘rich points’, which are moments in ethnographic research that are surprising in some sense to the researcher at the time of observation (Agar, 2006). It also involved choosing to distance myself after participating deeply, which proved necessary towards building more reflexivity into my perspective. In what follows, I outline my journey of participant observation through the fundamental ‘rich points’ (or ‘turning points’) in my process of simultaneously becoming a participant and uncovering the phenomenon of interest.

### 4.2 Becoming a participant

Although becoming a participant was both natural and necessary, I cannot claim that my decision to become a participant was initially premeditated. Rather, it happened in the process of discovering and exploring the space through my genuine stance as a teacher of mathematics who yearned for connection with other mathematics teachers. My quest of becoming a participant in MTBoS spanned several years and involved numerous pivotal moments, each of which in turn precipitated my sense of feeling as though I could
identify more and more as a MTBoSer, and then interestingly, less and less so. In what
follows, I share a chronological account of some of the most pivotal moments this
journey, including discovering the hashtag, trying to contribute, encountering MTBoS
participants within conference settings, being drawn into conversations online,
developing a digital persona, being referred to as ‘famous’, and finally, distancing myself.

As noted in the Introduction, my journey into MTBoS began with my curiosity into what
other mathematics teachers were doing in their classrooms based on what they were
posting publicly online in blogs and on Twitter. This curiosity drove me to searching the
Internet in early 2014 for a variety of search terms related to mathematics teaching such
as “mathematics teacher”, “math teaching”, or “innovative teacher”. Eventually, I was
subscribed to a set of about 68 blog sites managed by people dedicated to discussing
mathematics teaching. I opened my blog feed via Wordpress.com\[16\] daily, and regularly
read new blog posts made by this collection of people. These blog posts often included
hyper-linked text, as in Figure 4.1 below, that linked to other blog pages or resources.

![Figure 4.1](image)

**Figure 4.1  Example of blog post with hyperlinked text (Oliver, 2019, para. 2)**

The most memorable aspect of this initial journey was noticing that three hours had gone
by, I had not eaten, and my browser could hardly fit another tab because I had opened
so many in reading these posts. Blog posts generally range from short to long, and from
thin to thick in terms of density and level of reflection on issues pertaining to
mathematics teaching. A few months into exploring these blog posts, I noticed some of
the bloggers were referring to events that had occurred on Twitter, and in some cases,
were quoting tweets and responding to them in their blog posts. Some bloggers also had
embedded widgets that revealed their Twitter activity in real time. The recurrent

\[16\] Wordpress.com is a webservice that supports creating websites, but also includes a subscription
service to other blogs. It organizes blogs one subscribes to and notified of new material via email.
referencing to Twitter spurred my curiosity in exploring what was happening there. I was scratching the surface, and I knew there had to be more because there always seemed to be more whenever I followed another hyperlink.

Although I had joined Twitter in 2012 as part of a conference I had attended, I hadn’t used my Twitter account much in that time. I dusted off my account and updated my profile picture. I only had a small number of followers at the time and had not been following many others except for some mathematics teachers I had connected with at the conference. The first step to seeing anything of interest without directly searching for it on Twitter is to follow accounts that publish posts of interest. As noted earlier, following an account on Twitter is essentially a subscription that pushes their published content into a feed on one’s landing page where published activity of all one’s followers exists in a timeline fashion and is updated in real-time. Curating a list of followers who publish about a particular interest is one of the best ways to access content around a particular domain of interest. In this case, as may be expected, my domain of interest was that of mathematics teaching. So, my criteria for selecting users to follow involved inspecting their publications to confirm they primarily publish about mathematics teaching. As such, I embarked on my Twitter journey by taking a snowballing approach (Patton, 2002) to finding accounts to follow, which involved starting with a smaller set of confirmed users who fit the criteria, and using information listed on their profiles of who they follow to find further users who fit the sampling criteria. I began by following those who authored the blogs I had already been subscribed to (e.g., @ddmeyer, @Yummymath, @mathhombre, @fawnmpnguyen, @NatBanting, @MathMinds, @jensilvermath, @jreulback, @k8nowak, @mpershan, @cheesemonkeysf, etc.). I then searched through the lists of people they followed and chose to follow anyone who seemed to post only about mathematics teaching from those lists. I recursively repeated this until I had a list of about 350 accounts I was following, all of which seemed oriented around mathematics teaching in their contributions. In doing this, I dedicated my Twitter account solely to engagements around mathematics teaching, including my own participation.

As I navigated my Twitter feed regularly, I took note of things that caught my attention, such as the types of tasks that were being shared and discussed, moments in which conversations occurred, and values held about mathematics teaching seemingly revealed via posts. Mid-way through 2014, there was a tweet that caught my interest.
Many teachers are hard working and passionate. Just not aware of the advantages of #MTBoS They don't realize how amazing. (@jreulbach, July 28, 2014)

Although I had seen the MTBoS hashtag used before (usually at the end of tweets along with other hashtags used as organizational tools or descriptors), it stood out to me more than any other time because it was being used as a noun rather than an adjective. That is, MTBoS was being referred to as a living entity rather than a descriptor. This caught my attention because it was so different than most other instantiations of hashtags I had encountered. However, I still did not quite understand what it meant or stood for. My curiosity in the cryptic ambiguity and personification in this term led me to searching for other instantiations of it. Doing this led me down a rabbit-hole of posts both about mathematics teaching and about what MTBoS is. I learned that MTBoS stands for ‘Math Twitter Blog-o-Sphere’ and is an informally defined space for people interested in mathematics teaching to connect and share resources or ideas.

What is the MTBoS? Well, it stands for MathTwitterBlogOSphere, but it’s not an official entity or anything. It’s just a label that’s thrown around for math teachers and others in the math ed community who talk, share resources and build community on the internet. It’s not centrally organized, no one is in charge, and you don’t have to pay to join. Just teachers who like helping other teachers be a little better at their jobs. There’s also no pressure to contribute. If you’d just like to sit back, watch, and learn what you can, do it. (Kane, n.d., para. 1)

I also learned that ‘MTBoSers’ consider themselves as having a unique flavour in terms of their approaches to mathematics teaching, and generally tend to feel connected to each other as a community of educators.

How can you tell that a teacher isn’t part of your online/#MTBoS community? What are tell-tale signs? (@mpershan, July 28, 2014)

[In reply to mpershan’s above query] I think #MTBoS Ts are more likely to recognize there are other ways of teaching and are open to those. (@MaryBourassa, July 28, 2014)

As such, finding the MTBoS hashtag unlocked a whole avenue of investigation because it helped me navigate Twitter in efforts of finding more teachers of mathematics to follow. In turn, this increased the density of my sample space. By viewing accounts posting with the hashtag, I was able to continue a snowball approach until I followed about 500 users. Following these accounts regularly opened my view in a way that made me feel I was no longer scratching the surface, but that there was a collective living behind the surface.
As I encountered more examples of tweets about mathematics teaching, I began to adjust to and comprehend the linguistic norms in the space that at first came across as incredibly cryptic. Through peripheral participation (i.e., via lurking and actively searching or asking about unknown phrases), I began to develop a capacity to recognize what the cryptic phrases meant. Achieving this attunement involved attending to unrecognizable elements in tweets and pursuing my curiosities into what they meant via either searching or by asking questions. Since hashtags are automatically hyperlinked to a page that reveals all tweets that include the hashtag, perusing these helped illuminate the meanings of hashtags. When other abbreviations are unrecognizable, searching for other uses of them or asking the user who posted it proved useful in identifying what the abbreviation stood for. In the case of tweets about mathematics teaching, developing a capacity to make meaning of not only hashtags and abbreviations, but also general language around mathematics teaching used by users proved necessary.

For instance, the abbreviation SBG stands for Standards Based Grading, which exists in the domain of teaching in general, may be communicated with various hashtags.

SBGers do you just give standards quizzes (tests) or do you also give unit tests? #MTBoS #sbar #sbg #sbr #sbl (@LeslieVFL, July 29, 2014)

Sometimes, however, familiarity with mathematical language, not only teaching language was required. For instance, in the quote below, the reader needs to be familiar with the language of limits and difference quotients to parse the abbreviations.

AP Calc teachers: How much computing formal lim of the diff quotient derivatives do you have students do? #clacchat #mathchat #mtbos (@DamionBeth, July 25, 2014)

And, there could also be abbreviations from not only teaching in general or mathematics teaching in particular, but also from Twitter in general and local contexts more specifically. For instance, in the quote below, spiralling refers to the way content is ordered throughout a term, MFM1P and MPM2D are mathematics course names in the Ontario curriculum, and Ss & Ts stands for students and teachers.

[In reply to @MaryBourassa] #Spiralling MFM1P this year (MPM2D last year). Advantages: Ss & Ts think about connections btwn strands. #mtbos (@annarden, November 5, 2015)
The heavy use of cryptic abbreviations is strongly influenced by the constraint that Twitter imposes on its users with its limits on characters permitted per tweet. For many years, this limit was capped at 140 characters, and the abbreviations often used were created in response to mitigating this constraint while still communicating the desired message. In November of 2017, about a decade after its initiation, Twitter increased its character limit to 280 characters (Perez, 2017) with hyperlinks only counting as a maximum of 23 characters (Twitter, 2019b). Interestingly, this character limit increase had little impact on the average tweet size; however, it increased engagement on Twitter overall (Kastrenakes, 2018). Personally, I noticed users simply communicating the same sorts of messages but with fewer abbreviations. However, many of the abbreviations still remained in use. Regardless, one can quickly acclimatize to the meanings of abbreviations, but it takes time and investigation. I only noticed my ability to understand the tweets when I showed them to colleagues who did not understand them.

Although I was swiftly developing familiarity with abbreviations and norms in the community, I had yet to find ways to meaningfully contribute and be responded to. In fact, there are many who echo this struggle of joining the community. And, this became more public when a suggestion was made to ‘retire #MTBoS’ since it was not inclusive enough of a term, and that another hashtag should be used instead. Meyer (2017) made this suggestion under the premise that many may not know what MTBoS stands for, arguing that a hashtag such as #iteachmath could be more inclusive. His intent was framed with efforts of equity and access, but it came to be perceived as an insult towards those who had been putting time and effort to creating an inclusive space in MTBoS. Part of the issue around this, however, is more global and has to do with how large MTBoS has become in its 10 years of existence and how difficult the processes of upholding values becomes with such a rapid increase in size. Nonetheless, Meyer’s (2017) provocation resulted in great controversy in the network. Many were prompted to share about their experiences of struggling to participate in MTBoS.

I’ve lurked for years, worn MTBoS ribbons at NCTM, & went to TMC16. Still don’t feel like I’m part of the group. (@heyhyers, July 27, 2017)

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17 I acknowledge this is not everyone’s intent in participating in the space, but it was my intent.

18 MTBoS Directory has increased from about 200 listings in 2015 to over 800 in 2019.
I have followed #MTBoS for 6 months, and have wondered if I am smart enough to move from being a “stalker” (@allisonyokeley, July 27, 2017)

While others shared about how inclusive they feel MTBoS is, defending the accessibility and welcoming nature of the community they feel connected with.

But not everyone teaches math. I don’t want to exclude. #MTBoS includes math, twitter, and blogging. All inclusive. Not exclusive. (@jreulbach, July 27, 2017)

Part of what I love about #mtbos is threads for teaching AND threads for general love of math – feel like #iteachmath is only half of that (@cmuirmath, July 27, 2017)

Inclusivity has also remained a prevailing topic throughout MTBoS history.

Doesn’t #mtbos have intrinsic inclusivity? Discussion and interaction open to anyone with a twitter account (@teachbarefoot, October 20, 2013)

Others also noted that it is not really about the hashtag, it is more about what happens within conversations organized by the hashtags.

Genuine interest in a conversation is what makes the community, not a hashtag. (@TAnnalet, July 28, 2017)

However, the MTBoS name has attracted sentimental attachment over the years and has served to identify those who have grown together around mathematics teaching.

(I admit I underestimated and underappreciated the sentimental value our colleagues have attached to m t b o s. Big mistake on my part.) (@ddmeyer, July 31, 2017)

The result of this discussion provoked by Meyer's (2017) post was that tweets about mathematics teaching directed at other teachers of mathematics from then on typically included either #MTBoS, or #iteachmath, or both. Those who agreed with Meyer's arguments incorporated #iteachmath and those who felt more attached to #MTBoS continued to use it. And, some used both for greater visibility. Nonetheless, the point raised around difficulties in ‘joining’ and knowing that one has ‘joined’, even amidst many efforts made by others to welcome newcomers to the space, remained.

In my own experience, I also found it difficult to ‘join’ MTBoS when it was smaller in size in 2014. However, in 2014, I did not realize that ‘joining’ MTBoS was a more global issue and that others felt this same way. I merely struggled through it as an individual. Some
of my first tweets were very diverse. I was tweeting about things that were meaningful to me but that may have not been meaningful to others, or, that may simply not have been seen by others since my follower count was not very high. As I reflect on the posts I made before I found the MTBoS hashtag, they were strongly oriented around what I was learning about in my mathematics education coursework, and I was using hashtags without really knowing what they were meant for (as shown in Figure 4.3 below).

**Figure 4.2 Early tweets to which I received no response**

These attempts garnered no response. However, some of my first interactions with others came from my efforts at reaching out and responding to them (as in Figure 4.3).
Had the students measuring the steps in the stairwell today for slope... students love being out of the classroom. #mathchat #edchat

2:14 PM · Jan 22, 2014 · Twitter Web Client

@JudytaLarsen · Feb 1, 2014

@trevverreeh have you tried the Barbie bungee jump activity from NCTM?

@trevverreeh · Feb 1, 2014

@JudytaLarsen I have, don’t know how my students would react to Barbie’s.

@JudytaLarsen · Feb 3, 2014

@trevverreeh I guess you could use any object, but it’s a great problem that gets them out of the class... you teach gr??

@trevverreeh · Feb 3, 2014

@JudytaLarsen yes and yes.

Figure 4.3  Responding to others as my first form of interaction

After I found the MTBoS hashtag and regularly perused things being shared there with my newly refined view, I began to develop some traction in terms of connecting with other participants through some of the posts I made. This was especially the case when I referred to resources often talked about in the community (e.g., #visualpatterns), linked to highly followed people (e.g., @fawnpnguyen), used the MTBoS hashtag, and expressed my excitement as a teacher (in Figure 4.4 below).

visualpatterns.org --> this is so awesome
@fawnpnguyen! Can't wait to use it for sequences later this term! #mathed #mtbos

8:43 PM · Oct 10, 2014 · Twitter Web Client

@Veganmathbeagle · Oct 10, 2014

@JudytaLarsen don't wait for sequences! Use it now! Linear, quadratic, cubic
@fawnpnguyen

@JudytaLarsen · Oct 10, 2014

Ah! Why yes of course! Thanks for the nudge @Veganmathbeagle

Figure 4.4  My first-ever responded-to tweet
However, when I made attempts at asking MTBoS direct questions with the use of hashtags for visibility, they continued to receive no response. For instance, when I asked, “#mtbos #mathchat how have u used history of #math in ur teaching? Im doing a session on relation of #history and #mathed & need ideas soon” (@JudytaLarsen, February 14, 2015), I received no response. In fact, many questions directed to MTBoS as a whole go unanswered\(^{19}\). This continued to remain a mystery to me. However, while these examples indicate the difficulty in getting response from others in MTBoS, it is predicated on my assumption about response being the aim of participation, which is not always the case. I have spoken with contributors who use the hashtag to post their ideas, but who do not expect a response because they see it as a place to take notes for themselves rather than to communicate with others. Notwithstanding, many others have expressed their gratefulness for the interactions they find in MTBoS, and the capacity to solicit response may be involved in attracting continued participation, as it did for me. In my own personal journey of feeling more connected to MTBoS, in-person experiences with others were incredibly crucial in my ability to soliciting response online.

The most transformative moment in my own journey of becoming a MTBoS participant with the capacity to invoke interactions with others was during an in-person event self-organized by MTBoSers annually, referred to as ‘Twitter Math Camp’, or TMC for short. The first annual TMC was held in 2012 out of a desire for meeting each other in real life after having met online, and initially attracted 37 attendees from three countries (U.S., Canada, and England), none of whom had previously met each other in person (Henry, 2012; Shah, 2012; Waddell, 2019). The first meeting was held for free at the Mary Institute and Saint Louis Country Day School in St. Louis, MO and was organized voluntarily by Lisa Henry from Ohio and Shelli Temple from Oklahoma, who both had been engaging as mathematics teachers via Twitter and who aimed to initiate a space for mathematics teachers from Twitter to meet physically (Waddell, 2019). Waddell, who attended this meeting and later served on the organizing committees for subsequent meetings, indicates that “what made this conference unique is the teachers involved organized the conference events, planned the sessions, and knew each other solely through Twitter” (p. 9). They also actively engaged via social media by blogging and tweeting in relation to conference sessions. The attendees at this meeting created a

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\(^{19}\) In a search for tweets including “?” and “MTBoS” over a two-week period in 2017 included 50 out of 206 questions eliciting no response, and 65% attracting less than two replies (Larsen, 2017b).
website, www.twittermathcamp.com, and due to the synergy at the event, Lisa and Shelli committed to organizing another one in the following year. Organizing committees were formed from MTBoS participants who desired to be involved, and future meetings grew quickly in size, attracting 106 in 2013, 151 in 2014, 175 in 2015, 184 in 2016, 188 in 2017, and 190 in 2018 (Henry, 2018). Organizers kept costs of attendance either free or very low by using donated spaces. In 2016, a non-profit charity organization, TMathC, was formed so that organizers could pay for insurance and request grants. However, in all years, an effort was maintained to keep the conference small and affordable, with a cap of 200 attendees from 2016 onwards. To keep access to the conference equitable, the organization used a lottery approach for registration in 2017 onwards. The effort to keep the size small was intended to make it more likely for relationships during the conference to form and develop (Henry, personal communication, June 24, 2015).

Given the bottom-up emergent formation of this conference and the relevance of it to my study, I knew I had to go to the event when I learned about it in 2014. So, I planned my way there in July 2015. By then, it had become a relatively established conference with multiple concurrent sessions, plenaries, and short five-minute presentations. Excitedly, I packed my bags and travelled to Harvey Mudd College in Berkeley, CA, for my first TMC experience. My flight had been delayed, so by the time I arrived, the first sessions had commenced. I slipped into a morning session, titled “Creating a Culture of Exploratory Talk” (Statmore & Luzniak, 2015), and was pleasantly surprised with the amount of interactivity. We played math debate games in groups of four and discussed how we can use such strategies to motivate mathematical discourse in classrooms. It was a multi-day session that took up the first three hours of each of the three days, so we were able to build off the experiences in the first day throughout the conference. Participating in sessions was one way to form relationships with other attendees, but the diverse array of other conference activity contributed to this as well. For instance, since lunch was provided by food trucks, it provided plenty of serendipitous opportunities to converse with others, and they seemed overall approachable. Crossing paths afforded opportunities for forming relationships. One instance of this was when I met two prominent bloggers in a food truck line-up. At this point, I had heard of them because of their blogs but hadn’t recognized them since I did not know what they looked like in person. I was conversing with them without knowing about their respected social status nor how prominent their blogs were. As we engaged in small talk, multiple passersby
excitedly waved at them. I realized they were important, but I did not quite know why. It was in this moment I realized there was more to the social landscape to uncover.

I further attended to aspects of the social landscape when I engaged in a happenstance conversation with another prominent blogger during the group dinner. He had exclaimed that he felt others all had a ‘thing’ they were known for, but that he did not feel he had a ‘thing’. He pointed to others such as @fawnpnguyen who is known for her visualpatterns.org website, @ddmeyer for his 3-act-tasks, @mr_stadel for his estimation180.org website, @MathMinds for her number talk routines, and @mathequalslove for her interactive notebooks. Meanwhile, when I spoke to some of these people he referred to, they did not consider themselves as having offered anything particularly noteworthy either. And, this blogger was considered by others to be known for his Calculus-oriented virtual file cabinet of resources. That is, there seemed to be a prevailing sense of humbleness among those who were perceived by others as ‘popular’, each feeling as if they were just sharing whatever they felt was helpful for themselves, and hopeful it was helpful for others. Meanwhile, their online presence contributed to their popularity of becoming known for their contributions.

Without yet understanding this social attribute of humbleness and a strong orientation towards welcoming and embracing vulnerability, I had requested to have a moment of time to ask the whole group for volunteers to interview for my preliminary investigations. This was slotted into a plenary session that consisted of multiple members presenting their five-minute “My Favourites” in which they typically shared short examples from their teaching practices of things they were proud of. In my five-minute presentation, I introduced myself as a doctoral student and teacher of mathematics who was awe-inspired by the incredible work this group was producing online and that I hoped to be able to interview some of them. After this session, we were heading to the BBQ dinner. I naturally found others to carpool with, and in the carpool, one participant told me very directly, “I really liked your introduction, except I was very bothered with how you said, ‘you all are doing such great work,’ because you are not including yourself in this work” (fieldnotes, July 23, 2015). It struck me then just how important it was in the group to be welcoming and inclusive of others, and the notion of membership as being something that occurs by default through participating even in the most meagre manner. In other words, that membership is not about contribution, but rather, about showing up. In that
moment, I felt a sense of belonging (see Figure 4.5). It took this sort of interaction for me to gain a sense of connection to the community and to feel part of the MTBoS.

Another noteworthy aspect of TMC in 2015 that I had never experienced at a conference before was that during the conference sessions, most of the participants were tweeting about events while they were happening. Conferences often have ‘back-channels’ (Li & Greenhow, 2015) for participants, but they are not typically as consistently and prominently used by the majority of participants as they are at TMC events (Risser & Waddell, 2018). While some may consider it distracting, it was interesting that there were conversations about the presentations during the presentations. In this way, participants were able to communicate and make meaning about what was being presented in real-time. Some examples of this are exemplified in Figure 4.6.
Tweeting to the hashtag was another way to find belonging and to be valued as a contributing member. The idea could be posted to the hashtag and others could engage with it long after the presentation without having to find a time to fit in sharing an idea physically. In fact, Risser and Waddell (2018) found conference tweeting continuing long past the conference itself for TMC17. Tweeting about conference ideas and events drew focus to ideas rather than people, creating opportunities for participation regardless of what social status one had. I had never experienced such an engaging conference environment until then, nor had I been able to make as many Twitter connections with people I felt I could now more continuously communicate with.

My experiences at TMC in 2015 were so inspiring and energizing that I attended TMC again in 2016. In doing so, I developed an even stronger sense of connectedness with others, some of whom I had met in 2015, and others with whom I shared subsequent experiences. The conference organization was such that it allowed for many serendipitous physical encounters that were not only social in nature but also mathematical. In 2016, one member, @Trianglemancsd, with the help of others, had set up a variety of ‘play’ stations20 around the conference venue where participants would stop and explore the mathematical puzzles that were set up. These sites were intriguing because they mimicked the Twitter platform in terms of attracting social interactions through attractive content. Stations included a very large pattern machine (Figure 4.7a), a spinning turntable with sand (Figure 4.7b), geometric dance moves in a stairwell (Figure 4.7c), and a puzzle table (Figure 4.7d). Such provocations brought participants together and stimulated noticing and wondering in a mathematical manner.

20 The resources used for these stations, particularly in Figure 4.7a and d, were just coming to be available for purchase via talkingmathwithkids.com, designed and managed by @Trianglemancsd. The station in Figure 4.7c was co-developed with @mathinyourfeet and @maxrayrieck.
Aside from intriguing provocations and content that brought participants together, other bonding experiences also contributed to my sense of belonging in the community. Interestingly, these strong bonding experiences were not as closely related to mathematics teaching specifically, but rather, involved shared physical and interpersonal experiences. In particular, during a walk to a TMC16 conference dinner, a few of us witnessed a noncritical drive-by shooting. As a Canadian, this was quite traumatizing for me, but the support I received from others at the conference afterwards was invaluable. To this day, I continue to feel connected with those I experienced this event with. Given the heightened awareness around safety provoked by this occurrence, I felt very strongly supported by some of the participants, a few of them who even walked me home in the rain to make sure I was safe (Figure 4.8). By no means am I saying co-experienced traumatic events are necessary to foster relationships, but that the support and connection I felt through such in-person experiences indicated to me that the community cared about my well-being. It also further illuminated the fundamental complex structures within the collective that are driven by random local interactions between agents that may or may not be directly linked to professional practice, but that have a more global
effect on the collective. In turn, such random interactions engendered in me a sense of confidence in participating and sharing my ideas more freely and openly online. That is, I became more comfortable being vulnerable online after knowing how welcoming those who would be reading my tweets were.

Figure 4.8  Being walked home by conference participants

What is most interesting about these physical conference-related bonding experiences is that conference-related discussions continued online for months (if not years) after the conference. For instance, the #TMC15 hashtag spans from mid-2014 and continues through until 2019, with references to events from the conference still being made recently. That is, each of these events created a sort of ripple effect in terms of continuation of ideas from the conference experiences.

@AlexOverwijk @MaryBourassa At #TMC15 in LA we did an activity in your morning session around quadratics and memorizing words. Is that in a blog anywhere? (@stoodle, January 15, 2019)

In my case, I also found myself tweeting differently after the conference. As seen in Figure 4.9 below, my tweets after TMC16 had a new style. Namely, I began to include photos of mathematized real-life situations along the lines of the notice and wonder culture I experienced at the conference. In reflecting back on these posts, I do not remember making them intentionally, but perhaps a sort of ‘collective effervescence’ inspired me to continue connecting to those from MTBoS who attended TMC16.
Figure 4.9    My post-conference posts inspired by the noticewonder culture

Most importantly, these were made out of a sense of belonging I now felt. And, by no means was I the only one who felt a heightened sense of belonging after participating in the in-person experiences of TMC. Many others shared such sentiments.

I’m home, laying in bed, checking my twitter feed. For the 1st time I feel connected to #MTBoS. Thanks #tmc14 ! (@1mooreorless, July 27, 2014)

However, in-person experiences are definitely not the only opportunity for becoming connected with MTBoS. For myself, the in-person experiences helped me feel confident in sharing publicly from a more vulnerable position. That is, sharing things I was not yet convinced about instead of ‘proclaiming’ well-thought out ideas and taking a stance of ‘thinking with’ rather than ‘telling’. I also became more comfortable sharing reflections about my own practice-oriented experiences.

When asking for observations about them as learners…and me as ‘teacher’ . . . , there was actually a neat comment from one of the participants who said that as the ‘teacher’ I structured the mathematics - I’m still thinking on this...did I? And how so? (@JudytaLarsen, July 12, 2018)

This stance of ‘thinking with’ as I tweeted attracted new opportunities for me to feel the possibilities for learning as a teacher of mathematics and as a mathematics educator. This had to do with being drawn into conversations by either getting responses or responding to a tweet in which others responded to as well. Because I have Twitter set-up on my mobile device, interactions with my tweets initiate notifications on my phone that let me know what someone has responded. There have been many instances in which I was drawn into conversations on Twitter and distracted from whatever else I was doing as I thought deeply about how I could respond. In many cases, I found links to mathematics education research articles or resources to share in backing up my ideas and arguments. In many ways, the negotiation I experienced on Twitter felt similar to that
which I could have with mathematics education colleagues, except that I had time in between replies to look ideas up and ponder more deeply before publishing a response.

Through my participation in the online space, primarily through Twitter and a little bit through my blog, I found myself developing a digital persona. Although this was not intentional, it was a by-product of participating with the constraints I set for myself in limiting activity to issues only pertaining to mathematics teaching. As time progressed, I began tweeting more and more in ways that were oriented around the beliefs I began to hold towards educational practices. Namely, my involvement in observing and implementing practices from Liljedahl's (2016) 'building thinking classrooms' framework began to become evident in my tweets based on my physical experiences being more related to these sorts of practices (e.g., see Figure 4.10 below). In turn, my followers began to include more people who took interest in those sorts of engagement.

![Figure 4.10](image-url)

**Figure 4.10** Example of my increasing engagement around 'thinkingclassrooms'

While I remained cognizant of maintaining a digital persona of someone who posts about mathematics teaching more broadly, my account became more evidently oriented around certain practices more than others. This, in turn, served to develop new relationships with those who shared similar views, which is reflective of the tendency of Twitter as conducive for creating homophily, or 'echo chambers' (Colleoni et al., 2014). In my case, my sense of belonging expanded from closeness to ideas such as visual patterns, noticing and wondering, estimating, using Desmos, etc., to also including vertical non-permanent surfaces, visibly random groups, and rich tasks. Although these
views affected what I began to see, by this time, I had already developed a sense of what it meant to be an insider in MTBoS. And, as I developed a sense of belonging and gained more followers in my contributions to the space through sharing insights into my attempts and experiences with ‘thinking classrooms’, I realized others began to see me as someone who ‘had’ something, just as the blogger I spoke with in 2014 noted about others. This became most evident when I attended the NCTM conference in Seattle in November 2018. At this event, I was approached by a blogger whom I had met and spoken with in 2014 at my first TMC conference. He came to greet me and said, “I feel like I met you before you were famous” (fieldnotes, 2017). This resonated deeply for me because in that moment, I realized this was exactly how others must have felt when they were mysteriously deemed famous after going about their daily activity and engaging online around their practices and experiences. In my case, ‘being famous’ was not something I was yearning for but seemed to be a natural by-product of participation over time, and in turn, completed my journey towards becoming an insider. With my insider lens, I was now able to begin pursuing more rigorous study of MTBoS as a phenomenon of professional activity around mathematics teaching in general. Using this insider lens, I structured subsequent inquiry into the context, but at the same time, began participating less to distance myself from the phenomenon, so that I could observe more.

4.3 My presence in the study

My journey of becoming an insider corresponded with my process of identifying and accessing the phenomenon. Throughout this process, my insider and outsider perspectives oscillated as I moved between the etic and emic, participating as a teacher of mathematics but observing as a mathematics education researcher. Throughout my process of becoming an insider, I recursively reflected on activity, diligently noting my observations and saving evidence of encounters that spurred my observations. In doing so, I developed a capacity for both reflection-on-action, and eventually reflection-in-action (Schön, 1983) as I began to live as both a participant and an observer, at times almost simultaneously. Although I participated so much that I achieved being acclaimed as having ‘fame’, whatever that means, I cannot claim to ever have become a true insider because I have always traversed the insider-outsider perspectives in my participation. However, through this iterative approach in participation, I developed a sense of attunement to processes and values in the community. This growing sense of
attunement was necessarily intertwined with the growth in my personal sense of belonging in the community\textsuperscript{21}. However, as I felt closer to identifying as a MTBoSer, it became evident to me I had to step back and participate less, so that I could observe.

One aspect to this notion of stepping back pertained to the ongoing and shifting nature of MTBoS itself. By participating in the space for five years, I experienced changes in activity over time as they fluctuated in response to changing conditions. When I entered the space, there was a significant amount of engagement around ‘3-act-tasks’, ‘common core state standards’, and ‘which one doesn’t belong’ tasks. These were no longer as prominent in the content published five years later, but were now replaced with ideas pertaining to equity, social justice, noticing and wondering, and finding ways to empower students in the midst of the Republican political landscape, which came to reign after the 2016 US presidential election. Contextual landscapes for participants both in their broader situ and their more local MTBoS space continued to develop. As such, I realized that in order to more deeply attend to the processes involved in MTBoS, I needed to look at a more static portion of MTBoS activity and take on the role of observer more so than of participant. By choosing to become a participant, and then step back into an observer, I enriched my study by harnessing the sensitivities I developed through participation towards a more informed observatory stance in data collection and analysis. This decision to participate less came naturally as I dove into data analysis, but by doing so, I realized there were certain things I would have been unable to see without stepping back. Namely, by holding my emergent ‘concept image’\textsuperscript{22} of MTBoS still in my mind, I could attend to things that weren’t changing. In doing so, I further confirmed for myself just how fluid and adaptive it is to perturbations such as political events, curricular mandates, and other significant events that affect members. Although I already attuned to how MTBoS shifts and grows in terms of topics discussed and people who become prominent, finding a way to ‘freeze’ activity and my experience of it in the collective was a crucial component towards deeper inquiry into its processes. As such, my choice to step back during the final phase of this research was both natural and necessary.

\textsuperscript{21} I use the term ‘community’ here loosely because to me, it felt as though I was part of a community. However, I do not presume the MTBoS as a whole, to be a community per se.

\textsuperscript{22} I borrow this term from Tall and Vinner (1981), who use it to describe a residual mathematical concept image, and use it instead, as a way to describe the residual imagined ideas of MTBoS.
In summary, participant observation afforded me a deeper and closer view at the phenomenon, but my choice to step back was necessary in maintaining the integrity of the research and to bring analytical focus to the collective layer rather than an individual one. The nature of the phenomenon fundamentally required my participation, not only peripherally, but in a way that equipped me with the capacity to act within it. However, since my object of study was not around how to become a participant, but rather, on how individuals create emergence in MTBoS, my insider view only served to strengthen my approaches and interpretations. More specifically, it was used primarily in methodological choices of data collection and an enhanced ability for interpretation within analysis. My participation during data collection was minimal and therefore, did not significantly affect the results of this research. At best, it merely strengthened the integrity of the research by contributing to a more informed approach.
Chapter 5  Methods

The singular and complex nature of MTBoS invites methodological consideration in choosing pertinent data and suitable methods for analysis. Although ethnographic approaches were used to identify, access, and interpret activity in the MTBoS collective, complexity theory served as the primary theoretical framework in this study. As such, its tenets around conditions for complex emergence were fundamental in structuring methods towards answering the research questions. These questions (outlined in §3.4) are most concerned with identifying processes involved in complex emergence in MTBoS, but also reveal an overarching aim of developing an explanation for the sustainability of professional activity around mathematics teaching in MTBoS. In studying complex adaptive systems, Davis and Sumara indicate researchers must decide on the level of organization being attended to. To this end, although I primarily take interest in the collective layer in the MTBoS, I acknowledge it is fundamentally created by the individuals who form it. As such, the level of organization I am most interested in is the connective layer between the collective and individuals who form it; that is, to me, the source of emergence. To this end, I decided to first consider the collective and its emergence, then individuals and how they contribute to its emergence, and then at how these layers co-act to affect emergence in the collective.

My methodological decisions towards data collection and analysis were therefore aimed at conceptualizing the processes involved in how individual activity creates global patterns, and in turn, how these patterns direct further activity in the collective. Towards such an investigation, a selection of data had to be chosen. Aside from perspectives from complexity theory, which indicate that complex adaptive systems shift and transform over time, the specifics of data selection were mostly informed by my insider perspectives developed through ethnographic processes. These perspectives afforded me a broader view on the collective over time and attuned me to how frequently the collective shifts and transforms in response to changing conditions. Although shifts in the system are naturally unpredictable, my attunement as an insider allowed me to select a timeframe that most appropriately represented robust activity in the collective without including too much opportunity for it shifting or adapting to changing conditions within the timeframe. My insider perspectives also served in the interpretation of data.
The nature of the data itself also had to be considered since its features both contributed to and represented complex activity in the collective. As such, once data was collected, methods for analysis were not only informed by perspectives from complexity theory and my insider sensitivities towards interpretation, but also, perhaps more simply, by the nature of data itself. That is, the features available in data fundamentally guided how it could be parsed and analyzed. Therefore, methods for data collection and analysis were informed by tenets of complexity theory, insider perspectives through ethnography, and inherent structural features of data itself. In what follows, I outline the nature of data, the data selection process, the data sources selected, approaches taken towards analysis, and how these processes contributed to answering the research questions.

5.1 The nature of data

As with any context of study, data can be conceived of in various ways in terms of how it is chosen, collected, and analyzed. Aside from the research questions, features of the context fundamentally guide and constrain what counts as data and which kinds of data are pertinent to the research. In the case of Twitter, potential sources of data are constrained primarily by the dominant form of activity in the space, which in this case, is via tweets. If we consider the anatomy of a tweet, as shown in Figure 5.1 below, it contains various forms of information pertaining to the user who produced the tweet, the tweet content, and how the tweet was responded to.

Figure 5.1 Tweet anatomy
The information about the user account from which the tweet originates can reveal the followers the user has, the accounts they follow, the ‘likes’ they have made, brief biographical information, personal websites, a Twitter join date, and sometimes, a location. All this information is voluntarily provided by the user who manages the account and can be retrieved by clicking into their profile page. At times, it is anonymized if desired. In general, however, the primary mode of interaction on Twitter is not by clicking into other users’ profiles, but by viewing and interacting with tweets that are made by users one follows as they are published and collated together in one’s feed. When viewing activity on Twitter in this way, a participant most prominently sees the content that tweets feature, as is evident in the example shown in Figure 5.1 above or in any of the tweets exemplified in the rest of this thesis. Content may include text up to a maximum of 280 characters (previously 140 until November 2017), hyperlinks to external sources, hyperlinked hashtags, hyperlinked accounts of other users, hyperlinks to previous tweets published, and media in the form of either a series of images, a gif, or a short video that is under two minutes and 20 seconds (Twitter, 2019c).

In terms of feedback mechanisms, a tweet has the option of being responded to with ‘likes’, ‘retweets’ or ‘replies’, the numerical counts of which are tabulated underneath the tweet. While it is clear which users ‘replied’ to a tweet, it is more difficult to see all the users who ‘retweeted’ or ‘liked’ a tweet since there are limitations on this information. In terms of function, ‘likes’ are typically made when a user wants to bookmark the tweet in their ‘likes’ list, which also publicly available on their profiles, or if they want to communicate their appreciation of the tweet (Fussell, 2017). In making a ‘retweet’, a user shares the same content to their own followers as if they made the tweet, but it is indicated that it is a ‘retweet’ and the originator of the content is acknowledged. The user has the option to either directly ‘retweet’ or do so with a comment, and ‘retweet’ count only includes direct ‘retweets’. Further, the tweet may be ‘replied to’ by any users who can view it, including the originator of the tweet. Twitter organizes ‘replies’ into threads, and the ‘reply’ count includes the number of total threads. However, subsequent ‘replies’ may be made in response to any ‘reply’. This can result in a tree-like structure of responses, where each thread has a different ‘depth’, as shown in Figure 5.2 below.

23 Including hyperlinked account names (or ‘handles’) in a tweet is referred to as making a ‘mention’.
As may be evident, the features of the Twitter platform offer possibilities for activity that differ significantly from in-person settings. Aside from the multi-media capabilities, Twitter distorts temporality. Every time an utterance is made, it is public, permanent, and traceable. Once a tweet is published, it cannot be edited unless an author chooses to delete it, and it remains public unless deleted by its author or if the author decides to limit its visibility only to their followers by making their account private. This means tweets can be interacted with long after they have been made, and even if a tweet is deleted or not shown because an author has made their account private, any replies to the tweet remain public, ready for further interaction. Tweets are also often not directed at anyone in particular, but rather, to the vast void of potential respondents, or, an ‘imagined audience’ (e.g., Litt, 2012). They are also informal and fleeting in nature, offering a form of communication between the realms of publication and conversation.

If such features were available in a physical setting, it would be as if a participant were standing at a street corner proclaiming a statement with attached multimedia.

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24 Unlike traditional forms of publication, tweets do not undergo a review process and are not held to the same level of rigor. However, they are publicly accessible and traceable to their authors. This presents a dilemma for attribution. As Bruckman (2002) argues, “the Internet can be seen as a playground for amateur artists creating semi-published work” (p. 217). Through this lens, some level of disguise to authors of tweets is necessary depending on context. As Bruckman suggests, this may range from no disguise to heavy disguise. In this thesis, I have chosen to provide ‘light disguise’ to authors of tweets in the data set by not including their usernames or dates when presenting their work in the results chapters. This is because their tweets are being treated as data rather than as literature to be cited. Doing this protects authors from being associated with the analysis and focuses readers on the narrative. In areas where tweets are presented without analysis, usernames and dates are provided. However, no tweets are cited in the reference list. These choices are in line with the theoretical perspectives of this work as well as my ethical intuition.
visualizations and links to related content, and then their comment would remain floating in the air, available for others to interact with at any time after they have left. Now imagine this were scaled up to hundreds if not thousands of people doing this daily. This would result in a world polluted with floating content that no one person could ever glimpse into completely. Clearly, such a space would be physically impossible to create within an in-person setting. However, it is essentially what may be found on Twitter and other social media settings. The sheer mass of data created through such processes, however, poses a methodological challenge in studying it. Although there is a mass of data available, it is physically impossible to review it in its entirety because of its size and constant evolution, with new material being produced at an alarming rate.

The nature of this setting is therefore intriguing in terms of offering new possibilities for activity, but also challenging to navigate towards research efforts, as noted in the introduction to this chapter. As such, it invites methodological consideration in terms of how it is to be studied to yield insight into the emergence it holds space for. In this regard, I was most significantly guided in my research process not only by tenets of complexity theory and framings from related literature, but also with my insights from the ethnographic approach I took in becoming a participant over a five-year period. During this time, I developed a sensitivity to the history and development of the collective, which helped me make decisions about how to select and work with data in this space. Namely, how the data I collect could inform me of the inner workings of the collective in terms of what it is, how it is formed, and what it reveals about mathematics teacher professional activity. Ultimately, this involved selecting and segmenting a portion of MTBoS activity to preserve it in a specific timeframe that was short enough to capture MTBoS as relatively stable, but long enough to reveal its robust nature.

5.2 Data selection

As my participant observer view became more connected to the internal community of the MTBoS through the various experiences of engaging in the space, I began to realize that just like all complex systems, MTBoS was shifting and changing in response to outside conditions. These conditions included government elections, school shootings, mandates around the Curriculum State Standards, provocative occurrences at conferences or online, and other thwarting events. These events and perturbances ranged from small scale to large scale in terms of influence on the system. Because of
this in-flux ongoing nature of the MTBoS as well as the sheer density of data consistently being made available in this space, I had to ‘freeze’ a portion of MTBoS activity to analyze it more comprehensively. Doing this involved careful consideration of both which search term to use for tweet selection and a strategy for segmenting the data in terms of temporality in light of the shifting nature of the system.

Although it may seem obvious the search term would be the MTBoS hashtag, it is noteworthy to mention that users do not always include the MTBoS hashtag in all their posts. And, they typically only include the hashtag if they both identify with MTBoS and want to direct their tweet to participants in MTBoS. Some users do not use the hashtag at all once they have enough followers and feel their messages are seen by the followers they want to be seen by. Others may simply not know about the hashtag or choose to not use it. Alternatively, other hashtags may be used for discussing mathematics teaching, such as #mathchat, #mathed, or #iteachmath. Using my insider view, I developed the sensitivity that many of the other related hashtags were used for various forms of specificity (e.g., #msmathchat was used for a more synchronous chat around middle school mathematic teaching) and were often used in combination with #MTBoS when the user wanted their message to be seen by the MTBoS collective. However, as discussed in §4.2, there has been debate around the use of the MTBoS hashtag more recently. The hashtag #iteachmath was born out of this controversy (Meyer, 2017), which is driven by arguments around the ambiguity of MTBoS as a term, perceptions of it being unwelcoming despite strong efforts by various members to be welcoming, and considerations about equity. The result of this perturbation about choosing and identifying with a hashtag was that participants in the collective either continued to use #MTBoS or included both #MTBoS and #iteachmath in their tweets. Otherwise, those who identified with the reasoning behind #iteachmath along with newcomers who felt it was more appropriate for them, began to use that instead. However, the quantity of posts being made with the MTBoS hashtag did not diminish significantly and continued to thrive with content about mathematics teaching aimed at people interested in mathematics teaching. As such, I chose to keep #MTBoS as the primary search term for the purposes of this study.
In terms of segmenting a time for data collection, I initially made several different attempts at segmentation: by conversations within threads and across threads\textsuperscript{25}, by zooming in on key influencers and hashtags\textsuperscript{26} in different ranges of time, and by parsing it by certain search queries in various segments of time\textsuperscript{27}. The key issue with all of these in terms of capturing enough activity for the data to be ripe for analysis while limiting the data was around the shifting nature of the collective over time. While I had already determined that my inquiry was oriented towards the co-activity between the individual and the collective levels of organization, I had yet to determine an appropriate unit of time to constrain my view. In my preliminary work, I found that when creating network maps co-occurring hashtags in MTBoS tweets, single day collections offered too much divergence in prominent topics among days\textsuperscript{28} and a month of tweets was simply too large\textsuperscript{29} to manage with too many topics to consider. However, a week offered enough stability in terms of related topics without being either unmanageable or seemingly volatile in terms of susceptibility to that particular day’s events.

Further, in data segmenting, not only was time duration important to contemplate, but also the time placement within the academic school year. I decided on selecting a week in late September after playing around with capturing a variety of timeframes in which I explored hashtag relation graphs. While these did not give much information about depth of discussion or data richness, they indicated trending topics. Some timeframes had significant amounts of activity around certain time-specific hashtags such as if a conference was happening, if it was during school start-up, or during exam periods. Therefore, my aim was to avoid significant events that steer tweet content towards certain topics with the hope that the data collected would be relatively representative of more typical activity in MTBoS. Late September was chosen because it is after summer break, after the typical ‘back to school’ wave of tweets about starting up classrooms, but before any major events such as report cards that often sweep away teachers’ attention from November onwards. I wanted to avoid such timeframes since the discussions were

\textsuperscript{25} i.e., exploring replies stemming from a significantly responded to tweet (Larsen & Liljedahl, 2017).
\textsuperscript{26} i.e., creating network maps of hashtags used concurrently with #MTBoS (Larsen, 2017a)
\textsuperscript{27} i.e., collecting tweets that included both #MTBoS and a question mark symbol in its contents from a two-week-period to explore the nature of questions asked by participants (Larsen, 2017b).
\textsuperscript{28} For instance, Fridays often have many tweets pertaining to the fact that it is Friday.
\textsuperscript{29} Months of #MTBoS tweets within 2018 averaged at about 25,000 per month.
too obviously swayed by these influential events in the life of an academic year. As such, the primary source of data that I have used for this thesis is tweet data stemming from a collection of tweets that include the MTBoS hashtag from September 21, 2018 to September 28, 2018. Although the selection is in some ways arbitrary, and in other ways opportunistic, it serves as a snapshot of a relatively unperturbed state of MTBoS when the teaching term is starting to fall into its normative state and no significantly impactful events (such as political disruptions) seemed to be affecting the discussions. This choice was made with the intent of offering the most representative perspective on the space possible within an ever-emergent phenomenon. However, it should be noted that the primary intent of this thesis is to explore processes involved in emergence in the collective rather than what MTBoS as a collective is overall, which is never static.

5.3 Data sources

While the primary source of data for this thesis is taken from the collection of tweets identified in §5.2, information extending from these tweets as well as other supporting data sources were used. In what follows, I explain the construction of the core data set that was built from this collection of tweets from late September of 2018, and then explain supporting data sources such as field notes, saved tweets, and targeted searches, that at times extended outside of the initially selected time frame. As explained in §5.2, the primary source of data in this thesis is taken from a collection of tweets drawn from the 7-day period from September 21, 2018 to September 28, 2018. This data was collected by using a 'Twitter Archiving Google Sheet' (Hawksey, 2014) to archive tweets that included the search term 'MTBoS'. Due to Twitter rate limiting (Twitter, 2019a), it only allows applications to scrape data from the past 7 days (Twitter, 2019d). As such, at the end of September 28, 2018, I ran the search algorithm and the archiving sheet was populated with data on all tweets that included the term 'MTBoS' over the preceding 7-day period. The information included in this archive for each tweet included the username of the account that originated the tweet, the content of the tweet with links to media or webpages, the timestamp of the tweet, a hyperlink to the tweet, and other information about the users, hashtags, locations, and languages.

The initial archive of tweets with the search term MTBoS from this timeframe included 6146 tweets after removing instances of tweets from outside the date range and any that
were written in a language other than English or French\(^{30}\). These 6146 tweets were made by 2948 unique user accounts. However, 4653 of these were direct retweets, which are exact replicas of original tweets that were replicated in the data archive because they were made by different users than the original tweet was. These direct retweets were either replicas of tweets made from outside the date range, or from inside the date range. Direct retweets are different than quoted retweets, which are retweets where the new user links to the original user’s tweet but adds their own comment. Quoted tweets counted as original tweets in this dataset as they were nearly impossible to differentiate from original tweets in the way their content was presented in the archive sheet. However, although all 6146 of these tweets and retweets remained as study data, this collection had to be diminished to allow for deeper qualitative analysis.

While it was evident that a random selection of this data was necessary to make for pragmatic reasons due to the time required for analysis\(^{31}\), there remained a choice between either randomly selecting tweets from the entire archive captured, or only from the original tweets within the archive. Given that my primary interest (as informed by complexity thinking) was to uncover an ideational network that could represent a space of ideational possibility within the collective, only original tweets were sampled from. This choice was made to maximize the space of possibility rather than to account for temporally influenced factors. Including direct retweets in the sample space would have skewed the network towards the duplications arising from randomly selected retweets rather than from duplications in content and would also place emphasis on what a single user may encounter rather than on a snapshot of a sample space of ideas in the collective. However, retweets were not ignored in this research, and their influence is further explored in Chapter 8 where instead of being included in the core data set, they were accounted for by considering the retweet count on the original tweets that ended up being selected, which was more reflective of their influence on the ideational network. As such, with the primary aim of increasing the scope of inquiry and including as much diversity of content as possible within the formation of an ideational network, direct retweets were removed in the construction of the core data set, leaving 1493 original tweets made by 694 unique user accounts.

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\(^{30}\) Only two of the collected tweets were written in French, the rest were in English.

\(^{31}\) It took me about two weeks to analyze a sample of 100 tweets in a preliminary study.
Each of these 1493 tweets were assigned a unique ID value so that any analysis related to them could be traced to which tweet it related to. Although initial analysis was attempted on this set, it quickly became evident that 1493 tweets along with anything those tweets linked to was _still_ too much data to analyze rigorously with multiple stages of analysis. Since complex systems are typically scale-free networks in that they have similar structure at various levels of organization, taking a randomized 30% sample maintained the integrity of the data while making comprehensive qualitative analysis feasible. As such, a random sample was selected from the 1493 unique tweets by using a randomizing function in Excel to select 30% of these tweets. This resulted in a set of 444 unique tweets made by 322 unique user accounts\(^{32}\), which along with any other material that was linked to them, such as replies\(^{33}\) (before or after the tweet), webpages, and linked multimedia, composed the core data set for this thesis. The process of constructing this core data set is demonstrated in Figure 5.3.

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\(^{32}\) Note that the average number of tweets per user is lower in the random sample since such a high proportion of users who contributed the unique tweets had tweeted only once during the time period. This resulted in a higher proportion of users in the random sample who tweeted only once.

\(^{33}\) If a given tweet was a reply itself, the complete thread of which it was part of was considered.
however, additional information pertaining to them was gathered as needed via searching publicly available online content. Table 5.1 summarizes all information collected for each of these 444 tweet clusters in the core data set.

Table 5.1 Tweet information collected for each tweet cluster in core data set

<table>
<thead>
<tr>
<th>Information</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tweet ID for root tweet</td>
<td>From original numbering of the 1493 root tweets</td>
</tr>
<tr>
<td>Hyperlink to online version of root tweet</td>
<td>Auto archive spreadsheet</td>
</tr>
<tr>
<td>Root tweet content – body text</td>
<td>Auto archive spreadsheet</td>
</tr>
<tr>
<td>Root tweet content – hashtags included</td>
<td>Auto archive spreadsheet</td>
</tr>
<tr>
<td>Root tweet content – hyperlinks</td>
<td>In tweet content, explored online</td>
</tr>
<tr>
<td>Root tweet content – multimedia (images/videos)</td>
<td>In tweet content, explored online</td>
</tr>
<tr>
<td>Linked content – replies in thread of root tweet</td>
<td>Explored online via tweet hyperlink</td>
</tr>
<tr>
<td>Root tweet timestamp</td>
<td>Auto archive spreadsheet</td>
</tr>
<tr>
<td>Username for account posting the root tweet</td>
<td>Auto archive spreadsheet</td>
</tr>
<tr>
<td>Location of user of root tweet</td>
<td>Auto archive spreadsheet</td>
</tr>
<tr>
<td>Number of followers user of root tweet has</td>
<td>Auto archive spreadsheet</td>
</tr>
<tr>
<td>Number of accounts user of root tweet follows</td>
<td>Auto archive spreadsheet</td>
</tr>
<tr>
<td>Number of hashtags included in root tweet</td>
<td>Identified automatically via spreadsheet search function</td>
</tr>
<tr>
<td>Content in threaded replies and linked material</td>
<td>Explored online via tweet hyperlink</td>
</tr>
<tr>
<td>Number of likes*</td>
<td>Identified manually via tweet hyperlink</td>
</tr>
<tr>
<td>Number of retweets*</td>
<td>Identified manually via tweet hyperlink</td>
</tr>
<tr>
<td>Number of replies*</td>
<td>Identified manually via tweet hyperlink</td>
</tr>
</tbody>
</table>

*NOTE: number of likes, retweets, and replies were counted manually by considering their maximum values as made in the most responded to tweet in the cluster.

Although tweets most prominently feature content, they are created by people. These people are by nature participants in MTBoS through their choice to post publicly about issues related to mathematics teaching or about MTBoS and especially since they have included the MTBoS hashtag in the content of their tweet. While this is not the only way to participate, and participation also includes reading, liking, or retweeting content, the people who created and publicized the root tweets in the core data set made a public contribution to MTBoS, and in turn, to the data. As such, rather than calling them ‘participants’ or ‘subjects’, I refer to those who created the root tweets included in my data set primarily as ‘contributors’. However, when speaking more generally about them, I also refer to them as ‘users’ (see Glossary). So, in the collection of 444 root tweets, there were 322 unique contributors. Therefore, the contributors included in the data set were only those who contributed a tweet captured through the auto-archiving process. Given that the auto-archiving process only automatically captured user information for root tweets and not for anything else attached to them, adding information about other
involved users would have required identifying it through brute-force processes. Although such information could have helped develop an even more refined image of the social network, it did not seem necessary for the purposes of this research given the strong set of 322 contributors that was already available in the auto-archived data set. Instead, follow-up information was identified on an as-needed basis if it was deemed relevant and necessary within the exploration and interpretation of certain threads. However, the 322 contributors served as the primary data source for the social network.

As such, publicly available identifying features of the 322 contributors were included as data. These features included the number of accounts contributors followed, which accounts they followed, the number of accounts that followed them, which accounts followed them, their positions or roles (e.g., elementary teacher, secondary math teacher, math coach, professor, etc.), their websites or blogs, when their accounts were created, their listed locations, and the tweets they had liked, retweeted, or made. While the focus of this research is primarily on the collective layer of organization, including attributes of individuals contributed to the inquiry into how the collective emerges.

In summary, the 444 tweet clusters in the core data set composed the primary source of data. However, other sources supported the interpretation of the contents in these tweet clusters were also included as data. This included field notes and saved tweets made during the ethnographic process and information found through searches made on an as-needed basis in the interpretation of the data. All possible data sources and their uses are summarized in Table 5.2 below. As may be expected, the purpose of having more than a single form of data is for triangulation towards a more accurate and informed interpretation of the significant results these data emerged.

Table 5.2  Summary of data sources

<table>
<thead>
<tr>
<th>Data source</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core data set of tweet clusters from root tweets</td>
<td>Includes all information contained in or linked to the 30% selection of root tweets that included the search term MTBoS from September 21-28, 2018, including tweet content, content in replies and links, and information about contributors of these root tweets.</td>
</tr>
<tr>
<td>Supporting data set of tweets</td>
<td>The complete set of tweets that included the search term MTBoS with retweets before the 30% selection was made, as collected from September 21-28, 2018 and used when necessary to support inquiry into the full week of data.</td>
</tr>
<tr>
<td>Data source</td>
<td>Explanation</td>
</tr>
<tr>
<td>------------------------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Saved tweets and threads</td>
<td>Saved occurrences of tweets throughout the ethnographic process of participation between 2014 and 2019 based on instances that caught my interest as an observer and used as support in additionally exemplifying observations and findings in the analysis of the core data set.</td>
</tr>
<tr>
<td>Targeted searches</td>
<td>Information from searches made in the process of interpreting information found in the core data set, used to support findings.</td>
</tr>
<tr>
<td>Field notes</td>
<td>A collection of field notes made throughout the ethnographic process of participation between 2014 and 2019 including personal reflections on participation, notes about seemingly significant events, and notes from informal conversations with other MTBoS participants either in-person or online in my role as a participant observer. These notes are used in support of findings, to contextualize my perspective in this research overall, and in the interpretation of data in the core data set.</td>
</tr>
</tbody>
</table>

### 5.4 Parsing and analyzing the data

Given the systemic features available in tweets, data naturally provided three structural components around which it could be organized and considered (see Figure 5.4 below): tweet content, tweet contributors, and forms of response garnered by the tweet.

![Figure 5.4 Fundamental components of a tweet for organizing data](image)

Each of these offered both *quantitative* and *qualitative* information. Namely, contributors each had a certain number of followers and a certain number of people they followed but they also had qualitative attributes in their profiles. Content was mostly qualitative but could be categorized by types of content and tabulated quantitatively. And forms of response were primarily quantitative but could be explored qualitatively in rationale explaining why certain tweets received more response than others. As such, exploring this data set necessarily involved a mixed methods approach since quantitative features were helpful in organizing data towards qualitative analysis. Namely, data was organized
around each of the fundamental components available in a root tweet, offering three views: (1) one oriented around content, (2) the other oriented around contributors, and (3) another oriented around forms of response.

Orienting data around each of these views provided different windows on the space and broadly served to illuminate how the “ideational network . . . rides atop the social network” (Davis & Sumara, 2006, p. 143). Orienting around content offered a possibility to explore the ideational network. Orienting around contributors offered the possibility to explore the social network and its co-activity with the ideational network. And, orienting around forms of response offered a view on how the social and ideational networks work together towards emergence. Each of these three views also enlisted the use of other constructs identified in Chapter 3 within their respective analyses as necessary. Since pursuing analysis of data within each of these views involved significantly different processes and foci, they are only broadly outlined in what follows, and then more specific methods for analysis are detailed within each of the chapters that present the results stemming from inquiry into each view. As such, in what follows, I broadly explain my approach towards orienting data around each of these views, the general ways in which I analyzed the data in each view, and overall how I pursued cross-feature analysis. I then discuss how these general approaches contribute to and align with the research questions that guide this study.

**Orienting around content**

Content in tweets is by far the most qualitatively rich component in the data set overall and serves as a window into what the ideational network of MTBoS could be, even if it is only for this particular timeframe of data collection. Although Davis and Sumara theorize that there is an ideational network in a complex adaptive learning system, they do not provide guidance on how it may be constructed out of data. This may be because such a network could be challenging to construct from data available within self-organized in-person settings. However, the Twitter context offers a unique opportunity to explore ways in which an ideational network can be constructed because tweets create natural traces of ideational activity that are relatively accessible and uninfluenced through any sort of governing structure, unlike in physical settings. As such, ideational content in tweets can be used to build an ideational network that may serve to represent the ideational space of MTBoS in a certain slice of time. Although tweet content could have
been analyzed in other ways, for instance by examining tone, discourse, intentions, hashtags, or other attributes, my focus in analyzing tweet content was primarily aimed at uncovering an ideational network, as guided by tenets of complexity thinking. I also chose not to pursue automatic textual analysis, as other studies of the Twitter context do (e.g., Bruns & Stieglitz, 2013), because I wanted to account for more nuanced details within ideational content beyond that of commonly used words or reoccurring hashtags. Overall, the choices in building and examining an ideational network were ultimately driven by tenets of complexity theory and its interest in emergence.

As such, I designed a process of constructing an ideational network out of tweet content data that aimed to produce the most representative ideational network for the space and that resonated most closely with my personal ‘concept image’ of the ideas in MTBoS I had experienced as a participant. To this end, tweet clusters served as a unit of analysis and the ideas and relations between these ideas were identified and recorded in a way that would reveal the ideational network representative of the contents found in the 444 tweet clusters of the core data set. The global aim of this process was to avoid over-generalizing the nuanced nature of the ideas discussed within threads and across tweets, and instead, attend to their unique and elusive nature more directly. That is, rather than reducing tweet content to simpler thematic codes, I aimed to capture and account for the robust nature of how each tweet cluster presented a network of ideas.

To this end, each tweet cluster in the core data set was coded for ideas communicated by the root tweet as well as anything attached to the tweet, and codes were developed through an iterative process of attending to redundancies and diversities among ideas. Given that these ideas were fundamentally reifications of participation in that they had taken form through publication, I came to refer to them more specifically as ideational artefacts. In addition, not only did I consider the ideational artefacts that were being communicated in each tweet cluster, I also took interest in how these artefacts were being related with each other, which served to develop the overall network. As such, my process included identification of ideational relations that linked ideational artefacts in various ways such as through being referred to together in a statement, across media within the tweet, or through conversational means. In all cases of coding, my ethnographically informed insider view served as a guide towards making interpretive

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34 Reifications as in traces resulting from participation, as referred to by Wenger (1998).
judgements. Using this perspective enriched the analysis by allowing me to interpret the meanings of content beyond that of what was conveyed in its multi-medium form. However, in all cases, I kept track of supporting evidence that contributed to my coding choices, and at every stage, I questioned my interpretations so that they remained consistent within and across data points. Further details about this process of building the ideational network are revealed at the beginning of Chapter 6.

The resulting implication of coding for ideational artefacts and relations was that an ideational network could now be built from the combination of all relations between artefacts. This resulting ideational network revealed not only the most common ideational artefacts, but also those that were more central in their connections to other artefacts and those that were more isolated from other artefacts in the network. In complexity thinking terms, this means that an ideational network built in this way held the capacity to be examined according to sources of ideational coherence and randomness, which at the collective level, are necessary for complex emergence. As such, once the ideational network was built, processes for examining it were pursued. These necessarily involved using some of the quantitative features gleanable from the developed network to more deeply examine prominent sources of ideational coherence. Therefore, tenets of complexity thinking were used both to build the ideational network and to then to examine it further. More specific methodological details regarding these processes are outlined at the beginning of Chapter 6. However, what is important to note here is that the methods used in building and examining the ideational network were inherently emergent as they attended to the structures and features of the data.

**Orienting around contributors**

Aside from the content, which is the most prominent source of data stemming from a tweet, the contributor of the tweet is also an important factor and cannot be ignored. Although it was ignored in the analysis of tweet content, the second view of analysis attended to the role of contributors in carrying the contents of their tweets into the network. Contributors can affect how a tweet is interpreted, as well as if it comes to be seen by others in the network. That is, the features inherent in the Twitter environment dictate which content becomes more privileged, and much of this has to do with the contributor of the content. This is because contributors have various attributes that allow them to be either more or less visible and for them to see more or less activity in the
As such, the attributes available for each contributor create a possibility of organizing the data around their attributes to explore the effects of their positions in the social network. Although there are many attributes that can be used to distinguish between and categorize contributors, the purposes of this research aim to investigate how the MTBoS collective emerges through individual activity and how this activity is sustained. Contributor attributes are important in this regard because of who they are, what they see, and who sees them.

As such, these perspectives were used in orienting the data around attributes of contributors and then to analyze the content produced by contributors in each organization. To this end, each possible organizing attribute was considered and explored further. This involved identifying contributor roles for each of the 322 contributors to the root tweets in the data set to identify them by their self-reported occupations in relation to mathematics teaching. Namely, this included categories such as elementary teacher, secondary mathematics teacher, teacher leader, consultant, etc. Once categories were determined, tweet data was organized by each category and explored qualitatively by systematically looking for redundancies and diversities among tweets in categories as well as considering the ideational networks produced by each group of contributors. Results of this investigation were considered based on how strongly these categories affected significant changes in redundancies per category.

Additionally, guided specifically by the notion of a social network living under the ideational network, a social network was also constructed so that categories defining social positions could be created and the contents produced by each category could then be explored by attending to redundancies and diversities within and across categories. A very intentional choice was made in the construction of the social network to keep the social network limited to relations remaining strictly within the core data set. The Twitter platform naturally indicates social relationships through following relations. While these do not indicate inter-personal relationships per se, they indicate who sees who in the network. The fact that Twitter followings are unreciprocated means that one can follow others without being followed back. This creates two variables that define a contributor’s social location: their incoming followers, and their outgoing follows. And, to create the social network and the categories within the network, only those relationships that remained within the set of 322 contributors were considered. To achieve this, I used the application Twittonomy (Diginomy, 2018) to collect information about the accounts.
that each contributor followed, and then through a matching algorithm in Microsoft Excel, identified only those who were also contained in the set of contributors in the core data set. Through this process, I emerged a list of directed following relationships between the 322 contributors, which could then be mapped as a network diagram as well as a scatterplot indicating each contributor’s number of followers versus the number of following, the specifics of which are revealed in Chapter 7. For each of the identified regions, tweet content for those contributors was explored, and through attending to redundancies and diversities among these sets of tweets, themes were emerged to characterize the nature of content produced by each of the regions of participation.

This process of categorizing contributors by orienting features and analyzing their content contributions qualitatively by attending to the redundancies and diversities among tweets in each category and across categories illuminated the inter-woven effects that the social network has on the ideational network. That is, how the social attributes of contributors carry the ideational content and ultimately, how they affect emergence in the network. The choice to attend to redundancies and diversities among contributions within and across categories was made to highlight the possibilities for specialization in the network through individual agents who act in the collective. A combination of internal diversity and redundancy is a necessary condition for emergence, so attending the diversities and redundancies in relation to ideational content allows for illuminating the nature of specialization and how it prevails in MTBoS activity. Analytical rigour was reinforced by using iterative coding approaches within attending to redundancies and diversities within and across categories. This was selected to reflect the tenets of complexity thinking within the methodology and contributed to the development of themes that held the capacity for characterizing categories according to the redundancies within them and highlighting the diversities among them. Coding approaches were performed iteratively by developing codes and comparing them to previous codes, altering where necessary, until a point of saturation occurred. Further details on methods used to explore and analyze data oriented by contributors is provided within Chapter 7 where more detailed references can be made to results that directed my attention to certain features more than others.
**Orienting around forms of response**

Finally, not only are tweets created by contributors, they are also responded to by others in the system in various ways, indicating varying densities of neighbour interactions. The systemic features within the context of MTBoS lie in the constraints and capabilities the Twitter platform affords, which include the possibilities of responding through likes, through direct retweets, and through replies. Each tweet solicits a combination of responses in these forms and the responses are a natural mechanism for identifying various aspects of what makes a tweet, or an idea in a tweet, attractive to the audience of MTBoS itself. To further understand the features that make tweet content attractive, an investigation of tweets with various combinations of high values in each of these forms of response was pursued in this view on the data. More specifically, tweet clusters were coded with maximum response values in each form of response to signify and encapsulate how the system responds to the ideational network the tweet cluster portrays. Taking maximum values meant that if a root tweet was part of a thread where response values to the initial tweet in the cluster were higher, then these higher response values were taken. This was the case in only 27 out of the 444 tweet clusters. Alternatively, if a root tweet was a quoted retweet of something that attracted high response, then the values from the tweet referred to were taken. This was the case for only 12 out of the 444 tweet clusters. And, replies were counted by identifying all tweets linked to each other within a tweet cluster to capture both depth and breadth of response in that value (i.e., counting all possible tweets stemming from the initial tweet in the cluster as visualized in Figure 5.2). Counting these had to be done manually since Twitter only provides a count on the number of threads a tweet initiates rather than the number of replies in total it attracts. The choice to use maximum values was made to uphold fidelity to the ideational artefacts in that tweet cluster which elicited that level of response rather than prioritizing the contributor’s potential visibility as a factor in response. As such, the values selected for each of the 444 tweet clusters created three dimensions along which values for forms of response could be assigned: number of likes, number of retweets, and number of replies as prompted by the ideational network presented by the content revealed in each tweet cluster.

Each form of response attracted a certain distribution of values in which a level could be determined to be considered either as having low response or high response. As such, the various combinations of low and high response along each of the three dimensions
created several categories for forms of response. To acknowledge all possible categories, a Venn diagram approach was taken to structure the organization of data according to these categories. This Venn diagram, shown loosely in Figure 5.5 below, at first was used to differentiate between tweets that attracted absolutely no response along a dimension and those that received at least one or more in that dimension. However, it became quickly evident that doing this did not acknowledge the distributions of values in each dimension, and therefore, misinterpreted what counted as high and low. For instance, if a tweet received 12 likes, 2 retweets, and 1 reply, in the regular Venn diagram, it would be placed in the central intersectional region. However, it does not represent something that received much response in terms of replies or retweets because of how few of these it attracted, so it should actually be in the likes only region. As such, cut-off points were introduced to restrict regions to only including values that were at least at the cut-off point or higher. Doing this amplified the features of tweets within each category so that more appropriate generalizations could be made.

Figure 5.5  Venn diagram of options for modes of response

The Venn diagram approach allowed for viewing the collections of tweets pinned into each of the regions together. As such, the tweets from each region were analyzed according to their content with an intent to identify redundancies and diversities in tweets within and across each region. This process involved a similar method as in the analysis of tweet contents parsed by categories oriented by content and contributors, described earlier and explained in more detail in Chapter 8. This allowed for identifying attributes of tweets that related to certain kinds of response, which ultimately surmised in generalizations about how response mechanisms direct ideational emergence in the
network. A by-product of this analysis also revealed qualities in tweets that make them attract certain kinds of response.

**Cross-feature analyses**

Finally, cross-feature analyses were pursued to explore how the various results from each view on the data worked together to affect complex emergence in the network, particularly around ideas. This was done by taking the results from each of the above inquiries and relating them to each other. For instance, now that forms of response were explored, contributors could be reconsidered in terms of the forms of response they garnered. Likewise, forms of response could be reconsidered in terms of the contributors who most prominently were found in each category for forms of response. Similarly, contributors were related with content, and content was related with forms of response. Although all possible combinations of cross-analyses (listed on p. 272), were pursued and considered, only those which revealed strong patterns are presented in this thesis. Further details on these methods and the resulting implications stemming from these cross-analyses are revealed in Chapter 9 since the details of these methods depend greatly on the results presented in Chapter 6, Chapter 7, and Chapter 8.

However, it should be noted that the purpose of pursuing cross-feature analyses in light of complexity theory is driven by a sensitivity to considering the collective as composed of all of its features, which are necessarily inter-twined and codependent. With such prominent decentralized control in the network, with no governing body of any kind, the processes that direct ideational activity are worthy of examination. Cross-feature analyses therefore consider all influences on the ideational network working together to direct emergence within the ideational network, or rather, *ideational emergence*.

### 5.5 How methods serve to align with research questions

In summary, three perspectives on the data were taken that were naturally informed by the inherent systemic features in the Twitter environment. Rather than presupposing processes involved in ideational emergence, they were sought out through the use of iterative approaches driven by the basic tenets of conditions for complex emergence from complexity thinking that were used to guide and structure inquiry. In this way, the approaches involved in the analysis of this inherently unique data set were loosely guided by *analytic induction* (Patton, 2002), which allows for the use of *a priori*
theoretical constructs to guide the analyst in emerging subsequent theoretical contributions. However, some of the methods also necessarily involved the use of quantitative measures to parse and organize the multi-dimensional data before it could be qualitatively analysed. In such a way, a mixed methods approach was utilized. However, attention was drawn primarily to the results stemming from qualitative inquiry, and quantitative inquiry only served to help with data organization. The nature of the data ultimately directed the inquiry because it offered not only a linear transcript of text, but a multi-dimensional network of activity that required a variety of methods in order to analyze as comprehensively as possible towards the development of insights around ideational emergence in a complex adaptive learning organism. All inquiry was also fundamentally focused on the collective layer but involved analyzing individual activity in order to highlight how it contributes to collective patterns in the system as a whole. That is, the methods were designed to aim towards answering the research questions. Below, I reveal how each of the three abovementioned views contributes to each of the two research questions:

RQ1:  What is the nature of ‘MTBoS’ and how does it emerge from ‘the people of MTBoS’?

RQ2:  How and why does ‘MTBoS’ invoke a sustainable form of professional activity around mathematics teaching?

The first view, which only considers content and the ideational network that it creates, aims to illuminate RQ1 by revealing the nature of MTBoS and how it emerges from individuals, where individuals are defined by their contributions. Through considering the prominent ideational activity in the ideational network, it helps contribute to RQ2 by indicating which sorts of topics are involved in sustainable activity around mathematics teaching. And, these most prominent topics in the ideational network also illuminate the implications of this study for investigations around the nature of professional activity around mathematics teaching by highlighting the topics that seem to matter most to those seeking such professional activity. Namely, if professional activity is determined by the topics of interest, then the ideational network can inform those interested in professional activity around mathematics teaching by highlighting the kinds of topics teachers of mathematics naturally congregate around without external mandates.

The second view, which focuses on contributors and the social network among them as it relates with driving activity in the ideational network aims primarily to illuminate RQ1 by
further nuancing the nature of MTBoS as it emerges from individuals who hold different social positions in the network. By categorizing individual contributors based on their access and capacity in the network, the social landscape is revealed. Such a landscape necessarily affects the ideational space of MTBoS since ideas are published by people in activity with each other and their access and capacity in the network determines their potentialities for ideational activity. Developing such a view on contributors and their capacities in the social network also can also inform RQ2 because the sustainability of the collective hinges on the continued engagement of its users, who are inherently embedded and co-implicated by their social landscape. That is, since engagement in MTBoS activity is necessarily a social activity due to its primary endeavour being communication, therefore, it is possible that aspects in the social landscape may also be contributing to the ongoing sustainability of the collective. And, the social network can also illuminate implications arising from this study because the level of engagement in MTBoS is considerably different than in-person professional activity settings given its asynchronicity and publicity. As such, the social network is bound to reveal insights into professional activity around mathematics teaching.

The third view, which focuses on the feedback mechanisms available through the forms of response offered by the Twitter platform, can further inform each of the two research questions as it provides insight into the connective tissue among neighbour interactions in the collective. Namely, for RQ1, having deeper insights into content that receives feedback through forms of response can further nuance the sense of what comes to matter in MTBoS and how individuals form and direct activity in MTBoS; that is, not only through their contributions of content but also in their responses to content. Feedback mechanisms are also incredibly pertinent to answering RQ2, which is concerned with sustainability. Without adequate forms of response, contributors may not have the social incentives to continue engaging in such activity. And, systemically, feedback mechanisms also determine what comes to matter more in MTBoS as based on the forms of response certain ideas attract. Finally, considering how feedback mechanisms contribute to the ideational emergence in MTBoS may reveal implications for professional activity settings because part of designing spaces that allow for emergent activity, is setting constraints that foster such activity. Since MTBoS seems to reveal such an environment that attracts a significant amount of activity around topics related to mathematics teaching, it is interesting to consider how the forms of response available in
the environment value and direct certain ideas around mathematics teaching, and in what ways they do so.

Lastly, the cross-feature analyses aim to illuminate both of the research questions further by considering each view and what they indicate for the system when they are brought together. Further, through decentralized activity that ideationally orients around mathematics teaching as carried by certain contributors who hold capacity in the network and as responded to in certain ways through feedback mechanisms, new insights may be illuminated about the nature of professional activity around mathematics teaching. In doing so, explanations for why and how MTBoS is sustainable and how it emerges from ‘the people of MTBoS’ may be revealed. These are the aims of exploring the views on the data both with the grain and across the grain in my efforts towards responding to the research questions.

I now move into revealing the results of this research. In Chapter 6, I focus on the more specific methods and results pertaining to the view on data oriented by content. In Chapter 7, I report on the methods and results pertaining to the view on data oriented by contributors. And, in Chapter 8, I present the methods and results pertaining the view on data oriented by forms of response. Finally, in Chapter 9, I turn to attempts and results from the cross-feature analyses. In considering all of these views together, the research questions are addressed in light of the results in Chapter 10.
Chapter 6  An emergent ideational network

We have come to realize that neighbors in knowledge-oriented communities are not physical bodies or social groupings. That is, although undeniably important, personal and group interactions for their own sake may not be as vital or as useful as is commonly assumed. Rather, the neighbors that must interact with one another are ideas, hunches, queries, and other manners of representation. (Davis & Sumara, 2006, p. 142)

Given that content pervades the primary real estate in a tweet, my inquiry into MTBoS data began with an interest in the ideational interactions made available within the space. This interest was deeply guided by Davis and Sumara’s proposition of there being an “ideational network [that] rides atop the social network” (p. 143), where ideational interaction contributes to this network through the “bumping, colliding, and juxtaposition of ideas” (p. 142). As with any living organism, the ideational network of a complex system is ever-changing and constantly adapting to various conditions. So, any attempt at capturing an ideational network can only be a snapshot of its existence at a certain point in time. However, it may illuminate processes involved in its emergence.

While specific methods for uncovering an ideational network have not previously been explicated, developing a process for imaging even a small snapshot of the ideational network of MTBoS was crucial for my inquiry into the inner workings and implications of MTBoS. As such, I tailored an approach for creating an ideational network from MTBoS data by steering myself with the fundamental questions, “What ideas are being communicated?” and “How are these ideas interacting with each other?”

Through these guiding questions, I noticed that tweet clusters offer a synthesis of ideas woven together in various manners. Even a single tweet can include a rich amount of information that blends together contexts, practices, and resources through text, hyperlinks and media. To exemplify this process of identifying ideational artefacts, and the ideational relations between them, I present here one instance of a tweet in the data set which I coded with artefacts and relations in this way. As is seen in Figure 6.1 below, the tweet was coded with ideational artefacts of #playwithyourmath, activities after a test, games and puzzles, worksheets at desks, and engaging students. Some of these artefacts were being linked more prominently with each other than others. In this case, #playwithyourmath was being positioned as a task that engages students when they work on it as a worksheet at their desks, and as a game or puzzle that is done as an
activity after a test. So, games and puzzles were communicated as things to do at desks after a test and students were being engaged by games and puzzles after a test, specifically by the #playwithyourmath puzzle. However, it was not as prominently communicated that worksheets at desks were what engaged the students. Rather, they were engaged because of qualities in #playwithyourmath and because they were given time to play with a puzzle after a test. While this may be interpreted in different ways, my insider view contributed to the micro-decisions whenever there was more ambiguity and I regularly consulted my field notes during the process of coding ideational artefacts and the relations among them. Figure 6.1 shows how these relations were noted and represented visually for the given tweet.

Figure 6.1  Example ideational artefact analysis

While it may have been easier to use automatic textual analysis for each tweet that identified key words or hashtags and creating relationships between them, doing so would ignore the rich density of ideas presented in tweets that are at times very implicit, requiring an insider view for interpretation. For instance, the example seen in Figure 6.2 below explicitly links multiplication with patterns and relationships as well as @joboaler with visual number charts, and @mburnsmath with lined up equations.
Trying to build multiplication fluency across multiple grades using an activity that combines the search for patterns and relationships (like @joboaler’s visual number chart) with lined up equations (like @mburnsmath does). Thoughts? drive.google.com/open?id=1mLM0O ...
#mtbos #iteachmath

Figure 6.2  Exemplifying explicit and implicit ideational interaction

My insider view also allows me to use the historical occurrences I have experienced in MTBoS to know that the use of lined up equations is often seen as a number string (Fosnot & Dolk, 2002; Lambert, Imm, & Williams, 2017), which often involves practice approaches similar in structure to that of a number talk35 (Humphreys & Parker, 2015). Number talks are considered an instructional practice that helps students communicate their diverse mathematical approaches while building fluency and mental math abilities (Finkel, 2015) as well as empowering them with equity and access to mathematical ideas (Vanderwerf, 2016). The task in this tweet also involves notions from variation theory (Watson & Mason, 2006) in designing mathematical activities and reinforces the values of helping students visualize mathematical ideas, as seen in many other tasks shared in MTBoS, which comes from my experiences as a teacher of mathematics and mathematics education researcher. All of these explicit and implicit linkages were considered when making choices about identifying artefacts and relations between them. Although this involved developing interpretive capacity as an insider over time and tuning into my developing awareness towards issues in mathematics education more generally, I was able to interpret the content shared in MTBoS more comprehensively and in turn, to take note of features that otherwise may not have been surfaced.

Consequently, each tweet cluster was possible to be examined by identifying ideational relations and artefacts either explicitly through provided text or media, or implicitly by examining content through my ethnographically informed insider view. The resulting ideational relations and artefacts were then composed to create an ideational network representative of the data. Such a snapshot of the ideational space in MTBoS, albeit limited to one segment of time and not indicative of time-related shifts, allowed for a

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35 Numbertalks are instructional routines in which a teacher typically guides students in thinking through a prompt without telling them if something is correct or not, asking students to explain their thinking, and finding ways to represent that thinking for other students to ponder and contribute to.
closer dissection of it and its ongoing development as an organism within the given
timeframe. By acting much like an x-ray, it illuminated potential sources of coherence
and randomness in the system and opened a window for inquiry into processes related
to emergence within the ideational space of the collective. In what follows, I first reveal
the ideational network construction and then expose my exploration into prominent
features of this network that contributed to ideational emergence. The focus is not so
much on what constitutes the ideational network in this slice of time in MTBoS, but rather
on how its emergence behaves.

6.1 Building and examining the ideational network

Building the ideational network involved taking the composition of all ideational artefacts
and relations, which produced a network graph that visually depicted these in a
collective ideational view. To this end, for each ideational cluster, I included the
relationships identified between ideational artefacts that pertained to that root tweet ID in
a spreadsheet format. Relations were only repeated if they occurred again in another
tweet cluster. They were not repeated each time they were made within a tweet cluster
since a tweet cluster was the unit of analysis. The analysis of all 444 tweet clusters in
the core data set generated 1353 unique ideational artefacts with 4183 ideational
relations between them. These were then mapped using the Microsoft Excel NodeXL
Pro (Smith et al., 2010) add-in, which allows for network visualization and analysis
through various network centrality measures based on an initial list of relations.

More specifically, each relationship between ideational artefacts was entered in the
NodeXL Pro ‘Edge’ sheet, which allowed the add-in to tabulate a list of ‘Vertices’ from
these relations as well as centrality measures in the network such as degree,
betweenness, and PageRank. Degree calculates the number of edges that connect to
the vertex, betweenness measures the maximum number of pathways available in the
network that pass through the vertex, and PageRank calculates the importance of the
vertex in the network by not only considering the degree of the vertex, but also the
degrees of those it connects to (Golbeck, 2013). Edge recurrence was also calculated to
account for which relations occurred more frequently in the network. The NodeXL Pro
add-in also allowed for calculating various ‘groupings’, using for instance the Clauset-
Newman-Moore algorithm (Clauset, Newman, & Moore, 2004), which parsed vertices
into groups that contained denser networks. Since vertices represented ideational
artefacts, and edges represented ideational relations, these measures helped identify which artefacts were more prominent in these various manners both in the network overall and in the respective groupings identified. The measures and visualizations were also helpful in zooming into clusters that seemed important in the network.

Although such quantitative features were used for examining the constructed network, a priori theoretical concepts from complexity theory drove all analytical decisions. That is, a directive focus was taken to make analytical decisions that would help identify possibilities for emergence, and more specifically, coherence and randomness in the ideational network of the system. Overall, the broad pursuit of analysis involved first constructing the ideational network, then attending to features such as centrality measures, edge weightings, and groupings within the network to identify ways to parse the network, and then using these to identify and examine tweets that composed them qualitatively. The use of quantitative measures was reserved strictly for parsing the network into smaller components so that they could be further examined qualitatively. For instance, prominent vertices in the network could serve as sources of coherence in the network. By looking at the tweets that composed the most prominent edges or vertices and analyzing their content for redundancies and diversities to emerge themes, a more nuanced understanding of what contributed to their emergence could be surfaced. In this way, the quantitative features natural to the Twitter environment helped highlight and point to aspects inter-related with their emergence.

However, while such a process of generating a visualization for the ideational network should theoretically illuminate sources of coherence automatically, the initial version of its graph looked much like a constellation of stars (see Figure 6.3), where some patterns of cohesion were evident, but the density made it overall difficult to navigate.

Figure 6.3 First version of the ideational network graph
This made it necessary to apply various emphases. By enhancing features through the application used for graph construction (Smith et al., 2010), more structure became evident. Figure 6.4 shows the result of increasing edge thickness based on frequency, darkening vertices with higher betweenness\textsuperscript{36}, increasing vertex size based on degrees of occurrence, and bundling the edges\textsuperscript{37} to enhance visibility of network structures.

![Figure 6.4 Enhanced version of ideational network graph](image)

What became readily apparent in this process was that the dense set of ideational relations produced a completely connected\textsuperscript{38} network graph. While later attempts at pruning away weak relations or weak artefacts detached the network in various ways, it is interesting to note the overall ideational cohesion within MTBoS. This points to its nature as an ambiguously bound but organizationally closed system, a necessary condition of complex adaptive systems. It is ambiguously bound because boundaries are fluid and allow information to move between its system and contexts in which it lives, but organizationally closed because of its overall ideational cohesion.

Notwithstanding, after amplifying certain features in the network through visual means, it was still challenging to infer broader generalizations about possible sources of coherence and randomness that may have existed in this ideational space. It was simply

\begin{itemize}
  \item \textsuperscript{36} Betweenness centrality measures the number of shortest paths passing through a node. High betweenness centrality identifies 'brokers' in the network.
  \item \textsuperscript{37} Using a Harel Koren Fast Multi-scale (Koren, Carmel, & Harel, 2002) approach to visualization.
  \item \textsuperscript{38} Confirmed by applying the 'group by connected component' feature in NodeXL Pro.
\end{itemize}
too dense. As such, a further process was taken to identify clusters within the network that included more closely connected artefacts. This involved using the Clauset Newmann Moore Cluster Grouping algorithm\(^{39}\) (Clauset et al., 2004), which systematically parsed the network into cohesive groups. I then reviewed the artefacts in each group to determine labels\(^{40}\) that could be representative of the most prominent common attributes among artefacts in that group to highlight their coherence. One possible visualization for this grouping with such labels is shown in Figure 6.5 below.

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**Figure 6.5 Complete ideational network with cluster groupings**

Another visualization for this same graph is seen in Figure 6.6, in which inter-group relations have been collapsed into thinner and thicker connectors to represent the volume of those inter-group relations. Ideationally, these views on the network reveal an evident decentralized organization since several different ‘hubs’ of various strength emerge without any clear direction from any governing body.

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\(^{39}\) The Clauset Newman Moore algorithm systematically removes nodes with high betweenness, which is a measure the number of shortest paths in a network that pass through the node. Such nodes serve as ‘bridges’ between clusters in the network. By removing these, sub-clusters that are more connected among each other than with others in the network can be identified.

\(^{40}\) This process of using quantitative approaches to identify clustered components within data, and then applying qualitative tactics for labelling such clusters is loosely relatable to methods such as in exploratory factor analysis, where statistical methods cluster factors together and an analyst then determines labels for such clusters based on the most common attributes among them.
An even more collapsed view of this same network graph with groups and relations between groups collapsed into vertices is shown in Figure 6.7, where it reveals these various topic ‘hubs’ in relation to each other.

Such ideational network visualizations reveal a strong connectivity among ideational artefacts within MTBoS as well as a tendency for some artefacts to relate more
frequently with other artefacts\textsuperscript{41}. For instance, at the broadest level of organization shown in Figure 6.7, ‘sensemaking activities’, ‘engaging students’, and ‘real world math’ were most frequently related with each other, and ‘assessment’ and ‘meaningful homework’ were quite often related as well. However, ‘assessment’ and ‘curriculum’ were less so. Similarly, ‘problem sets and activities’\textsuperscript{42} were less commonly related with ‘engaging students’ but more so related with ‘real world math’ and ‘making math moments matter’. Such broad patterns pointed to general attributes of the ideational space, indicating opportunities for ideational coherence in the system. And, these cursory investigations of the ideational network implied broadly that MTBoS is oriented around progressive views about mathematics education, with a focus on using real world contexts, involving students in sensemaking, engaging students, and finding ways to implement assessment strategies. However, to refine such implications and to establish their validity, closer examination of the space was evidently warranted.

Although examining the ideational network in the ways revealed above indicates some general patterns of coherence within the ideational network, the inner processes involved in their emergence is yet unrevealed. Therefore, an altogether different way of examining the network was pursued. Rather than clumping ideational artefacts together and trying to account for the entire network as a whole, a process of ‘pruning’\textsuperscript{43} away less frequent ideational activity was undertaken. While decisions around how to prune were primarily data driven, they were also fundamentally interwoven with my insider view and the underlying tenets of complexity theory\textsuperscript{44}. At times, pruning decisions were pragmatic\textsuperscript{45} and even aesthetic\textsuperscript{46} when otherwise arbitrary, but always guided by the question, “Does this representation resonate with my insider-view on MTBoS?” and, “Does this pruning afford a view that reveals the system’s complexity?” Given the

\footnotesize
\textsuperscript{41} Such a decentralized network structure was evident at various scales, with similar network arrangements appearing at various levels of organization, a notion described by Davis & Sumara (2006) as scale-free nested organization that is a necessary condition for complex systems.

\textsuperscript{42} Note that many MTBoSers refer to mathematical tasks as ‘activities’, but they often additionally include indications of how the task is to be implemented or how it was implemented. As such, I use the term ‘activities’ to refer to such instances of tasks with implied pedagogy.

\textsuperscript{43} The term ‘pruning’ is used to describe attempts at viewing only the thickest veins in the complex system and reflects the biological nature of the system’s organization.

\textsuperscript{44} That is, to attend to both sources of coherence and randomness.

\textsuperscript{45} The sheer mass of data available via social media makes it challenging to approach qualitatively.

\textsuperscript{46} Some argue that research is fundamentally an aesthetic process that necessarily involves subjectivity, perspective, and reflexivity (e.g., Dixson, Chapman, & Hill, 2005).
ideational network composes of both relations and artefacts, two approaches to pruning were available: one that privileged prominent relations, and one that privileged prominent artefacts. These approaches resulted in two smaller networks, one revealing prominence around ideational relations, and the other around ideational artefacts.

Pruning the ideational network in these ways allowed for a closer inspection of prominent sources of coherence in the ideational network and a window into ways in which they emerged. To this end, since emergence hinges on the presence of both redundancy and diversity within neighbour interactions, an interest was taken in the redundancies and diversities among tweet clusters that composed relations, and among relations that composed artefacts. As such, my key guiding questions within each parsing were, “How do the tweets contribute to this relation or artefact?” and, “What redundancies and diversities are evident within this group of tweets in how they contribute to the relation, or in how this group of relations contribute to an artefact?”

While relations were only coded a maximum of once per tweet cluster, they could repeat across tweet clusters. Therefore, some relations were more prominent in the network than others. Out of the 4183 relations (including repeats), 3424 of them were unique, meaning 759 relations were redundant. Repeated relations ranged up to a maximum of 14 reoccurrences. Overall, the distribution of reoccurrence for relations was very bottom heavy in that 85% of the them occurred either once or twice, and only five reoccurred more than ten times. This distribution is shown in Figure 6.8.

![Figure 6.8](image)

**Figure 6.8  Relation reoccurrence in increasing order**

The most prominent relational occurrences were between ‘engaging students’ and ‘fun activities’ (14 occurrences), ‘fun activities’ and ‘hands-on learning’ (13 occurrences), ‘fun activities’ and ‘games and puzzles’ (12 occurrences), and ‘hands on learning’ and ‘using manipulatives’ (12 occurrences). In contrast, single-occurring relations included ‘writing on desks with dryerase’ and ‘expression mats’, ‘worksheets’ and ‘non-routine problems’,
and ‘being a good teacher’ and ‘work life balance’. Since prominently reoccurring relations were of interest, pruning involved systematically removing relations that occurred fewer times until a graph arose in which individual artefacts and relations were sparse enough to be identifiable, while robust enough to offer insights into the network.

Beginning with the complete network with all metrics and grouping algorithms as in Figure 6.5, relations were removed one by one. Since the maximum reoccurrence of relations was only 14, stepping up by single values was most appropriate. However, removing relations resulted in many disjoint components within original groupings, so new grouping calculations were made at each stage to improve graph visibility. Figure 6.9 below shows the network pruned to include only relations that occurred 3 or more times (~10% of relations) with new groupings and group names applied, and Figure 6.10 shows the network pruned to includes relations that occurred 6 or more times (~5% of relations). At 7 or more reoccurring relations (~2% of relations), the graph (seen in Figure 6.11) was no longer robust enough, leaving only ten relations.

Figure 6.9  Ideational network pruned to include 3 or more relations

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47 Grouping calculations were always made using the Clauset-Newmann Moore Grouping Algorithm via the NodeXL Pro Excel Add-in.
Since the graph pruned to include relations reoccurring 6 or more times included 22 relations (Figure 6.10), it was deemed robust enough to use as a source for further inquiry. These top 22 relations along with their frequencies are shown in Table 6.1.

**Table 6.1  Top 5% of ideational relations with frequencies**

<table>
<thead>
<tr>
<th>Artefact Relations</th>
<th>Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 engaging students</td>
<td>fun activities</td>
</tr>
<tr>
<td>2 fun activities</td>
<td>hands on learning</td>
</tr>
<tr>
<td>3 fun activities</td>
<td>games and puzzles</td>
</tr>
<tr>
<td>4 hands on learning</td>
<td>using manipulatives</td>
</tr>
<tr>
<td>5 engaging students</td>
<td>hands on learning</td>
</tr>
<tr>
<td>6 engaging students</td>
<td>games and puzzles</td>
</tr>
<tr>
<td>7 conferences</td>
<td>NCTM</td>
</tr>
</tbody>
</table>
Each of these 22 relations were then used to identify contributing tweets towards further analysis, involving an examination of redundancies and diversities among tweets contributing to each relation, the results of which are presented in §6.2.

Further, I also explored prominent ideational artefacts. To this end, an alternative approach to pruning the ideational network was taken to emphasize the ideational artefacts that occurred most frequently in the data. Artefacts were coded a maximum of once per tweet cluster but could repeat across tweet clusters, allowing for the possibility of repetition. Out of the 1353 artefacts, 398 of them had occurred in more than one tweet cluster, and these repeated occurrences spanned up to a maximum of 77 times. The distribution of reoccurrence for artefacts was very bottom heavy just like for relations, and most (about 90% or more) of the artefacts occurred either once or twice with only a few having very high recurrence. This distribution is shown in Figure 6.12.

![Artefact Recurrence in Increasing Order](image)

**Figure 6.12** Artefact reoccurrence in increasing order
The most prominent ideational artefacts included ‘engaging students’ (77 occurrences), ‘fun activities’ (46 occurrences), ‘mtbos’\(^{48}\) (45 occurrences), and ‘games and puzzles’ (41 occurrences). In contrast, some examples of ideational artefacts that only occurred once include ‘scientific notation’, ‘angle pair hopscotch’, and ‘math fair’. The top fifteen reoccurring ideational artefacts are shown in Table 6.2.

### Table 6.2  Top fifteen ideational artefacts by frequency of occurrence

<table>
<thead>
<tr>
<th>Ideational artefact</th>
<th>Frequency of occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>engaging students</td>
<td>77</td>
</tr>
<tr>
<td>fun activities</td>
<td>46</td>
</tr>
<tr>
<td>mtbos</td>
<td>45</td>
</tr>
<tr>
<td>geometry</td>
<td>44</td>
</tr>
<tr>
<td>games and puzzles</td>
<td>41</td>
</tr>
<tr>
<td>using technology</td>
<td>39</td>
</tr>
<tr>
<td>visualizing math concepts</td>
<td>38</td>
</tr>
<tr>
<td>hands on learning</td>
<td>36</td>
</tr>
<tr>
<td>real life examples</td>
<td>36</td>
</tr>
<tr>
<td>algebra</td>
<td>33</td>
</tr>
<tr>
<td>problem solving</td>
<td>33</td>
</tr>
<tr>
<td>using manipulatives</td>
<td>33</td>
</tr>
<tr>
<td>activities</td>
<td>29</td>
</tr>
<tr>
<td>assessment</td>
<td>28</td>
</tr>
<tr>
<td>noticewonder</td>
<td>26</td>
</tr>
</tbody>
</table>

Since prominent artefacts were of most importance in this approach to pruning, artefacts that occurred fewer times were systematically removed until a graph in which individual artefacts and their associated relations could be visually identified while remaining robust enough to offer insights into the network. However, since removing artefacts also removed relations stemming out from them, a secondary process of adding in relations around top artefacts was performed. This was because the interest in artefacts is fundamentally interwoven with how those artefacts exist in relation to other artefacts.

The first pruning began with the complete network, with all metrics and grouping algorithms applied as in Figure 6.5 presented in §6.1. From this graph, fewer occurring artefacts were removed systematically by leaving only the top 10%, then 5%, then 2.5%,

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\(^{48}\) ‘Mtbos’ as an ideational artefact was only coded as such if it was being referred to as an idea, hence the lower-case presentation of it. It was not included as an ideational artefact if it was being used as an organizing hashtag since it would have been then included in every tweet in the data set based on it being the key search criteria in data selection.
and then 1% of artefacts until it was possible to see individual artefacts and relations. Figure 6.13 shows the top 5%, and Figure 6.14 shows the top 1%.

As evident in the above figures, a closer examination of the pruned network was not possible until all artefacts and relations were more available to view distinctly, which only began to occur in the top 1% of artefacts shown in Figure 6.14. As such, the top 15 artefacts (just over 1%) as listed in Table 6.2 were used as the source from which further inquiry into contributing factors to network prominence could be pursued.

These artefacts were used to find prominent relations stemming out of them back into the graph to enhance the view on them and the ways in which they were composed by relations. That is, for each of the 15 top artefacts, a list of which other artefacts they related to from the entire corpus of artefacts was identified and examined for frequency
of occurrence. For example, in the case of the ‘noticewonder’ artefact, it related to a total of 74 other artefacts (as seen in Figure 6.15 below).

![Figure 6.15 All relations stemming from 'noticewonder'](#)

However, many of these relations occurred only once (49 of the 74 to be specific). By limiting the relation thickness to include only stemming relations that occurred 3 or more times, the graph became a little more readable (as seen in Figure 6.16 below).

![Figure 6.16 Only relations of thickness 3 or more stemming from 'noticewonder'](#)

In efforts to keep the resulting graph readable and relevant to including only the most prominent artefacts and their corresponding relations, a limiting criterion was placed on the frequency of occurrence for stemming relations that were included. This was done systematically by increasing the limiting value for relation frequency for stemming relations for each of the top 15 artefacts until components in the entire graph were possible to view distinctly. In the end, the most desirable graph occurred when including
stemming relations that reoccurred 3 or more times in relation to each of the top 15 artefacts. And, in order to enhance node visibility, a grouping algorithm that identified ‘motifs’ was used\textsuperscript{49}. This resulting graph is shown in Figure 6.17 below.

Figure 6.17  Top 15 artefacts with stemming relations of thickness 3 or more

This graph (Figure 6.17) of the top 15 artefacts along with any relations occurring 3 or more times stemming out was used to structure further inquiry into how each of these artefacts emerged from the data. Since ideational artefacts are composed by the ideational relations around them, my inquiry here was not so much around what these particular artefacts were, but more so about how they emerged through the relations that formed them. As such, the analysis of these artefacts involved inspecting the relations around them based on categories developed for classifying relations.

Once these two ‘prunings’ were made (to highlight top relations, and then top artefacts), analyses of the redundancies and diversities among tweets composing each of the top relations and then artefacts were carried out. Observations from these analyses contributed to the development of themes to represent the dimensions of possibility in how ideational relations, and in turn, ideational artefacts can live in the ideational

\textsuperscript{49} Motif simplification is a technique that groups nodes into common patterns that group by network structures such as with fan clusters, connector links, and cliques (Dunne & Shneiderman, 2013).
network. These results are revealed and discussed in what follows, with ideational relations explored in §6.2 and ideational artefacts explored in §6.3.

6.2 Exploring ideational relations

Towards a deeper investigation of the top 22 reoccurring ideational relations (shown in Table 6.1 and Figure 6.10), two lenses of observation were taken: one aiming to identify ways in which each relation was made by tweet clusters, and the other to identify sources of redundancy and diversity among how tweet clusters contributed to each relation. Each of the two lenses of observation resulted in two further dimensions, meant to offer contrasting (but not completely distinct) categories. In terms of how relations were made, they were made either more explicitly, with specific terminology being used to draw ideational artefacts together, or implicitly, with more inference about associated meanings required. In terms of possibilities for redundancy or diversity, relations could either be created through more redundancy, with repetition of associated meanings across tweets, or more diversity, with noticeable variances in associated meanings across tweets. Relations that offered redundancy were considered as having ideational convergence; in that, the ways in which the ideational artefacts were positioned together continued to support the same meanings each time the relation occurred. Consequently, relations that offered more diversity were considered as having more ideational divergence; in that, the ways in which the ideational artefacts were being brought together varied in meaning each time the relation occurred. Since each of these could occur explicitly or implicitly, relations could have either explicit or implicit ideational convergence or divergence. These four options are shown in Table 6.3 below.

<table>
<thead>
<tr>
<th>Table 6.3 Ways for relations to be made</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relation made explicitly</td>
</tr>
<tr>
<td>More redundancy among tweets in how relation is made</td>
</tr>
<tr>
<td>More diversity among tweets in how relation is made</td>
</tr>
</tbody>
</table>

These options offer dimensions of possibility for ideational relations but are not meant to hold any specific relations static to any one category. Rather, they serve as a dynamic framework with which to observe how ideational relations exist within a fixed period of time inside a necessarily dynamic ideational network. They are also not intended to be mutually exclusive since there can always be some diversity within something with a lot
of redundancy, and vice versa. Likewise, some implicit aspects can exist when much is explicit, and vice versa. While tweet clusters did not always solely contribute either explicitly or implicitly, or with ideational convergence or ideational divergence to a relation, it was possible to identify them as being predominantly representative of one or the other within the data snapshot. Overall, most of the top 22 relations involved strong ideational convergence, with 16 out of the 22 evidencing ideationally convergent meanings. Half of these were made explicitly, and the other half implicitly. The other 6 were more prone to evidencing ideational divergence, half of which were made more explicitly, and the other half implicitly. These are summarized in Table 6.4 below, and the relations that most prominently exemplify each of these categories along with indications of affordances and limitations found in each category are discussed in what follows.

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<th>Table 6.4</th>
<th>Summary of categorized artefact relations</th>
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<td><strong>Frequency</strong></td>
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<tr>
<td>fun activities</td>
<td>games and puzzles</td>
</tr>
<tr>
<td>creating new activity resources</td>
<td>real life examples</td>
</tr>
<tr>
<td>desmos</td>
<td>desmos activity</td>
</tr>
<tr>
<td>fun activities</td>
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<td>great ideas and inspiration</td>
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<td>noticewonder</td>
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<td>visible student thinking</td>
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<tr>
<td><strong>Artefact relations with implicit ideational convergence</strong></td>
<td><strong>Frequency</strong></td>
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<tr>
<td>engaging students</td>
<td>fun activities</td>
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<tr>
<td>fun activities</td>
<td>hands on learning</td>
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<tr>
<td>hands on learning</td>
<td>using manipulatives</td>
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<tr>
<td>engaging students</td>
<td>hands on learning</td>
</tr>
<tr>
<td>engaging students</td>
<td>games and puzzles</td>
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<tr>
<td>games and puzzles</td>
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<tr>
<td>geometry</td>
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<td>noticewonder</td>
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<td><strong>Artefact relations with explicit ideational divergence</strong></td>
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<tr>
<td><strong>Artefact relations with implicit ideational divergence</strong></td>
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<tr>
<td>engaging students</td>
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<td>engaging students</td>
<td>students discussing and debating</td>
</tr>
<tr>
<td>classroom community</td>
<td>classroom culture</td>
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</tbody>
</table>
Explicit ideational convergence

Relations identified as most prominently exhibiting explicit ideational convergence indicated in Table 6.4 involved ideationally convergent meanings in how artefacts were being related with each other and overall tendencies towards using explicit terminology to relate the two artefacts. Relations made in such ways afforded replication and reoccurrence of redundant forms of activity with slight variances. That is, they allowed for generativity of new material that continued to support the explicitly convergent meanings found within the relations. Also, these relations were often referred to by users who were responding to others, thus further solidifying their explicit convergence. The power of such relations was their ability to thrive and flourish, but their danger existed in the possibility of them being overly prescriptive and causing potentially useful diversities to be ignored when offered. In this data set, these dangers were not particularly evident, and most examples supported the generative capacities of ideational content. For example, ‘fun activities’ and ‘games and puzzles’ was one prominent relation with explicit ideational convergence. All tweet data forming this relation included explicit evidence that students were having (or were going to have) ‘fun’\(^{50}\) with mathematical ‘games or puzzles’, as implied through media included. Although some were more implicitly portraying ‘games or puzzles’ as ‘fun’ (e.g., Figure 6.18a), several provided explicit indication that ‘games and puzzles’ are ‘fun’ (e.g., Figure 6.18b).

![Figure 6.18](image)

\(50\) Although ‘fun’ is an ambiguous term, in this context, ‘fun activities’ is used in the sense that it is a non-traditional task often done in spare time outside of content delivery and aims to entertain students, giving them a productive break and keeping them occupied. It is often hands-on and engaging, and sometimes involves more mathematical content and other times less.
suggestions for a fun game involving basic subtraction facts?” This request resulted in a multitude of responses that provided diverse examples and resources that redundantly supported the ideational convergence around how ‘games and puzzles’ make learning ‘fun’. Although what counted as ‘fun’ was not explicitly discussed, the repetition of ‘games and puzzles’ being positioned as ‘fun activities’ gave the relation generative potential. It carved a pathway for new content to be produced in a similar fashion.

Another prominent relation with explicit ideational convergence that held generative potential was between ‘real life examples’ and ‘creating new activity resources’. Every contributing tweet cluster in this relation included an image of a ‘real-life example’ and then explicit reference to it as a resource for a potential ‘new activity’. The overwhelming consistency in posts explicitly suggesting a ‘real-life example’ become a ‘new activity resource’ is revealed in Figure 6.1951.

Noticeably, each tweet shown presented a ‘real-life example’ and suggested how it could be used for ‘new activities’ in practice. This indicated strong ideational convergence

51 The number of examples presented here serves to illustrate the strong commonalities among these posts rather than to draw attention to the examples individually.
around how using ‘real-life examples’ for ‘creating new activity resources’ was a valued form of contribution. Many of the posts in this relation also invoked practices prevalent in the community such as ‘visualpatterns,’ ‘noticewonder,’ ‘3 act tasks,’ ‘desmos activities,’ and ‘tmwyk’ (talking math with your kids). The recurrence in how these posts were made created a consistency that communicated an explicit ideational convergence in meaning around how MTBoS is a place for sharing and ‘creating new activity resources’ that involve ‘real-life examples’ that intend to spur mathematical observations. Through this recurrence, mathematics was presented as a domain that extends beyond the walls of the classroom, requiring observation; and that mathematical observation can be provoked in the classroom through tasks that summon the outside world.

A focus on observation in many of the resources publicized in MTBoS space stems in part from the prominence of the ‘noticewonder’ approach to mathematical thinking and learning. Noticewonder52 was perhaps one of the most frequent forms of activity both within the contents of the core data set and throughout my insider encounters in the MTBoS space. It was found to be closely related with ‘instructional routines’ through explicit ideational convergence due to its strong links with routines such as math talks53, number talks54, and problem strings55. These routines often involve annotating student thinking, sometimes revealed in tweets (Figure 6.20). The goal of such routines is generally to value diverse student approaches to ‘noticing and wondering’.

52 Noticewonder refers to a pedagogical strategy for beginning problem-solving tasks through attending to what is noticed and wondered about without presupposing a solution. It can be used as an instructional routine, a warm-up, or general thinking strategy (NCTM, n.d.; Ray-Riek, 2013), and also by teachers in developing practice (Fukawa-Connelly, Klein, Silverman, & Shumar, 2018).

53 Math talk is an umbrella term for a variety of strategies that encourage students to share and discuss their understandings of mathematical concepts (Luce, 2017).

54 Number talk is a specific math talk routine aimed at building mental fluency and flexibility as prompted by numerical provocations and teacher questioning (Humphreys & Parker, 2015).

55 Problem string is a series of related problems sequenced to illuminate properties (Harris, n.d.).
‘Noticewonder’ was therefore not only used as a descriptor in terms of the practice of having students ‘notice and wonder’, but was also used as a prompt provoking other users to ‘notice and wonder’ (see Figure 6.21).

While there was some diversity around how posts were made, there was clear redundancy around how ‘noticewonder’ was being used as an ‘instructional routine’, creating ideational convergence around ‘noticewonder’ being an ‘instructional routine’ itself. This was made explicit through tweets that indicated learning about these strategies at professional development events. For example, one contributor tweeted, “Leaders engaged in the ‘notice and wonder’ routine. Thank you @saravdwrf for facilitating.” In another example, a contributor posted a projected slide they saw during a professional learning event that included a bullet point that stated, “Pose a picture, [and] have students notice mathematics or reason and estimate to respond to a prompt.”

This redundancy was made even more explicit in one of the tweet clusters in this relation where a contributor asked about the origins of ‘noticewonder’, which initiated a rather lengthy reply thread about what ‘noticewonder’ is, what it isn’t, and where resources for it can be found. The user had asked, “Hi #mtbos I’m looking for the origins of #NoticeWonder preferably in the form of a citation. Can you help? Cc @nomad_penguin
@cmmteach @MFAnnie." Out of several of the responses to this request, one was made by a founding advocate of ‘noticewonder’, who responded with a very explicit statement about how ‘noticewonder’ relates with ‘instructional routines’.

From convos with folks, I know it often starts as an explicit instructional routine (usually in math class), but then morphs into an all-encompassing state of being. More than one building leader has said to me, ‘We #NoticeWonder about EVERYTHING!’

As such, although the relationship between ‘noticewonder’ and ‘instructional routines’ attracted a bit of diversity, there was strong redundancy around the associated meanings within its ideational relationship. When it was questioned, there was explicit redirection back to the ideational convergence in meaning within the relation, which was something that occurred in all relations with explicit ideational convergence.

**Implicit ideational convergence**

The explicit nature of related artefacts, though, was not always evident even when there was a prominence of redundancy. In some of the relations, links were more implicit requiring inference through attached media, interpreting nuanced language, and using an insider view. Although meanings in these cases were convergent and often co-defining, they had to be inferred. These more covert relations continued to contribute to the generativity of redundant material similarly as with the more explicitly convergent relations, maintaining a tendency towards recurrence and replication. However, the redundancies here were less actively acknowledged. Instead, the meanings seemed to have become taken-as-shared over time through established patterns of activity. The danger with this was that engagement could continue with implied assumptions, causing any diversities in interpretation to remain invisible.

One relation that attracted a lot of redundancy among tweets and related ideas implicitly was in ‘games and puzzles’ and ‘problem solving’. These were co-defining in nature and revealed a clear pattern of activity. Most of these tweets included a picture of a ‘problem or puzzle’ and linked it implicitly with a ‘problem solving’ process. Although ‘games or puzzles’ and ‘problem solving’ aren’t exactly mutually exclusive, they were being treated interchangeably in these tweets, which may be because the examples in provided indeed lived in the intersection of being ‘games or puzzles’ and ‘problems’. However, within this confounding of meanings, a new related meaning emerged of not revealing answers to problems that seem like games or puzzles. For instance, one tweet explicitly
stated, “please do not tweet the answer” (Figure 6.22c below), and another one had a small conversation about how having a solution booklet is “not fair” (transcript below).

User 1: I have this puzzle! (I also have solutions somewhere.)

User 2: Greg feels that's not fair. Terrific puzzle though. Found it in the toy section, as I've found others before.

User 1: Solutions are handy when you spill it accidentally. (Voice of experience.)

So, sharing puzzles, games, or problems without revealing solutions was a clear ideational convergence in this relation, which was in some sense, emergent.

Another emergent feature in the ‘games and puzzles’ and ‘problem solving’ relation was around who problems were intended for. There was a taken-as-shared implicit meaning of how ‘games or puzzles’ were ‘problems to solve’ for both users themselves and their students and that ‘games or puzzles’ were recreational in nature. For instance, the tweet shown in Figure 6.22a was directed at MTBoSers presumed to be teacher colleagues who may use it as a #lessonstarter. The tweet shown in Figure 6.22b was indicative of it being used in a classroom as a special weekly problem. And, the tweet in Figure 6.22c was directly aimed at other users as a challenge. The intended audience varied.

![Image](image-url)

My husband is home from shearing in Qld and has found my new, incredibly frustrating puzzle I discovered at Salvos yesterday. #handsonlearning #lessonstarter #MTBoS All the pieces look identical (in 2 sets) BUT...

We are doing a Difference Sudoku from @richmaths for this week's problem solver. I love this twist. #TeachMath #MTBoS #Teach180 #TeachAlgebra #VTeD (rich.maths.org/content/id/579 ...)

Can you solve this in 10 minutes or less? If you solve this, reply with a GIF!!! (Please do not tweet the answer. )

#math #maths #MTBoS

![Image](image-url)

(a) (b) (c)

Figure 6.22  Examples of games or puzzles as problem solving

However, the redundancies here implicitly communicated the ideational convergence that when ‘games or puzzles’ were shared, they were open to being treated either as opportunities to problem solve online or to use as resources to engage students. And, the redundancy of this in tweets served as an implicit invitation to problem solve and to share examples of games or puzzles in action.
Further, a slightly different form of implicit ideational convergence was seen in the relation between ‘noticewonder’ and ‘real-life examples’. While ‘noticewonder’ was explicitly related with being an ‘instructional routine’ (exemplified earlier), it was also generative of ‘real-life’ prompts implicitly aimed to be used as ‘noticewonder’ activities. The examples acted similarly as in the explicit relation between ‘real-life examples’ and ‘creating new activity resources’ in which real-life scenarios were used as the basis for new activities. Therefore, most tweets in this relation were a subset of those presented earlier and shown in Figure 6.19. However, the most interesting variance in this relation was that the associative relationship between ‘real-life examples’ and ‘noticewonder’ were so redundant and taken-as-shared in the MTBoS space that at times they initiated jest (Figure 6.23a). Although the initial tweet in this example did not explicitly refer to the umbrella image as a ‘noticewonder’ prompt, it alluded to ‘wondering’, which seemed to signal activity indicating familiarity with the ‘noticewonder’ routine. One user responded, “The spokes don’t line up, which probably makes it stronger” along with a GIF that reveals the jest in the interaction (Figure 6.23b).

![Initial tweet](image1)
![One response](image2)

**Figure 6.23 Noticewonder about this umbrella example**

Overall, there was a lack of using explicit means to establish the use of ‘real-life examples’ as ‘noticewonder’ prompts, but it was evident there was implicit ideational convergence around how ‘real-life examples’ were useful as ‘noticewonder’ prompts. A newcomer may not have picked up on the underlying meanings that remained implicit in these interactions, but the redundancy was indicative of a taken-as-shared meaning among contributors that ‘real-life examples’ invoked ‘noticing and wondering’. Perhaps the relation became so strong over years that it was no longer necessary to make explicit. Instead, the patterns of activity were recurrent and generative of new examples that continued to uphold the same implied meanings within the relation.
Another relation that involved implicit ideational convergence that bred emergent taken-as-shared meanings and new (but familiar) content was around ‘engaging students’ and ‘hands on learning’. ‘Engaging students’ was used to indicate evidence of students engaging (or intentions of engaging them) either in thinking, doing, or discussing mathematics, and was very often implied through the way language was used and the media provided. Most commonly, this was seen around ‘fun activities’, ‘hands on learning’, and ‘games and puzzles’. As is likely evident, many of these overlapped since ‘games and puzzles’ are by nature often ‘hands-on’ and ‘fun activities’ can have components related to ‘games or puzzles’. A typical tweet structure in this relation was to state excitement about something that occurred in class, indicating the task and posting media revealing of students engaging in it, often with ‘hands-on activities’ (e.g., Figure 6.24). Several examples are shown together below to reveal redundancies in how tweets were presented and how they contributed to an implicit ideational convergence in the relation’s meanings.

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56 For instance, tone, sentence structure, and use of exclamation marks often implied excitement about the fact that students were ‘engaged’.
The prevailing implicit ideational convergence in these relations based on the recurrent elements across tweets communicated that students will ‘engage’ if the activities are ‘fun’\(^{57}\), ‘hands-on’\(^{58}\), or involve a ‘game’ component. In terms of ‘engagement’, while there was no explicit discussion of ‘engagement’ or specific attributes of the activities in this data set, it seemed generally agreed upon that students who looked like they were working, were ‘engaged’\(^{59}\), and that the ‘hands-on’ ‘fun activities’ or ‘puzzles’ were ‘engaging’ in and of themselves. As with other relations identified as having implicit ideational convergence, meanings continued to be supported with new tweets that sustained the redundant patterns of activity in the relation.

**Explicit ideational divergence**

Unlike the relations that attracted a lot of redundancy (and therefore ideational convergence), other relations attracted more diversity (and therefore ideational divergence). Diversity does not mean the artefacts being related were not actually related with each other, but rather, that when brought together, diversities in meaning surfaced. These diversities were sometimes more explicit and other times less so. In some relations, ideational divergence was evident across tweets, and made explicit occasionally. In other relations, this divergence was more explicit and held a tendency towards negotiation\(^{60}\). Negotiation brought ideational divergence to the forefront and afforded development of new meanings, thus pushing ideational boundaries within relations. However, even within instances of negotiation, potential disparities in meaning could remain irreconcilable because of the limitations of asynchronous textual and audio-visual interaction found in social media settings. Therefore, some relations with explicit ideational divergence were more generative and in different ways, than others.

One example of a relation that exemplified explicit ideational divergence was between ‘groupwork’ and ‘students discussing and debating’. While it was not questioned whether these went hand in hand, there was a striking diversity in the approaches expressed

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\(^{57}\) Activities were identified as ‘fun’ particularly when they seemed to be a special event in the classroom (e.g., Figure 6.24a and c).

\(^{58}\) Activities were considered ‘hands-on’ if they involved students using their hands.

\(^{59}\) I emphasize this because there is research that indicates that students can look like they are engaged, but are not necessarily engaging in their mathematical task (Liljedahl & Allan, 2013b).

\(^{60}\) Negotiation as in ‘negotiation of meaning’, a term borrowed from Wenger (1998), used here to refer to conversations that aim to reconcile diversities in meaning.
around how to get ‘students discussing and debating’ within ‘groupwork’ settings. For instance, one teacher revealed how they prompted students to ‘discuss and debate’ within an activity that involved ‘groupwork’ by explicitly stating they need to prove order.

#clotheslinemath today. Proving how to organize mixed numbers, fractions, improper fractions, decimals, and percents from least to greatest. #bfc530 #mtbos #iteachmath [In image: “You must prove to me why your order is correct.”]

Another explained, “I like to use a document camera to show what a particular group has done so that all students can compare their groups.” And, another indicated that they have a poster with prompts to help students know how to ‘discuss and debate’ within ‘groups’. While it was evident these teachers were structuring activities to elicit students’ ‘discussing and debating’, there seemed to be incongruence around how ‘discussing and debating’ was to be achieved within ‘group work’ settings. This was most evident and explicit within threaded responses. For example, one contributor indicated their students were having rich mathematical conversations when they were given time to ‘discuss’ a test without pencils in ‘groups’ prior to writing it.

For tests, I dedicate the first 5 minutes to have students hold the tests and talk to their group about strategies (w/out pencils). This lowers their test anxiety AND when you walk around, you hear some of the richest math conversations. #iteachmath

This tweet triggered hundreds of responses with great diversity including supportive comments, questions about how and why this teacher does this, and statements about why this would not work. Below are a few such responses made by various users.

I like this idea a lot. Do you ask students to only talk strategy and not solutions?

HS geo had their first proofs test today. After 15 minutes of independent work, I said, “ok, phone a friend...or several”. They jumped in to two groups and had the best discussions I’ve heard this week.

Tfs! This year’s lessons have all consisted mostly of student collaboration. Was wondering how tests would be affected. First test is Monday. Going to try this.

This would not lower test anxiety for all students. For my son, who has anxiety and exec fn issues, (& an IEP) this could increase anxiety, esp if done without warning or choice.
The diversities around how students should and should not be directed to ‘discuss and debate’ within ‘groupwork’ situations were so disparate that it led to active negotiation. This made the ideational divergence even more explicit and allowed for attempts at providing more diverse perspectives and experiences within subsequent responses.

There was evidently interest in how to achieve the pairing in this relation within constraints of classroom contexts and amidst the institutional worlds that teachers live in. There also was a willingness to negotiate ways in which this relation could be achieved.

Further, another relation that explicitly involved diversity in meanings that resulted in a different kind of negotiation was between ‘geometry’ and ‘problem solving’. The intersection between these initially seemed obvious because ‘geometry’ offers rich grounds for ‘problem-solving’ opportunities. However, various diversities existed. Firstly, the ways in which ‘geometry’ was presented as ‘problem solving’ differed around who problems were intended for and whether the focus was mathematical or didactical. The focus could be purely on the problem itself with an interest in how to solve it (Figure 6.25a, b & c), or on the choice of problem towards teaching specific mathematical content (Figure 6.25d). This afforded two roles: that of being a problem solver, and that of being a teacher. In both cases, tweets about ‘problem solving’ in ‘geometry’ generally attracted a significant amount of negotiation in which users shared diverse strategies towards solving the problem or finding an appropriate problem to use with students. This occurred even when the problem was not directed with a question (as in Figure 6.25a).
Further, mathematical negotiation often preceded pedagogical negotiation. For instance, the responses to the prompt seen in Figure 6.25c about the ant on the box problem ranged significantly in approaches and visualizations initially, but also spurred conversations about how to guide students to see the solution without telling it to them.

User 2: I would open it up as a paper model.

User 3: Me too, it’s hard to get students to get there without showing them, so a good follow-up question is to ask how far along the opposite side the ant reaches before turning onto the other face.

Online problem-solving engagements such as this were often followed by reflections about teaching mathematics, a phenomenon which may be supported by instances of teacher change occurring when teachers engage in mathematical experiences as learners, which gives them a different perspective than when teaching, before unpacking mathematics teaching (Liljedahl, 2010). An example of such a shift is seen in the interaction below.

User 4: I feel like I would be using guess and check which I know is not the most efficient way but it’s were I start. I would also draw a line diagonally across the top face as I am estimating it is the fastest. The ant has to stay on the surface correct?

User 161: Yes, the ant is restricted to the surface. I also used guess-and-check with what I thought were the three candidates, but I was mistaken.

61 User 1 refers to the user who contributed the initial root tweet to which replies were made.
User 4: So the quickest route isn’t straight up from A and then diagonally across to B?

User 1: It’s not!

User 4: Oh I see. The front face is smaller bc it’s only 3 inches wide. So go diagonally across the front face then straight back to point B!

User 4: What I love with what we are doing here is exactly what we need to get kids doing. I have entered into this convo and made a lot of conjectures about which route is shortest without doing any calculations or use any formulas yet. It created curiosity! All can enter in!

Further, there was also diversity around what a ‘geometry’ problem is. For instance, in the responses to the question about lessons/activities for area and perimeter seen in Figure 6.25d, what was considered a ‘problem’ varied between being treated as a rich task (Figure 6.26a & b), a worksheet activity problem (Figure 6.26c), a formula-deriving structure (Figure 6.26d), or real-life visualization examples (Figure 6.26e & f).

Figure 6.26 Variance in what counts as a ‘problem’ in geometry

This diversity created rich soil for generating new material that was acceptably different than existing material rather than continuing to generate slightly varied replicas of past contributions as seen in the ideationally convergent relations. That is, meanings in the
relation were redundant enough to create and maintain the relation but involved enough diversity to produce novel content. As such, in this relation, explicit ideational divergence stemmed primarily from variance in what users interpreted ‘problem solving’ in ‘geometry’ to be. There was explicit diversity in the ways offered to solve given problems, ways used to ‘engage students’ in ‘problem solving’, and approaches used to select appropriate ‘geometry’ problems. The ideational divergence was not negative as it was not rooted in disagreement, but rather, diversities allowed for generativity of new material. This contrasts with the replication seen in some of the ideationally convergent relations. Also, the diversity in this relation was revealing of how many choices teachers have in lesson design and the dimensions of possibility around ‘problem solving’.

In contrast to the recreational and productive negotiation spurred by the ideational divergence within the ‘geometry’ and ‘problem solving’, the relation between ‘NCTM’ and ‘conferences’ was generative of more negative and frustration-driven negotiation. Although it was generally agreed that ‘NCTM’ offers ‘conferences’, tweet clusters contributing to this topic involved great diversity in opinions about NCTM conferences and their structures. While there was some consistency among a few of the tweets that excitedly announced the next upcoming NCTM conference (e.g., “NCTM Regional Conference in Seattle this fall!”), there were several directions for diversity that moved beyond advertising the event. One of these directions was around deciding whether to attend the conference. For instance, in opposition to the typical excitement-laden conference announcements, one user expressed hesitation and sought advice about if it is worth their time and money to attend an NCTM conference (See Figure 6.27).

Figure 6.27  Hesitation about attending NCTM conference
In this tweet, the teacher linked a blog post written by Dan Meyer\textsuperscript{62} (2016) that presented various benefits and drawbacks of NCTM conferences as well as of their journals and other subscriptions. In the linked blog post, Meyer (2016) critiqued the unused ‘potential energy’ found at NCTM conferences arguing that ideas don’t necessarily have anywhere to continue past the conference, whereas they do on blogs and on Twitter. The responses to the initial tweet as well as in the plethora of comments to Meyer’s (2016) linked blog post indicated a wide range of positions about benefits and drawbacks of what NCTM offers. For instance, one respondent to the tweet in Figure 6.27 indicated, “One reason I go is for the energy of the event . . . for the individual people from across the country I meet for 5 minutes; for the dialogues with those I’ve known for years”. Another respondent to Meyer’s (2016) blog post supported Meyer’s point about finding ways to continue conversations beyond the NCTM conferences.

There were several K-2 talks I attended at NTCM that I would love “to continue the conversation.” I tried Twitter, but the sessions had low turnout so there was no interaction. (Meyer, 2016, para. 25)

Meyer (2016) also linked to a poll in which he asked how likely MTBoS participants were to attend an NCTM Annual conference, attracting hundreds of responses with a wide range of diversity in opinions, many of which straddled the tensions between cost and feelings of connection to the community. Interestingly, the poll indicated it was more likely for those involved in MTBoS to attend NCTM Annual, mostly because of the connections they created online that they wanted to extend in person, as was evident in the replies to initiating tweet. However, cost remained a prevailing issue.

The tension around cost was prominent not only in this tweet cluster, but also in others. The most marked one was initiated by a user who felt fees for speakers to present at the conferences were unreasonable and sought advice. The contributor claimed, “@NCTM @robertqberry Really? $341 for lead speaker registration? At this cost, during 3 school days, on the other coast, do you really want practicing teachers to attend? #unreasonable.” Over 100 responses to this provocation ranged from justifying why these fees were necessary, to arguing for change in NCTM structures with an advocacy for accommodating practicing teachers to have their voices heard and participation valued. Other instances of negotiation also revolved around issues of cost, and at times

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\textsuperscript{62} Dan Meyer is generally considered to have high authority in the math twitter blogosphere due to his prominent contributions and long-lasting initiatives.
involved discussions around establishing a MTBoS booth in a self-organized manner around the constraints of being subject to pay a $300 fine if they do not have enough volunteers manning the booth. Financial pressures can hamper such self-organization, and contributors were using the space to work around such barriers. That is, the online collective was finding ways to self-organize within a centrally organized event through harnessing the possibilities of negotiation within explicitly divergent ideational relations such as between ‘NCTM’ and ‘conferences’.

The explicit ideational divergence that surfaced in this relation invoked debates around professional activity, related costs, and associated benefits to which there were no clear resolutions. The powerful result of this relation is that when divergence among ideas surfaced explicitly, it spurred significant discussion among members. Moreover, these debates emerged opportunities for the collective to be reflexive, in a sense of self-evaluation, and paired with the opportunity for discussion arising from explicating differences, such instances followed subsequently with the power to invoke action.

**Implicit ideational divergence**

Finally, while diversity was sometimes made explicit within relations exhibiting explicit ideational divergence, it was at other times more implicit. Akin to their more explicit cousin, relations in this category evidenced diversity in communicated meanings. However, these diversities were only possible to infer through careful inspection. That is, on the surface, they may have been treated as ideationally convergent, but upon further inspection, ideational divergence became evident. The lack of explicit acknowledgement of diversities prevented negotiation or emergence of new meanings, leaving the potentialities available but unseized and dormant during the time frame of data collection. The affordances within these relations, therefore, involved their potentiality for emergence, but the barriers were in a lack of attention to the differences in meanings. To some extent, these barriers are inherent to the limitations of the social media space in terms of content visibility and tools for communication.

For example, one relation that involved implicit ideational divergence was between ‘engaging students’ and ‘problem solving’. In this relation, there was evident incongruence among interpretations of what was considered ‘problem solving’ and how ‘engaging students’ in ‘problem solving’ was to be achieved. In one case, a request for ‘problem solving’ activities for area and perimeter for high school students elicited
diversity in what was regarded as a ‘problem’. This diversity was also revealed earlier in the examples shown in Figure 6.26 within the ‘geometry’ and ‘problem solving’ relation where problem solving was considered in different ways: either as rich tasks, worksheet activities, formula-deriving structures, or real-life visualizations. However, unlike in the ‘geometry’ and ‘problem solving’ relation, which invoked explicit ideational divergence that spurred negotiation around solving geometry problems, there was a more implicit ideational divergence around ‘engaging students’ in ‘problem solving’. While it may be argued that ‘problem solving’ is naturally multi-dimensional, an evident space of possibility for diversity existed around how ‘problem solving’ was to be implemented to ‘engage students’. However, it was not explicitly addressed in the data set captured.

In some tweet clusters, students were ‘engaged’ in ‘problem solving’ by being given a problem handout and sitting together to discuss the problem as in Figure 6.28a. Alternatively, students were ‘engaged’ in ‘problem solving’ by ‘instructional routines’ that supported multiple approaches to thinking through an array design problem that students notated in individual notebooks as seen in Figure 6.28b. Or, via technological devices that students used either individually or collaboratively as in Figure 6.28c with minimal teacher intervention. And possibly, through being invited to try solving a puzzle on dry-erase boards individually during their free time as in Figure 6.28d.

![Figure 6.28](image-url)
Across these different manners of implementation, there were nuanced differences in what counted as ‘problem solving’. While these tweets hide a lot of implementation details, they offer a space of possibility for what ‘problem solving’ situations that ‘engage students’ could look like. Such possibilities could have been negotiated, but were not. Although the ideational divergence widened the space of possibility in the relation, it remained implicit and did not surface through explicit negotiation. In other words, while there was potential for discussing meanings, the potential was not acted on in the data. Instead, the notion of ‘engaging students’ in ‘problem solving’ was taken-as-shared.

Another example of implicit ideational divergence was in the ‘classroom community’ and ‘classroom culture’ relation. These artefacts were often used interchangeably but seemed to hold different meanings in each instantiation. For instance, as seen in Figure 6.29a, practices such as #vnps and #vrg could allow for students to be ‘helpful and proud’ and make it a ‘better place to learn’. This tweet implied that positive ‘classroom community’ and ‘culture’ could be supported through involvement of #vnps and #vrg in practice, which aim to make the classroom a place where students care for each other. Another way to achieve this, seen in Figure 6.29b, was by infusing classroom culture with materials that value students’ names to help with building community. And, as seen in Figure 6.29c, positive ‘classroom culture’ and ‘community’ was also considered to be supported by greeting students daily as they enter class.

![Figure 6.29](image1.png)

While ‘classroom culture’ and ‘community’ were implicit in the above examples, and were not specifically differentiated between, approaches to achieving their existence

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63 VNPS stands for vertical non-permanent surfaces, as introduced by Liljedahl (2016, 2019).
64 VRG stands for visibly random groups, as introduced by Liljedahl (2014b, 2016).
were revealed through diverse means. As such, opportunities to negotiate different ways in which positive ‘classroom culture’ and ‘community’ could be instilled were available. One tweet even indicated an attempt at such a negotiation by asking what it takes to change a ‘classroom culture’.

It seems to me that a lot of what makes various teaching things work/not work is class culture. How long does it take to create such a culture? How long does it take to *change* such a culture? If you’re visiting another’s class, can you work with the existing culture? #MTBoS

Although this provocation elicited some response and shed light on the dimensions of possibility for negotiation, the meanings around what sorts of ‘classroom culture’ build ‘classroom community’ were not explicitly negotiated. Rather, responses moved no further than stating how challenging it is to change ‘classroom culture’. As such, the possibilities for negotiation remained dormant. This is significant because it represents unseized potential for the kind of activity that could be occurring, as seen in the explicit ideational divergence section.

**Summary of dimensions of possibility for ideational relations**

Overall, ideational relations have been identified as possible to be made either explicitly or implicitly and with evidencing ideational convergence or divergence. The distinction (albeit fluid) between ideational convergence and ideational divergence highlights the contrast between relations that exhibited a lot of redundancy and those that exhibited more diversity in the meanings attributed to their co-existence within a relation. And, explicit or implicit aspects within relations reflect the degree to which emergent meanings within relations were *mobilized*. Within these options and their respective combinations, various potentialities and limitations for emergence surfaced.

Namely, emergence within ideationally convergent relations occurred through the replication of ideas with similar formats and minimal variance. The consistency in form and function of these contributions afforded continued engagement in similar means, feeding explicitly or implicitly into the establishment of taken-as-shared meanings. Such recurrent consistency in meanings within a relation created an opportunity for reification. I use the term reification here similarly to Wenger (1998), who defines it as a process of attaining ‘thingness’, but I reserve my use of the term to describe ideational meanings that have become so strongly reinforced that their sense of ‘thingness’ is significantly
evident. Reification can therefore develop implicitly through replication within relations or explicitly through acknowledgement. In such ways, it can carve out pathways for emergence of coherence in the collective as well as helping newcomers acclimatize to the norms of the system through the stability it offers. However, a tendency towards reification holds limitations around lack of diversity, which may obfuscate nuanced differences by reinforcing them in ways that make them remain taken-as-shared. When ideas are considered established, they can remain unquestioned and unchallenged, which limits the creative potentialities afforded by the presence of diversity.

Ideational divergence, on the other hand, employed diversities that created opportunities for negotiation. As noted earlier, I borrow the term negotiation from Wenger (1998), who theorizes about the negotiation of meaning as a practice of the social production of meaning. However, since I am using it in a complex systems perspective, I use negotiation to refer to the opportunities for ideational mediation within the system among participating agents who aim towards an objective of communicating each others' perspectives. Negotiation naturally became possible within relations of ideational divergence because the diversities in meaning allowed for mediating diverse perspectives within the relations. Without adequate diversity, there simply was nothing to negotiate. So, ideational divergence was necessary for negotiation, but negotiation did not always occur when there was ideational divergence. Rather, it remained merely an opportunity for negotiation. When negotiation happened, though, opportunities for generativity of novel material prevailed. Without negotiation, such opportunities were unseized and risked being interpreted as taken-as-shared, which allowed diversities to remain hidden, preventing new material from emerging. The diversities were also sometimes too disparate to reconcile even if negotiation occurred, which is in part reflective of the limitations of the social media space as a site for activity.

As such, the systemic features in MTBoS offered possibilities for ideational convergence and divergence to prevail either in explicit or implicit manners. Since ideational convergence created opportunities for reification and ideational divergence created opportunities for negotiation, these opportunities became more evident when relations were contributed to explicitly. That is, when either redundancies or diversities were

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65 This is to distinguish from the natural opportunities for every action within Twitter to have 'thingness' in the sense that there is a trace of activity within each action, which would be too broad a treatment of the term reification in this setting.
actively reified or negotiated, respectively. On the other hand, implicit relations remained as unseized opportunities that held potential active energy towards reification or negation, and therefore remained inert. I introduce the terms active and inert here to describe the resulting effects of explicit and implicit relations, respectively. Although negotiation and reification did not always occur because of ideational convergence or divergence, they were more likely to occur actively when the relations were made explicitly, and inertly when relations were made implicitly. Therefore, ideational convergence steered towards reification and ideational divergence steered towards negotiation, but the steering was related to how actively or inertly this occurred.

Further, each of these combinations could presumably transform into each other given systemic conditions that pull either towards reification or negotiation depending on how diversities and redundancies are mobilized in the system. Since these categories are fluid and can evolve into each other at any moment with changes to the system's context or composition, they may be represented as in the diagram in Figure 6.30 below.

![Diagram showing dimensions of possibility for ideational relations]

**Figure 6.30  Dimensions of possibility for ideational relations**

However, the scope of this research makes it impossible to examine ways in which these categories operate and morph into each other over time since the data does not reflect change over time. Notwithstanding, the existence of the possibilities for transformation between categories is supported by my insider observations of the collective as it has grown and developed over time. For example, while the relation between ‘noticewonder’ and ‘real-life examples’ acted in implicit ideational convergence in this data set, my insider experience made me privy to seeing this relation in both explicit ideational convergence and explicit ideational divergence as it has been at times reified actively and other times negotiated actively. The systemic opportunities afforded by decentralized organization and neighbour interactions allow for such shifts to occur, which are more deeply explored in subsequent chapters.
6.3 Exploring ideational artefacts

Before moving out of the ideational space, I first turn to how ideational relations compose to define ideational artefacts. It turns out that ideational artefacts have various possibilities for how they can live in the ideational network and how they contribute to ideational emergence. Once the top 15 reoccurring ideational artefacts (shown in Table 6.2) were selected for closer examination, sources of redundancy and diversity were sought out among the relations around each of these artefacts. To this end, the dynamic framework for identifying types of relations that was presented in Table 6.3 and exemplified throughout §6.2 served as an analytical tool for exploring the relations around artefacts seen in Figure 6.17. More specifically, relations were identified as either explicit or implicit, and composed with either ideational convergence or divergence. As such, relations could be identified as offering potentialities towards either reification or negotiation that was either active or inert. By determining the kinds of relations made around each artefact, it was possible to identify various manners in which ideational artefacts could emerge and live in the ideational space in a given timeframe. After each relation was coded in this manner from the tweets, it was then coloured accordingly on the ideational network, a visual rendering of which is seen in Figure 6.31.

Doing this allowed for an investigation into the diversities and redundancies around each artefact. This involved looking at each artefact's coded subgraph, and then sorting the artefact subgraphs into categories based on redundancies and diversities in their
structures. Since most of the relations in this pruned network were ideationally convergent in nature, the differentiating features between artefacts existed in how much ideational divergence they held opportunity for. Some artefacts had at least one source of explicit ideational divergence (Figure 6.32), others had some sources of implicit ideational divergence (Figure 6.33), and many had no sources of ideational divergence, only varying degrees of ideational convergence (Figure 6.34).

Figure 6.32  Ideational artefacts with at least one source of explicit ideational divergence

Figure 6.33  Ideational artefacts with no sources of explicit ideational divergence but at least one source of implicit ideational divergence

Figure 6.34  Ideational artefacts with no sources of ideational divergence
This left ideational artefacts on a spectrum between having adaptive capacity through ideational divergence, and stability through ideational convergence. By considering tendencies in relations towards negotiation or reification with surrounding artefacts, four categorizations for artefacts emerged and are revealed in Table 6.5 below.

**Table 6.5  Categories of ideational artefacts**

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reified Artefact</strong></td>
<td>Ideational artefact with most of its surrounding relations in <em>explicit ideational convergence</em>, offering directions for active reification, with no surrounding relations in <em>ideational divergence</em></td>
</tr>
<tr>
<td><strong>Reifiable Artefact</strong></td>
<td>Ideational artefact with most of its surrounding relations in <em>implicit ideational convergence</em>, offering directions of inert reification, with no surrounding relations in <em>ideational divergence</em></td>
</tr>
<tr>
<td><strong>Negotiable Artefact</strong></td>
<td>Ideational artefact with several of its surrounding relations in <em>implicit ideational divergence</em>, offering directions of inert negotiation, with the possibility of some surrounding relations also being <em>ideationally convergent</em></td>
</tr>
<tr>
<td><strong>Negotiated Artefact</strong></td>
<td>Ideational artefact with a mix of types of surrounding relations, but with at least one in <em>explicit ideational divergence</em>, offering direction for active negotiation</td>
</tr>
</tbody>
</table>

Just as with ideational relations, these serve as dimensions of possibility meant to allow for transitivity between categories depending on emergent activity in the system. Their purpose is descriptive to classify artefacts within a constrained segment of time and their existence within the dataset in this thesis is discussed and exemplified in what follows.

**Reified Artefacts**

Reified artefacts involved a prominence of surrounding relations that created opportunities, and many times evidence, for active reification. This occurred through anchoring the artefact’s meaning towards those artefacts with which it had a relation of explicit ideational convergence. That is, the reified artefact became stabilized by the strong relations it held with certain other artefacts that emerged active reification.

For instance, the ideational network graph surrounding the ‘mtbos’ artefact (in Figure 6.35 below) shows how it lived in explicit ideational convergence with all the other artefacts around it such as it was explicitly associated with being ‘welcoming’, a place for ‘great ideas and inspiration’, and for ‘finding resources’. Informed by each of these directions, ‘mtbos’ was reified in the data as a welcoming Twitter community that offers great ideas and inspiration for teachers as well as pre-service teachers and is a good
place to find resources and participate in real-time chats. While these did not all occur simultaneously, the composition of tweets formed this coherence.

Figure 6.35  Ideational subgraph of ‘mtbos’

The active reification around what ‘mtbos’ is defines it explicitly in ways that could easily indicate to newcomers the values and practices ‘mtbos’ supports. These reifications of the nature of the ‘mtbos’ were made by individuals on behalf of the collective.

Welcome! Just tweet here when you are looking for something and someone will help out! #mtbos

Twitter can be used as a way to find powerful and engaging resources to use during student teaching! #NCSUNCCMathEd #MTBoS

Welcome to Twitter, @ogarrmath! I love chatting about students’ math opportunities. We have a vibrant math teacher Twitter community. Check out #MTBoS (math twitter blogosphere) and #ITeachMath

Contributors continued to reinforce these meanings actively through the production of content that indicated ‘mtbos’ was a positive place for resource exchange and connection with other people interested in improving mathematics teaching. This message was made explicit, and therefore accessible to the wider public.

The other artefact that involved a significant amount of active reification in its outwardly stemming relations was that of ‘games and puzzles’, which held stability through explicit ideational convergence towards how it related with ‘fun activities’ that could be used as ‘review activities’ that involve ‘algebra’. However, it also had some directions, such as that of it involving ‘groupwork’ or ‘problem solving’ that were evidenced only with implicit

66 While MTBoS was ideationally convergent in this particular data capture, sometimes it attracts more ideational divergence as based on my insider view. Also, only tweets that referred to MTBoS as a community or collective were included in the ‘mtbos’ artefact, hence the lowercase treatment.
ideational convergence. This means that the ‘games and puzzles’ artefact was actively pulled towards certain anchors of stability, but more loosely towards others. The more active relations established meanings publicly while the more inert relations continued predominantly with replication of redundant material.

Overall, both reified artefacts, ‘mtbos’ and ‘hands-on learning’, offered emergence through replication and acknowledgement of it, where new emergent material continued to support established meanings. Such strong reification afforded the possibilities for taken-as-shared meanings to become sturdily established, evidencing ideational coherence, a form of collective identity for the system. The space of possibility, however, was constrained and attracted little negotiation, only reification.

**Reifiable Artefacts**

The process of replication of similar content with slight variance was also evident within artefacts that had more implicit ideational convergence in surrounding relations. The sources of potential reification remained inert and were not made as public as their more active counterparts. Tweets within reifiable artefacts also had minimal diversity in relational meanings, and in turn did not attract much novelty. Instead, they revealed consistency in the pathways for participation, offering structures for engagement that were potentially noticeable but not made explicit. While some reifiable artefacts had more space for active reification than others, overall, reifiable artefacts remained inert in terms of public acknowledgement of taken-as-shared meanings that they continued to support. That is, the primary form of emergence within reifiable artefacts was replication.

For instance, ‘hands-on learning’ provided an avenue for users to tweet media that involved students ‘engaging’ in activities with ‘hands-on learning’. An overwhelming amount of replication, or redundancy, pervaded the tweets in this artefact with tweet after tweet involving media that showed students using their hands (Figure 6.36 below). However, this redundant pattern of activity was not being explicitly addressed. Rather, the redundant activity of including students using their hands in the mathematics classroom seemed to act as a formula for making tweets about classroom activities.
Figure 6.36  Examples of hands-on learning as recurrent

All other artefacts that ‘hands-on learning’ related to were also in implicit ideational convergence as seen in Figure 6.37a below. ‘Hands-on learning’ was therefore being inertly reified as linked with ‘using manipulatives’, ‘visualizing mathematical concepts’, ‘groupwork’, ‘geometry’, ‘fun activities’, ‘engaging students’, and ‘games and puzzles’.

Figure 6.37  Ideational subgraphs of hands-on learning and real-life examples

‘Real-life-examples’ (Figure 6.37b above) was the other reifiable artefact that also had mostly implicit ideational convergence surrounding it. In this case, a few directions had more explicit manifestations of convergence, such as towards ‘creating new activity resources’ and ‘data’. However, the dominant behaviour in ‘real-life examples’ still tended towards the repetitive structures of replication characteristic of inert reification.

Both of these reifiable artefacts exemplified taken-as-shared values in MTBoS that emerged from replication of similar content being produced in similar ways with slight variations. Although they were not made explicit in this data set, they very well could have been. So, they contributed to the ideational coherence of MTBoS, defining it as a space for sharing about ‘hands-on learning’ activities and ‘real-life examples’ in practice. However, rather than this being public, it required inference via participation.
**Negotiable Artefacts**

Inference was also necessary within negotiable artefacts because many of the relations around these artefacts were implicit, remaining taken-as-shared without explicit acknowledgement. The key distinction between negotiable artefacts and reifiable artefacts was that negotiable artefacts had some sources of implicit ideational divergence. The diversity available in these relations, however, remained inert and taken-as-shared. So, while negotiable artefacts acted very much like reifiable artefacts in terms of having a strong characteristic of repetition and recurrence of redundancies, they held a potentiality for ideational divergence that made negotiation more available. So, directions of ideational convergence maintained the stability and coherence of the negotiable artefacts, and directions of ideational divergence offered space for negotiation. While active negotiation did not arise, there was enough diversity for it to arise. Some negotiable artefacts seemed more prone to this than others.

For instance, the ‘fun activities’ artefact (Figure 6.38) offered several directions for ideational convergence and one with ideational divergence. It was supporting its association with ‘student choice’, ‘using manipulatives’, and ‘games and puzzles’ actively and with ‘real-life examples’, ‘hands-on learning’, ‘groupwork’, and ‘engaging students’ inertly. These were all artefacts that contributed to defining the meaning of what ‘fun activities’ could involve. The ways in which these prevailed were similar to that seen in reified and reifiable artefacts. However, the one direction ‘fun activities’ had that offered space for potential negotiation was in its relation with ‘activities’.

![Artefact subgraph for fun activities](image)
'Activities' generally involved lessons that included special features such as ‘real-life examples’, ‘pair work’, ‘noticing and wondering’, and general strategies for ‘engaging students’ in learning content. ‘Fun activities’, on the other hand, were reserved for things that were ‘engaging’, but not necessarily curricular. The intent was slightly different because ‘fun activities’ aimed to entertain students (Figure 6.39a), and ‘activities’ aimed to target learning objectives in manners that could be engaging (Figure 6.39b). Also, in cases where ‘fun activities’ overlapped with ‘activities’ (in that they were both ‘fun’ and curricular), there were differences in what attributes contributed to making the activities ‘fun’. On one hand, ‘activities’ could be ‘fun’ because they were conceptually interesting (Figure 6.35c). On the other, they could be ‘fun’ because they were ‘hands-on’ (Figure 6.39d), or because they were embedded into a ‘game’ (Figure 6.39b).

So much going on in math workshop today! STEM lego challenges, place value towers, lesson work on Freckle, and number sense videos on ChatterKid! #LRelem #ipaded #lego #iteachmath #STEM #mtbos @LEGO_Group @learnfreckle @LCPS_Math

Rapid fire review of the unit circle (<3333) and exact trig values in non AP Calculus today! Hula hoops & all! #MTBoS qivemeasine.blogspot.com/2018/09/unit-c ...

Exploring properties of 2D shapes & really looking at the process expectation of #reasoning & #proving. Some incredibly passionate #mathematicians in The 107.

Middles school kids need to play and explore too! Polydrons are so much fun. @swuttig #iTeachMath #MTBoS

Figure 6.39 Examples of ‘fun’ activities

As such, there was a potential space of possibility for reifying or negotiating what makes an activity ‘fun’ and whether ‘fun activities’ are curricular or non-curricular. But this space of possibility was not actively explored. Instead, the diversity within the relation contributed to implicit ideational divergence, serving as inert negotiation.
The space of possibility for negotiation was even more prevalent in the ‘assessment’ artefact (Figure 6.40), where there were a few directions for ideational convergence, and several for implicit ideational divergence. In terms of convergence, ‘assessment’ offered potential for active reification with ‘feedback’ and ‘formative assessment’, inert reification with ‘student self-reflection’, and inert negotiation with ‘retakes’, ‘growthmindset’, ‘mastery’, and ‘using technology’. Its convergent relations with ‘feedback’ and ‘formative assessment’ provided stability, defining what is important in ‘assessment’ within MTBoS. These directions, along with ‘student self-reflection’, provided anchoring structures that helped ‘assessment’ have coherence in the space.

However, as may be expected, the contextual landscape teachers typically live in create tensions between ‘assessment’ and curricular mandates, state testing impositions, and efforts towards reform approaches to teaching. Some of these are irreconcilable and contributed to ideational divergence around ‘assessment’, particularly around the pragmatics of implementing ‘assessment’ practices amidst other reform-oriented practices evidently valued in MTBoS. Valuing an idea is one thing but putting it into practice is another. As such, the relations surrounding ‘assessment’ that contributed inert negotiation pertained mostly to pragmatics of implementation.

Figure 6.40  Artefact subgraph for assessment

For example, there was diversity in stances towards ‘retakes’ in ‘assessment’ practices, with some agreement about providing ‘retakes’, and some resistance.

#GrowthMindset? . . . maybe it wasn’t the students fault, but the test didn’t provide opportunities to “show what you know.” How can we both improve? [image with student work on an assessment with a blank answer and comment asking to retake the quiz after more learning]
When a student comes to me a day before a test to say they aren’t ready and request to take it another day, I say they can’t earn full credit.. torn bc I know learning happens at wildly different paces, but they need to be prepared to handle deadlines #MTBoS #iteachmath

Interestingly, the idea of ‘retakes’ in ‘assessment’ was situated with justifications rooted in relations between ‘assessment’ and ‘growthmindset’. ‘Growthmindset’, being a very active and popular hashtag in MTBoS and a term coined by Dweck (2008) and advocated for by influencers such as Boaler (2015). It permeates throughout the fabric of much of the content in MTBoS and was evidently agreed upon as being important based on the way it is referred to, supported, and recurrent in the data. However, the ways in which users posted about using ‘growthmindset’ approaches within ‘assessment’ practices varied, particularly around the locus of responsibility. This was most evident within tweets that involved ‘assessment’, ‘growthmindset’, and ‘feedback’. For example, one teacher indicated their efforts at providing students with ‘feedback’ on summative assessments indicating they are employing principles of ‘growthmindset’ in their stance towards highlighting opportunities for improvement.

Grading my first real SBG summative assessment in AP Stats. Loving the work and detail Ss put into their work! Also amazingly satisfying just writing feedback and noting areas of strengths and needed improvement instead of points. #MTBoS

Since ‘feedback’ was a source of explicit ideational convergence, holding strength in its importance and consistency, it pulled the ideational space towards the convergence amidst its divergence. Further, in a case where a focus on ‘feedback’ was not as evident, in that it involved verbal feedback rather than written feedback, there was more ideational divergence around the use of ‘growthmindset’ principles within ‘assessments’.

For tests, I dedicate the first 5 minutes to have students hold the tests and talk to their group about strategies (w/out pencils). This lowers their test anxiety AND when you walk around, you hear some of the richest math conversations. #iteachmath . . . I let them look at the test and talk about how to solve those problems. Ex: "#1 is easy, let's skip talking about that one. #2 we're going to get common denominators, draw a number line,..." etc.

Without the stability of the ideational convergence with ‘feedback’, this provocative tweet made everything negotiable. As expected, it received a significant amount of attention in the form of comments either affirming the idea, asking questions about implementation,
or revealing expressions of worry that it is not an appropriate manner of assessment. Although the notion of ‘growthmindset’ wasn’t explicitly referred to here, it was an underlying aspect to the practice suggested in the contribution. It also illuminated the diversity in how some respondents viewed ‘assessment’ as either individual fixed-point progress reports, or a valuable component to student learning journeys. The notion of ‘feedback’ being verbal and possibly not structured the same way for each student, removed the source of stability that could make an idea more familiar and possibly less resistance-provoking. However, the lack of ideational convergence around the artefact contributed to a more prominent ideational divergence. This opened the space of possibility for actively negotiating how ‘growthmindset’ could be used within ‘assessment’ structures in mathematics. Although active negotiation about how ‘growthmindset’ notions could be used to structure ‘assessment’ was not enacted in this data set, it remained available through the diversities that emerged within the relation. That is, the soil was rich with diversity and redundancy, albeit not actively seized.

Negotiable artefacts therefore involved sources of instability through ideational divergence in some directions but remained anchored in sources of stability through ideational convergence in other directions. Sources of stability, such as the ideas about ‘feedback’ within ‘assessment’, contributed a sense of familiarity amidst the ideational divergence it could be paired with, such as with ‘growthmindset’. This allowed for emergence of new material that did not attract active negotiation, but that seemed to remain taken-as-shared. Alternatively, when such sources of stability were not present, and more of the tweet contents were negotiable with less redundancy, there was a greater tendency towards negotiation. Although the negotiation remained inert in negotiable artefacts, it is interesting to observe the fickle balance between redundancy and diversity in what it breeds in combinations that involve both sources of ideational convergence and divergence, and how it creates potentialities for negotiation.

**Negotiated Artefacts**

Finally, negotiated artefacts involved at least one direction for active negotiation. This means they could solicit any of the opportunities available in negotiable or reifiable artefacts, but also could create space for active negotiation, which not available in other

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67 Some responses to this tweet were revealed earlier, but the tweet is revisited here to examine it as ‘growthmindset’ in ‘assessment’ rather than ‘students discussing and debating’ in ‘groupwork’.
kinds of artefacts. This has to do with the diverse array of ideational relations that negotiated artefacts involved; some with more opportunity for negotiation than others. In particular, ‘visualizing math concepts’ and ‘geometry’ mostly involved reification except for one source of active negotiation they had; and, ‘problem solving’, ‘noticewonder’, and ‘using manipulatives’ had a combination of all types of relations. Sources of active and inert negotiation offered space for new ideational opportunities towards emergence while active and inert reification created stability. The diversity afforded by these combinations of opportunities for both reification and negotiation around artefacts created rich grounds for emergence. Interestingly, none of the instances of active negotiation invoked only one or the other, they always involved both. This points to the necessity of having sources of both stability and instability to emerge novel ideational material.

One example of this was around ‘visualizing mathematical concepts’. Although this artefact had several directions in which it evidenced reification, such as towards ‘statistics’ and ‘drawing’, it had a strong source of negotiation towards ‘fractions’. The sources of reification indicated the importance of the artefact and defined its stable features. Namely, it was actively reified with ‘statistics’, particularly in relation to how important visualize statistical content was, especially if related to ‘social justice’ content. It was also directly referred to as related with ‘drawing’ in that drawing is an important aspect of visualizing mathematical content.

What are the different ways to visually represent division? If you are going to draw an example let’s use 15 divided by 3 for consistency. #mtbos #iteachmath #elemmathchat #msmathchat

Other directions of reification were more inert and involved artefacts, such as ‘being a good teacher’ in that helping students ‘visualize mathematics’ seemed to be considered an aspect of ‘being a good teacher’. For instance, helping a student understand the concept of independence in probability was done by providing the student with a visualization opportunity (Figure 6.41). These sources of reification established what was taken-as-shared about ‘visualizing mathematical concepts’.
However, the one significant source of active negotiation, was in relation with ‘fractions’ because it became actively discussed and recurrently exemplified that there are many ways ‘fractions’ may be ‘visualized’. A dominant approach among the diverse manners of ‘visually representing’ ‘fractions’ was to use ‘area models’. For instance, an ‘area model’ was used to reveal how fractions can be multiplied visually using a grid array approach along with base ten cubes (Figure 6.42a). It was also used for finding ways to prove division of fractions visually, with evidence of teachers investigating diverse ways of representing $1 \div \frac{3}{2} = \frac{2}{3}$ (Figure 6.42b and Figure 6.42c). Although an area model approach was redundant among many tweets in this relation, there were diverse ways of using the area models to represent fraction operations in visual ways.

And, this diversity was actively reinforced within responses to a request to help identify ways to support an eighth grader develop a stronger understanding of fractions.

Brainstorm with me, #MTBoS: Let's say you’re trying to do triage with an 8th grader who has an incredibly shaky foundation in fractions. What big ideas do you focus on? Can't do everything. What ideas about fractions are most essential for high school success?
The responses to this query indicated significant diversity in approaches to visualizing fractions, indicating ideational divergence that was explicitly acknowledged throughout the comments forming the relation. For instance, one thread involved active negotiation around the use of fraction strips; not only were fraction strips introduced, but multiple ways of using fraction strips to adapt to the learners’ needs were exemplified.

User 2: Get out some fraction strips (blank strips of paper) and fold, fold, fold! I like using strips because they transition nicely to using a number line. If you can get at the concept of a fraction, things will start to fall into place. Keep us posted!

User 1: This has been such an effective intervention. We made the strips last week and he noticed some things like equivalence that seemed like new discoveries. "Oh, 6/12 is 1/2 because 6 is half of 12!" This week we’re doing lots of scale factor stuff, so I am using the strips to help him with 6* 1/3. I hold up the entire strip and say "If this is 6 inches long...[fold fold fold] how long is this piece?" Using that strategy this AM, I got him to solve 32 * 1/8 mentally and then extended it to 32 * 5/8. Even if he only retains the unit fraction strategy, that's a big win for someone who didn't know that 6/12 = 1/2 last week

User 2: That is fantastic!!! A next step might be having him draw tape diagrams/bar models (an awesome representation for the strips).

Other responses included area models such as cake and traybake examples.

User 3: My brother in law baked a cake to help me when I was 11 😊

User 4: I do a lot of cake stuff. Dividing up a traybake. So far, it's the best method I can find for explaining what multiplying fractions means.

Some users also referenced specific resources that included ready-made ‘visualizations’ for ‘fractions’ they found useful with students, such as @gfletchy’s visual fraction progressions and @mburnsmath online fraction tool. And, interestingly, most of the responses included not only a brief explanation of their suggestion, but also rationales for why their suggestion had worked for them and their students. Rationales often included reference to how the visualizations were useful for future mathematical topics.

User 5: I think visual equivalency and scale. This will translate well to fractional variables (scaling) as well as similarity in HS geometry. When students can work with equivalent fractions they can add, subtract and have a stronger sense of numbers in general.
So, the topic of ‘fractions’ offered a space of possibility in which active negotiation could take place around identification of various ways to ‘visually represent’ ‘fractions’ towards supporting ‘conceptual understanding’. The negotiation did not stem from disagreement, but rather out of a seeming curiosity around brainstorming diverse ways in which fractions can be visualized and rationales for them so that different options could be available to use with students for whom certain ways may not work as well. Thus, an ideational space of possibility was opened around pedagogical and didactical aspects of teaching ‘fractions’ through ‘visual’ means. Systemically, the sources of reification brought purpose, prominence, and familiarity to the artefact while active negotiation allowed for emergence of new material. This emergence may not have been possible without the sources of reification that also prevailed around the artefact.

**Summary of dimensions of possibility for artefacts**

Overall, ideational artefacts had varying amounts of ideational divergence and convergence in their surrounding ideational relations, and therefore elicited either more negotiation or reification in various directions. Some of these directions were either more active or more inert. Active relations involved explicit acknowledgements of either diversities or redundancies among relational meanings. Inert relations created potentials for either negotiation or reification that remained unseized. Ideational artefacts had various combinations of such relations surrounding them, making them either reified, reifiable, negotiable, or negotiated. Each of these possibilities bred a different kind of emergence within the collective, revealing how artefacts can involve both sources of stability and provocation, each pulling the meaning in an artefact in different ways.

Reified artefacts tended to involve replication as well as acknowledgement of what was being replicated. Surrounded primarily by sources of explicit ideational convergence, emergent material continued to support well-established meanings, thus maintaining consistency in how the artefact was interpreted within the collective. This afforded the possibility for taken-as-shared meanings to surface explicitly and be actively acknowledged, which often resulted in these reified artefacts becoming points of ideational coherence in the collective, and in turn, a form of identification for MTBoS. The drawback for reified artefacts, however, was their lack of openness to expanding the space of possibility through negotiation, limiting their capacity to emerge novel material. Rather, emergence took form through reinforcement, replication, and reiteration.
Reifiable artefacts had a similar nature as reified artefacts, but their sources of stability were less explicitly acknowledged. While they similarly continued to support taken-as-shared meanings through implicit manners, these meanings were not always explicit. However, they were redundant enough for the possibility to become explicit to exist. Therefore, much of the reification around these artefacts remained inert and the primary forms of emergence involved replication and reiteration. For instance, the multitude of tweets that included images of ‘real-life examples’ alluding to how ‘fun’ and ‘engaging’ they are for students and how easily ‘new activity resources’ can be created with ‘real-life examples’ formed a source of replicable activity. Artefacts, such as ‘real-life-examples’, that involved a few sources of explicit ideational convergence in addition to the dominance of implicit ideational convergence had more acknowledgement of the artefact’s meanings in certain directions, such as towards ‘creating new activity resources’ with ‘real-life examples’. However, with little evident diversity, the space of possibility for negotiation remained limited, steering the ideational space primarily towards reification of taken-as-shared meanings. And, rather than being public, this reification required inference through participation. Notwithstanding, even though reification did not emerge significantly novel material, it evidently served as a sort of ‘lifeline’ for the collective in terms of offering a way for contributors to continue participating in the space, in turn upholding reified and reifiable ideational activity.

Negotiable artefacts, on the other hand, involved some directions of ideational divergence in addition to the convergence seen in the reified and reifiable artefacts. With the introduction of some divergence, albeit implicit, these artefacts revealed the fickle balance between ideational convergence and divergence necessary for movement away from replication towards negotiation and the emergence of novel material rather than reinforcement of existing patterns of participation. Since negotiable artefacts involved both sources of convergence and implicit divergence, they contributed both stability in meanings related to the artefact as well as space for provocation to challenge this stability. As such, negotiable artefacts afforded more potential for negotiation, and therefore emergence of novel material. However, since the divergence remained veiled and seemingly unseen, contributions typically treated divergent ideas as taken-as-shared, evoking little negotiation. Instead, emergence continued to involve replication and reiteration, creating the mirage that the meanings were consistent even though in certain ways, they were not. Therefore, negotiable artefacts created opportunity for
negotiation; and, this space for negotiation was made more available when less ideational convergence was involved. Perhaps such activity would surface given an appropriate provocation or context.

Negotiated artefacts, however, evidenced the moments when active negotiation did surface. These artefacts involved at least one source of explicit ideational divergence, which pushed towards negotiation, amidst other sources of convergence. The sources of convergence offered points of stability, but the sources of divergence attracted negotiation, which often resulted in emergence of new material, allowing the ideational network to adapt to changing circumstances. For instance, when participants needed to find ways to teach fractions when other ways failed them, the system was robust enough to provide them with a variety of ways to visualize fractions. Or, when a user noted they could not figure out how to solve a problem, the system responded with wide diversity of ways in which one could not only approach the problem, but also how to teach the problem without giving the answer away. So, the ideational divergence within relations, allowed for adaptability and emergence of novel material, as seen through negotiation during moments of necessary adaptation in response to various provocations. Collectively, this contributed to the ideational randomness available within the system.

Since moments of active negotiation involved not only sources of divergence but also convergence, evidently both were necessary. Ideational convergence infused the space with purpose, prominence, and familiarity of topics through the redundancies it involved, and ideational divergence brought enough diversity to breed negotiation. The diversity evidenced active negotiation in the directions of divergence that were made explicit. In turn, this increased the likelihood of emergence of novel material, an event that was generally rare. Active negotiation capable of generating novel content was therefore a product of a sort of 'perfect storm' between ideational divergence and convergence.

Given that ideational relations can morph between being active and inert, and living in reification or negotiation, this implies that ideational artefacts can also be transient like this since they are built from relations. That is, small perturbations such as individual comments or statements could cause a relation stemming from an artefact to shift from convergence to divergence, or vice versa. For instance, an artefact could be reified, but then random occurrences cause it to become more taken-as-shared, and therefore hold more inert reification around it, making it reifiable. Or, it may be reified but a strong
provocation from a user tips its convergence to divergence in a certain direction, pulling it into being negotiated. While this was not possible to observe in this data set, complex systems are by nature constantly evolving and responding to changing conditions. As such, there is a dynamic aspect to the possibilities for artefacts to live between reification and negotiation, as pictured in Figure 6.43 below.

Figure 6.43  Dynamic nature of possibilities for artefacts

6.4 Ideational emergence: Reification and negotiation

Overall, through a prominence of redundancy within ideational relations, reified and reifiable artefacts precipitated emergence through replication and reiteration. This involved forming and reinforcing taken-as-shared ideas, which became so prominent that they became sources of ideational coherence, as in, ideas that MTBoS could be identified with. These forms of emergence were also evident in artefacts that veiled their diversities, such as in negotiable artefacts. However, beyond a minimum amount of reification, which offers familiarity, emergence through negotiation was more available when explicit sources of ideational divergence were involved. This sometimes resulted in active negotiation, which created space for emergence of the as-yet unimagined, pushing the ideational boundaries in the system. Interestingly, the most prominent form of ideational activity in the ideational network of MTBoS was that of reification; negotiation was much rarer. Having entered the phenomenon with hopes of finding and examining negotiation, this was a surprising result. However, this points to the importance of both reification and negotiation for emergence within the MTBoS system.

Reification not only allows for emergence through replication of the taken-as-shared, it also makes room for ways in which participation can continue with little significant diversity. Perhaps the tendency of desires for ‘fitting in’ contribute to this draw towards conformity, or alignment, in ideational behaviour. In fact, the notion of an ‘echo chamber’ is often used to describe communities on Twitter (e.g., Barberá, Jost, Nagler, Tucker, &
Bonneau, 2015; Himelboim, McCreery, & Smith, 2013), which further supports the strong tendency towards reification evidenced here. Although reification on its own is devoid of negotiation, it seems to be a necessary aspect in negotiation. This is because no negotiation in the data set occurred without at least some source of reification, even though reification occurred without negotiation. This suggests that reification holds purpose in the processes involved in ideational emergence. By contributing to the formation of fundamental qualities that define the sources of ideational coherence in the collective, reification makes it identifiable. In other words, the collective gains an identity through the artefacts it reifies. While these identities can come about through past negotiations, it is the reified that gives the ideational space stability, and therefore an identity to come back to. As such, reification holds the power to create coherence, a sort of sameness that invites taken-as-shared language and ideational familiarity. It attracts redundant behaviour, and although this means it lacks a tendency towards emergence of novel ideas, it can serve as a gateway for entry. The strong redundancy that reification offers, regardless of whether it is actively or inertly acknowledged, allows contributors to identify ways in which to engage. In this way, it creates a baseline of ideational cohesion for the collective and defines its identity.

Negotiation, however, layers onto reification, creating opportunities for novelty. The stability offered by reification is necessary to make the instabilities offered of negotiation productive. In other words, without points of coherence, the randomness offered through negotiation has nothing to build from. For instance, when negotiation occurred around fraction modelling strategies, many of the notions around fractions had been reified. But the provocation around how to help an eighth grader understand fractions surfaced diversities among contributors who in turn engaged in negotiating their various approaches. If they had not shared redundancies around having experiences that led them to developing ideas for teaching fractions, these negotiations and the products that emerged out of them may not have arisen. Their redundancies needed a provocation to be mobilized, which allowed for their diversities to surface and become negotiated. The initial condition was a redundancy, something that was reified, which in this case was visualizing fraction operations. The provocation around it made these redundancies visible, and in turn, shed light on the diversities they could now see and interact with, resulting in negotiation. As such, negotiation requires reification. Reification gives the
system stability, but negotiation offers adaptation; both are necessary in the life of a complex system. And, negotiation creates opportunities for emergence.

However, the mere presence of both ideational divergence and convergence was not enough for emergence of novel material through negotiation. Some artefacts had both sources of ideational divergence and convergence and did not involve active negotiation. Therefore, other factors are evidently involved in emergence. Given the infrequency of active negotiation that was generative of new ideas and materials, further investigation into the factors involved in such instances of collective creativity is warranted. Namely, the social aspects necessarily involved in neighbour interactions have not yet been considered and are explored in the following chapter.
Chapter 7 The underlying social network

By underscoring the conceptual character of the agents in a knowledge-producing system, we do not mean to ignore or minimize the role of social interaction. The ideational network rides atop the social network. The point is that they should not be collapsed. (Davis & Sumara, 2006, p. 143)

While the ideational network developed in Chapter 6 represents a global view on the ideational neighbour interactions in MTBoS, such a global view ignores the social relations that support and drive these ideational neighbour interactions. Social relations form a dense underlying social network in MTBoS. While ideational content in tweets is important, tweets are ultimately made by people who have various redundancies and diversities and who presumably spend time reading the material publicized by those they follow. Because of the decentralized organization in MTBoS, there is no guarantee that anyone will see a tweet made by a particular user. As such, each user has a unique view into the ideational space of MTBoS depending on the social relations they choose to create and maintain through both following other users and being followed by others. Since following relationships need not be reciprocated, it is possible to follow others but not be followed back and vice versa. However, the likelihood of a tweet being seen and possibly interacted with increases depending on the number of followers a user has (as described in §5.1). Users typically choose who to follow based on personal interests as well as through random encounters with content (e.g., via retweeted content by those one follows, or through random searches). To be followed, a user needs to publicize content others find useful or interesting in some way.

As such, follower relationships, and therefore social locations, are constantly in flux, contributing to the alluring potential of having the power to shape one’s social location through who one follows and how one tweets. This creates a fascinating dynamic for neighbour interactions because ideational bumping becomes limited by the social relationships that determine what ideational material some users are privy to and what ideational material users choose to publicize (potentially on the grounds of their awareness of who their followers are). Since neighbour interactions are a necessary condition for emergence and social relations allow for neighbour interactions to occur,

68 I use the term ‘social location’ as a way to describe one’s placement in the social network as will be more specifically defined in §7.1.
the ways in which users in diverse social locations engage in contributing ideational material are essential to explore towards uncovering the driving forces behind emergence in the ideational network. My curiosity therefore subsequently turned to the social network after exploring the ideational network.

The initial grounds were rich. One of the fundamental systemic conditions necessary for complex emergence is that of diversity and redundancy among agents. Since the people who contribute ideational material in MTBoS live within their own contexts, they inherently have diversities from which they draw on towards their endeavours of contributing ideational material. In the case of the core data set used in this thesis, the 322 accounts (61% female, 32% male, and 7% groups or organizations) that contributed the 444 tweets used for analysis ranged widely in terms of their affiliations with mathematics teaching. About 64% of them were schoolteachers of mathematics (41% secondary, 13% middle, and 9% elementary), 19% were leaders or coaches for mathematics teachers, 5% were professors (mostly of mathematics education), 2% were pre-service teachers, 7% were organizations or groups, and 4% were either non-mathematical or unknown. They resided in a variety of locations around the world (the majority being from US and Canada, but also including those from Australia, India, France, Spain, Korea, and Pakistan) and ranged vastly in their time on Twitter (join dates between 2007 and 2018), in the number of followers they had (between 1 and 151537), and in the number of users they followed (between 9 and 11636). At a socio-cultural level, this implies there was ample space for legitimate peripheral participation as well as participation anywhere along the trajectory towards an elusive core member participation within MTBoS.

Amidst this potential space for diversity among agents, MTBoS fundamentally attracts agents who are primarily interested in mathematics teaching as evidenced by the predominant user occupation being involved in mathematics teaching, which serves as a unifying source of redundancy among those who participate in MTBoS. Other more specific redundancies can also be found among agents and likely contribute to some users wanting to follow certain other users with whom they share interests. For instance, if a user is interested solely in elementary mathematics teaching, it is likely they want to

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69 My interest in social relations here was not necessarily interpersonal in nature, but rather, systemic. In other words, my focus was primarily attuned to how the underlying social network contributed to the ideational possibilities of the collective, and therefore, to emergence.
follow others who share elementary-related interests. However, merely sharing redundancy around interests is not enough to engender a neighbour interaction\textsuperscript{70}, one must also be seen. Being visible is therefore the minimum requirement for a neighbour interaction to occur. As such, my primary interest in the social network pertained to investigating the possibilities for neighbour interactions given the landscape of diversities and redundancies in how users engaged and interacted in the ideational space. As such, my attention was drawn towards the diverse manners of engagement in the system by users according to their respective social locations, which evidently privileged their access to ideational neighbour interactions. By considering tweet visibility through follower relationships as an orienting attribute for social location, I pursued investigation of the nature of neighbour interactions in the space. Thus, I aimed to bridge the social and the ideational in a manner that was primarily concerned with the ideational system, rather than an interpersonal one. As such, my inquiry into the social network was driven by the key questions, “What social locations exist within this network?”, and “How do social locations drive ideational emergence?”

To this end, I first constructed a social network for follower relations based on the core data set. Once such a network was available, I used the prominent features of the network to organize tweet data and investigate the intertwined relationship between social location within the collective and ideational quality; and, therefore, the nature of neighbour interactions. Although this was done only for a small snapshot of data representing a slice of time in the life of MTBoS, it aimed to illuminate the processes involved in the social landscape that contribute to the ideational space. In what follows, I first present the construction of the social network as built on follower relations, and then reveal prominent features in this network that contributed to identifying social locations. Finally, I develop a conceptualization of how these social locations play a role in ideational emergence in the collective and what they mean for participation and neighbour interactions.

\textsuperscript{70} In the context of this thesis, neighbour interactions involve ideational bumping, whether it is visible or invisible. For example, in reading someone’s tweet, my ideas bump with theirs.
7.1 Building and examining the social network

With a primary interest of identifying the space of possible neighbour interactions within the core data set, a process for uncovering the social network behind the 444 core data set tweets made by the 322 core data set contributors was developed. Although such a global view is not typically available for users, it was constructed in an effort towards considering the social landscape as a whole. Rather than generating a network built from all the users that each of the 322 contributors followed or were followed by, the social network developed in this thesis was created as a closed set. This means the follower relations were only included if they were within the set of 322 contributors, and follower relations with users outside the data set were ignored.

This way, the social network revealed social visibility within the selected subset of MTBoS users without skewing it with information from outside the network. For instance, perhaps a user primarily tweets about sports and has many sports-related connections on Twitter, but tweets occasionally about mathematics teaching because they also teach math once a week. If all follower relationships were considered, such a user would be treated as a core participant rather than a more peripheral one because of the high number of relations they have. However, because they do not share many relations with others in MTBoS, by considering only relations within the data set, this user would be treated as having a more peripheral social location, which is a more accurate representation of their engagement in the MTBoS space. So, the primary interest in building a social network was to map out which contributors from the data set were able to see the tweets of which other contributors within the data set as opposed to which contributors were more visible globally within all of the Twitter space.

More specifically, the process of constructing the social network involved identifying a list of all users that each of the 322 contributors to the data set followed. This was done by using the Twitonomy application (Twitonomy, 2019) to generate and download a spreadsheet with this information for each user. A matching algorithm was then run to select only the users followed by each contributor who were also found in the original data set of contributors. A contributor relation list was then created that identified follower relations within the closed set of contributors. A total of 10458 following relations were found between the 322 contributors and were subsequently mapped as a network graph using the NodeXL Pro application. Metrics of degree centrality were used to place
user nodes on colour and sizing scales, and reciprocal relationships were coloured with black rather than grey. This graph is shown in Figure 7.1, and is evidently very dense.

![Figure 7.1: Social network from closed set of contributors](image)

**Figure 7.1** Social network from closed set of contributors

The overwhelming *density* of the underlying social network was indeed surprising because it was very well connected even though it was constructed only from users who made 30% of the unique tweets within one week of MTBoS activity. Considering *scale-free* attributes of complex systems, one can only imagine the density in the social relations within the larger collective of people who tweet using the MTBoS hashtag. The density seen in this social network reveals the possibility for a *community* to exist within the space, a community built on shared ideational neighbour interactions made available through overlapping followers. However, at this scale, it is unclear how a contributor’s social location can co-act with the nature of ideational neighbour interactions they make.

To pursue inquiry into the social network, dynamic regions of social location in which contributors could be identified within the data set were necessary to determine. Tweets made by users in these various social locations could then be examined for the redundancies and diversities within these regions to characterize the space of possibility for how social location may co-act with ideational activity and neighbour interactions. Since there are two dimensions to social following, the number of contributors one is followed by, and the number of contributors one follows, a scatterplot was used to represent each of the 322 contributors in the data set in terms of these values. So, contributors were mapped as datapoints on a scatterplot that related their in-degrees...
(number of contributors following them) and their out-degrees (number of contributors they were following). This scatterplot is shown in Figure 7.2.

![Scatterplot of contributor in-degrees and out-degrees](image)

**Figure 7.2** Scatterplot of contributor in-degrees and out-degrees

As is evident on the scatterplot above, the largest density of contributors existed within the lower ranges of both dimensions. There was also a tendency towards a linear regression. This is expected since it is typical for users to reciprocate followings unless they are vigilant about keeping their feed restricted in some way. What is more pertinent to the investigation, however, is to consider the diverse social locations available in the network and what they illuminate about neighbour interactions.

Assuming a contributor can have either a relatively low or high value in each of the two dimensions, this leaves four options for social location: low in-degree and low out-degree (LL), low in-degree and high out-degree (LH), high in-degree and low out-degree (HL), and high in-degree and high out-degree (HH). To establish what counts as ‘high’ or ‘low’ in each dimension, cut values were identified. To this end, distributions of each list of in- and out-degree values were examined. Their long-tail power law distributions (Figure 7.3 below) led to selecting the top 20% of values in each dimension to define high values.

![In-degrees in increasing order](image) ![Out-degrees in increasing order](image)

**Figure 7.3** Contributor social in-degree and out-degree distributions
This resulted in identifying contributors with high in-degree as those who had 56 or more followers within the set, and high out-degree as those who followed 49 or more contributors within the set. As such, three regions were demarcated to include contributors with significantly high values of in- or out-degrees, leaving a lower region that included contributors with significantly lower in- and out-degrees. Each of these four regions of social location included a collection of contributors along with a set of any tweets that they made (as shown in Figure 7.4 below).

Figure 7.4  Contributor in- and out-degrees divided into social locations

While each of the three higher regions were reasonable in size for analysis, the lower left region, which was very densely populated, required a random sampling to make the data set manageable for qualitative analysis. Rather than sampling from the entire 287 tweets made by those 230 contributors, a buffer region was introduced to reduce the sample space to include only tweets made by contributors who were closer to the densest part of the region and not near its boundary. To this end, a buffer region was created that included the top 20% of values under each degree boundary, leaving the lower region to include contributors with in-degrees less than 44 and out-degrees less than 39. This resulted in a sample space of 259 tweets (made by 209 contributors), of which 60 tweets (made by 57 contributors) were randomly selected. The aim of having a sample size of 60 was to have it be a similar size to the other denser regions. This sampling is revealed in Figure 7.5 below where ‘x’ marks indicate the 57 contributors in the random sample.
These sets of tweets served to represent the content produced within each region of social location. Although the buffer could evidently have been extended into the LH and the HL regions, this would have cut down the already small data sets to 22 contributors with 26 tweets in the LH region and 17 contributors with 47 tweets in the HL region. While this could be desirable given that those in a buffer region may involve combinations of features found at the extremes around them, it proved better in this case to maintain the density of the data sets rather than reduce them for this purpose. This was particularly because upon cursory investigation of the contents within each region, significant differences in engagement were evident between regions regardless of including or excluding the buffer regions. As such, in this research, the contributors located in each region serve to represent that region with the exception of those within the buffer region, and the tweets made by contributors as indicated in Figure 7.5 above serve to represent the engagements made by contributors in each region.

As such, these four regions created an organizing framework that allowed for an investigation of the redundancies and diversities among evident traits found within regions and across regions. To this end, there were three evident traits available to investigate: agent profiles (i.e., job titles), ideational content (i.e., prominent ideational artefacts in their contributions), and ways in which these agents contributed the ideational content. While the first two of these were explored, they did not on their own
yield evident insights. This is perhaps because personal contexts and ideational contents are not nearly as pertinent to neighbour interactions as the nature of how contributors choose to make their tweets. Personal contexts were broadly determined through agent profiles, which identified them as either math teachers (elementary, middle, or secondary), math teacher leaders, coaches, or professors, organizations or groups, or other. Table 7.1 below tabulates these results.

Table 7.1 Distribution of contributor job titles for each social location

<table>
<thead>
<tr>
<th>Job Title Category</th>
<th>LL Contributors (total 209)</th>
<th>LH Contributors (total 27)</th>
<th>HL Contributors (total 27)</th>
<th>HH Contributors (total 39)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math teachers elementary</td>
<td>10%</td>
<td>4%</td>
<td>15%</td>
<td>5%</td>
</tr>
<tr>
<td>Math teachers middle</td>
<td>13%</td>
<td>19%</td>
<td>7%</td>
<td>13%</td>
</tr>
<tr>
<td>Math teachers secondary</td>
<td>39%</td>
<td>37%</td>
<td>41%</td>
<td>38%</td>
</tr>
<tr>
<td>Math teacher leaders/ coaches/consultants/education professors</td>
<td>21%</td>
<td>37%</td>
<td>33%</td>
<td>33%</td>
</tr>
<tr>
<td>Organizations/groups around math teaching</td>
<td>7%</td>
<td>4%</td>
<td>4%</td>
<td>10%</td>
</tr>
<tr>
<td>Other/non-math</td>
<td>9%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Note: Percentages represent portion of accounts in each social location with the given job title category. And, the highest percentage across job title categories are bolded for reference.

Although there were some general tendencies, such as for those with lower out-degree to be schoolteachers and those with more out-degree to be leaders or organizations, these tendencies were not significant, and not justifiable by locations. All four regions included a mixture of job titles related to mathematics teaching, with a prominence of schoolteachers (particularly secondary math teachers) and math teacher leaders, as congruent with overall dataset averages. This view supports the overall diversity among agents, which is typically assumed within complex systems. As such, this view did not offer insight into the nature of ideational emergence in relation to the social network.

Further, ideational content was investigated in relation to the tweets made by contributors in each region of social location. Namely, the frequencies of ideational artefacts were considered within each region and compared across regions. Although nuanced differences existed, there was a high amount of overlap between regions. And, seemingly disparate regions involved similar artefacts. For instance, those in the lowest
region of followers had many overlapping prominent ideational artefacts with those in the highest region of followers. Lists of the top ten ideational artefacts by frequency of occurrence within each region is shown in Figure 7.6 below.

<table>
<thead>
<tr>
<th>Ideational Artefact</th>
<th>Frequency</th>
<th>Ideational Artefact</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>hands on learning</td>
<td>6</td>
<td>mtbos</td>
<td>16</td>
</tr>
<tr>
<td>visualizing math concepts</td>
<td>6</td>
<td>real life examples</td>
<td>8</td>
</tr>
<tr>
<td>multiplication</td>
<td>5</td>
<td>algebra</td>
<td>7</td>
</tr>
<tr>
<td>using manipulatives</td>
<td>5</td>
<td>geometry</td>
<td>7</td>
</tr>
<tr>
<td>engaging students</td>
<td>4</td>
<td>social justice</td>
<td>7</td>
</tr>
<tr>
<td>games and puzzles</td>
<td>4</td>
<td>visualizing math concepts</td>
<td>7</td>
</tr>
<tr>
<td>review activities</td>
<td>4</td>
<td>hands on learning</td>
<td>6</td>
</tr>
<tr>
<td>teacher life</td>
<td>4</td>
<td>noticing wonder</td>
<td>6</td>
</tr>
<tr>
<td>being a good teacher</td>
<td>3</td>
<td>engaging students</td>
<td>5</td>
</tr>
<tr>
<td>conceptual understanding</td>
<td>3</td>
<td>fun activities</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ideational Artefact</th>
<th>Frequency</th>
<th>Ideational Artefact</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>engaging students</td>
<td>13</td>
<td>real life examples</td>
<td>11</td>
</tr>
<tr>
<td>fun activities</td>
<td>11</td>
<td>engaging students</td>
<td>9</td>
</tr>
<tr>
<td>using technology</td>
<td>11</td>
<td>mtbos</td>
<td>9</td>
</tr>
<tr>
<td>geometry</td>
<td>10</td>
<td>fun activities</td>
<td>8</td>
</tr>
<tr>
<td>algebra</td>
<td>6</td>
<td>geometry</td>
<td>8</td>
</tr>
<tr>
<td>conceptual understanding</td>
<td>6</td>
<td>mashupmath</td>
<td>8</td>
</tr>
<tr>
<td>activities</td>
<td>5</td>
<td>activities</td>
<td>6</td>
</tr>
<tr>
<td>hands on learning</td>
<td>5</td>
<td>creating new activity resources</td>
<td>6</td>
</tr>
<tr>
<td>problem solving</td>
<td>5</td>
<td>games and puzzles</td>
<td>6</td>
</tr>
<tr>
<td>assessment</td>
<td>4</td>
<td>using technology</td>
<td>6</td>
</tr>
</tbody>
</table>

Figure 7.6  Top ten ideational artefacts by region of social location

As may be observed in Figure 7.6 above, each region seems to contribute very similar ideational artefacts. While some artefacts were more prominent in certain regions, the artefacts themselves did not indicate how social location played a role in ideational emergence. For instance, by noticing that 'real-life examples' were more prominent in more highly followed contributors than in less highly followed contributors, and that 'hands-on learning' was more prominent in less highly followed contributors than in those more highly followed, it did not illuminate how they differed. Looking only at the
ideational artefacts publicized by contributors in various social locations also did not involve the social nature of their contributions.

Therefore, my central focus of analysis turned to exploring how the ideational content found in the tweets made by contributors in each region of social location was contributed in terms of the *forms and modes of engagement*\(^7\) used in publicizing the content\(^2\). Towards this end, an open coding process was undertaken within the tweets included in each of the four regions (with the random sampling in the lower left region as described earlier). The process involved iterative coding with attention to redundancies and diversities within and across regions, and was guided by the underlying question, “How is the ideational content being delivered by this contributor in this tweet?” This allowed for attending to the forms and modes of engagement.

Coding began with the tweets made in the high in-degree and high out-degree region because it presumably offered all possibilities that could be available in other regions. New codes were generated when necessary until a saturation point was reached in that every tweet fit into at least one coding. Overlap was permitted, and some tweets were coded with several codes if the tweet revealed multiple forms of engagement. The coding was re-checked with a second pass to make sure no new codes could be seen other than those that had emerged and that every tweet fit with the codes it was tagged with. This set of codes for forms of engagement was then used as an a priori framework to code tweets in other regions, continuing with iterative coding for redundancy and diversity through tweet clusters in all other categories. Once no new codes were found and each tweet in each of the categories was tagged with at least one of the codes, a final set of codes was determined. These served as a space of possibility for forms of engagement in MTBoS in general since they emerged from coding more than half the tweets in the dataset with saturation. To explore how these forms of engagement compared across regions of social location, the proportions of each code within each region were identified and compared as percentages across regions. In doing this, significant differences became evident and the prominent attributes identified for each region of social location were used to characterize and infer the modes of engagement.

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\(^{71}\) I draw on Remillard’s (2012) distinction between *forms and modes of address*: *forms of address* involve “particular ‘looks’ or formats that reflect and reinforce the mode of address” (p. 106).

\(^{72}\) Although tweet content message categories have been more globally developed (e.g., Naaman et al., 2010), they have not been specific to activity in the MTBoS.
predominantly involved in social locations. In what follows, the emergent forms of engagement are first briefly described. Then, each region of social location is presented in terms of the prominent forms of engagement found in the tweets made by their contributors. From the characterizations of each social location, modes of engagement available in MTBoS are then identified. Finally, more global implications are drawn together to conceptualize how social locations intertwine with modes of engagement to support the neighbour interactions that drive ideational activity in MTBoS.

7.2 Identifying forms of engagement

The set of codes indicating possibilities for forms of engagement emerged from the data evidenced saturation across all 217 examined tweets. These eight forms of engagement revealed ways in which tweets were made: ‘soliciting advice or resources’, ‘contributing resource of teaching idea’, ‘revealing practice’, sharing accomplishments’, ‘endorsing’, ‘signaling identity’, advocating an opinion or stance’, and ‘building community’. Each of these are briefly described and exemplified in what follows, in no particular order.

1. Soliciting advice or resources

While MTBoS is often considered a place to solicit advice and to ask about resources for mathematics teaching, only about 18% of the 217 tweets examined evidenced explicit requests for advice or resources. This points to how directly asking for resources is not a dominant form of engagement, but rather, is one option available to participants. Queries ranged from specific questions about content (e.g., Figure 7.7a), requests for specific activities for given topics (e.g., Figure 7.7c), seeking advice around classroom practice (e.g., Figure 7.7d), and even asking others to help locate past resources that were shared in MTBoS (e.g., Figure 7.7b).
Figure 7.7  Examples of soliciting advice or resources

Although there was diversity in what was being asked about, the redundancy among requests for help involved a general sense of urgency and utility. Contributors seemed to be seeking resources they could use for specific purposes. While it did occur, it was not common to see queries about broader meta-level questions that indicated pondering about teaching practices and approaches towards certain mathematical topics.

2. Contributing resources or teaching ideas

Although some contributors asked directly for advice or resources, many instead chose to contribute resources or teaching ideas. In fact, 41% of the 217 of the tweets explored involved some sort of sharing of resources, which was the most highly prominent form of engagement. Many of these tweets were also overlapping with other forms of engagement such as with revealing practice, sharing accomplishments, and signaling identity while sharing a resource or teaching idea. For example, a resource could be shared on its own as in the example in Figure 7.8a, or it could be shared in context of teaching as in the example in Figure 7.8b.

Figure 7.8  Examples of contributing resources or teaching ideas

Either way, whether resources or teaching ideas were explicitly revealed with lots of supporting information, or less explicitly with mere pointers to what the idea entailed, the redundancy was that they involved resource sharing.
3. Revealing practice

Details from practice evidencing implementation of teaching ideas were also at times exposed within tweets or in the blog links included. Whether the details disclosed were comprehensive enough to make the contribution useful for others as a resource for implementation or not, 24% of the 217 tweets explored revealed some specifics from practice. Sometimes these share-outs were specific and detailed (e.g., Figure 7.9a) while other times they were more vague (e.g., Figure 7.9b).

This begs the question of who these tweets were written for since details help others be able to use the idea, while vagueness is a kind of self-report that leaves the intent of the post unclear. And, revealing details from practice can engender trust in the content posted because it gives the reader the ability to see how a task was carried out and not only what the task was (Larsen & Parrish, in press). However, in some cases, revealing practice seemed to be a case of meforming (Naaman et al., 2010), or self-promotion, which is generally very prominent in Twitter activity. Nonetheless, revealing practice was a prominent form of engagement overall in this data set, which supports the commonly attributed identity of MTBoS as a space for tweeting about mathematics teaching.

4. Sharing accomplishments

About 17% of the 217 tweets involved some indication of pride and excitement in the language used in reference to something that was achieved. Most often, these involved successes experienced with student learning (e.g., Figure 7.10a), but also involved personal and professional accomplishments (e.g., Figure 7.10b).
A S was struggling with the concept of independence in probability because all the examples with cards were independent. We decided to find some events in a deck of cards that were NOT independent. We came up with “being red” and “having a face with two eyes”. #statschat

Huge congrats to our friend and colleague @olopez8MS for winning Teacher of the Year for 2018. #relentless #burtonstory #mathconceptions #mtbos It is both an honor and a pleasure, and we're so proud!

Figure 7.10  Examples of sharing accomplishments

Sharing accomplishments seemed to offer a way for contributors to make their successes known and to engage in the space in a way that would reveal their capacity for accomplishment, and therefore support their status as being valuable.

5. Endorsing

Not only did tweets involve sharing personal accomplishments, they also involved endorsement. In 23% of the 217 tweets, some form of endorsement of either a resource or another user was made by sharing forms of support for that which was being endorsed. Quite often, endorsements involved not only a resource, but also acknowledgement of the user who shared or produced the resource (e.g., Figure 7.11).

Figure 7.11  Examples of endorsement

Although endorsement occurred in all regions of social location, endorsement was slightly more prominent in terms of frequency of occurrence among the higher in-degree followers and more influential since they were consequently seen by more followers.

6. Signaling identity

In some cases, there was a sense that a contributor was trying to signal an identity by posting something evidently valued within the community, such as via revealing practice, contributing a resource or idea, or advocating an opinion or stance. In turn, they were
indicating who they were in relation to these values. While this may not have been intentional, in my view as an insider perceiving the tweet, it seemed as though the contributor was not tweeting content just for themselves or for those they think they can help by offering resources, but rather, for revealing that they know the prominent ideas in MTBoS and can capacity to invoke them. For instance, in Figure 7.12, the contributor revealed that they explicitly valued growth mindset culture in their classroom by showing the sign they hung in their classroom. This was an honest statement and could have come from a place of excitement, but also communicates their values, which align with the prominent #growthmindset movement in the ideational space of MTBoS.

Figure 7.12  Example of signaling identity

Signaling identity was sometimes also more nuanced in that the way a contributor formed a tweet indicated an implied awareness of insider trends. For instance, by making a #noticewonder tweet that included a real-life example (e.g., Figure 6.19a), which is found in the ideational network to be a strong source of ideational coherence. Signaling identity was also not always directly linked with revealing practice. At times, it was done through indicating one is attending or leading a particular event that seems important and ideationally aligned with the collective, promoting oneself by revealing that one plays with certain puzzles at home, asking about specific resources that are common in the collective, and name dropping ideas or users commonly addressed in the collective. Signaling identity, therefore, allowed for ideational alignment with sources of ideational coherence.

7. Advocating an opinion or stance

Further, some tweets firmly indicated an opinion or stance in relation to issues around teaching mathematics. That is, they did not necessarily share a resource or discuss how a lesson went, but rather, wrote more critically and more metacognitively in reflection about the issues. 16% of the 217 tweets involved this sort of activity. Some came across as more heated, while others were tamer and more indirect.
Infuriated—giving a departmentally mandated benchmark the 3rd week of Alg2 which includes logarithms, imaginary numbers, and trigonometric functions in quadrants other than the first. Waste of time, demoralizing for students. Have to wonder who this serves. #mtbos

I can just enjoy the entirety of their work and feel like I’m conversing with them. Yes, I’m writing more, but I feel my grading means more now to my students. A refreshing change I needed. Proud to be their teacher this year! #MTBoS

8. Building community

Finally, some tweets were clearly aimed at building community online within MTBoS. 16% of the 217 tweets involved this sort of activity. This included aims to involve users in online initiatives (e.g., Figure 7.13a), physical MTBoS meet-ups (Figure 7.13b), and in relation to sharing about MTBoS to non-users (Figure 7.13c).

The explicit efforts at building community were interesting in that they did not pertain to mathematics teaching per se, but rather, served as a meta-level form of engagement in which awareness of a community was apparent.

7.3 Exploring forms of engagement in social locations

Each of these eight forms of engagement existed within each of the four regions of social location, but to varying degrees. As Remillard’s (2012) phrasing around forms and modes suggests, forms of engagement contribute to modes of engagement. As such, by considering the various manners in which forms of engagement dominated each social location, characterizations of the prevailing modes of engagement elicited by contributors in each social location were established. Notably, while these characterizations necessarily gloss over variances within each region by focusing primarily on common features, these characterizations point to trends within each region.
One way to compare varying levels of forms of engagement across social locations was to consider the proportions of tweets within each social location that involved each form of engagement, as presented in Table 7.2 earlier. A summary of these eight forms of engagement and their frequencies of occurrence in the data are shown in Table 7.2.

**Table 7.2  Occurrence of forms of engagement by social location**

<table>
<thead>
<tr>
<th>Form of Engagement</th>
<th>LOW in-deg.</th>
<th>LOW out-deg.</th>
<th>HIGH in-deg.</th>
<th>HIGH out-deg.</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>soliciting advice or resources</td>
<td>11/60 (18%)</td>
<td>8/31 (26%)</td>
<td>12/60 (20%)</td>
<td>9/66 (14%)</td>
<td>40/217 (18%)</td>
</tr>
<tr>
<td>contributing resource or teaching idea</td>
<td>24/60 (40%)</td>
<td>3/31 (10%)</td>
<td>32/60 (53%)</td>
<td>29/66 (44%)</td>
<td>88/217 (41%)</td>
</tr>
<tr>
<td>revealing practice</td>
<td>19/60 (32%)</td>
<td>15/31 (48%)</td>
<td>4/60 (7%)</td>
<td>14/66 (21%)</td>
<td>52/217 (24%)</td>
</tr>
<tr>
<td>sharing accomplishments</td>
<td>16/60 (27%)</td>
<td>11/31 (35%)</td>
<td>5/60 (8%)</td>
<td>5/66 (8%)</td>
<td>37/217 (17%)</td>
</tr>
<tr>
<td>endorsing</td>
<td>10/60 (17%)</td>
<td>8/31 (26%)</td>
<td>8/60 (13%)</td>
<td>23/66 (35%)</td>
<td>49/217 (23%)</td>
</tr>
<tr>
<td>signaling identity</td>
<td>16/60 (27%)</td>
<td>13/31 (42%)</td>
<td>7/60 (12%)</td>
<td>23/66 (35%)</td>
<td>59/217 (27%)</td>
</tr>
<tr>
<td>advocating an opinion or stance</td>
<td>7/60 (12%)</td>
<td>2/31 (6%)</td>
<td>10/60 (17%)</td>
<td>15/66 (23%)</td>
<td>34/217 (16%)</td>
</tr>
<tr>
<td>building community</td>
<td>3/60 (5%)</td>
<td>3/31 (10%)</td>
<td>12/60 (20%)</td>
<td>17/66 (26%)</td>
<td>35/217 (16%)</td>
</tr>
</tbody>
</table>

NOTE: Some tweets involved multiple forms of engagement, hence the percentages sum to more than 100%, but all tweets were found to evidence at least one of these forms of engagement.

Another way to compare these levels was to consider the proportions of tweets coded with each form of engagement that originated from each social location (in Figure 7.14).
This view shows the percent distributions of each of the four regions of social location among tweets coded with each of the eight forms of engagement. Percentages indicate the proportions of tweets contributors from each social location made for that form of engagement out of all instances of that form of engagement. In this way, it shows the forms of engagement each of the social locations were most prominent in. Evidently, each form of engagement involved contributors from some social locations more than others. By taking the top social locations that prevailed in each of the forms of engagement, it became possible to see how some social locations dominated certain forms of engagement more than others. For instance, the engagement through building community was primarily contributed to by those with high in-degree and high out-degree. To this end, each form of engagement was identified as having a primary contributing social location and a secondary one based on Figure 7.14 above. Taken together with the information provided in Table 7.2 earlier to cross-check prominence, Table 7.3 below summarizes the forms of engagement identified as most important within each social location, by primarily drawing on information in Figure 7.14.

Table 7.3  Summary of forms of engagement prominent in each region of social location

<table>
<thead>
<tr>
<th>Regions of social location</th>
<th>Prominent forms of engagement by contributors in each social location</th>
</tr>
</thead>
</table>
| LOW in-degree/LOW out-degree (LL) | Sharing accomplishments 34% *(secondary)*  
Revealing practice 29% *(secondary)*  
*Note: There were no primary forms of engagement in this category because as shown in Figure 7.14, none of the forms of engagement had this social location as primary.* |
| LOW in-degree/HIGH out-degree (LH) | Revealing practice 45%*  
Sharing accomplishments 45%  
Signaling identity 36%*  
Soliciting advice or resources 33%  
Endorsing 28% *(secondary)* |
| HIGH in-degree/LOW out-degree (HL) | Contributing resources or teaching ideas 36%*  
Building community 33% *(secondary)*  
Advocating an opinion or stance 29% *(secondary)*  
Soliciting advice or resources 26% *(secondary)* |
| HIGH in-degree/HIGH out-degree (HH) | Building community 43%  
Advocating an opinion or stance 40%  
Endorsing 38%*  
Contributing resources or teaching ideas 30% *(secondary)* |

Notes: Percentages represent the number of tweets coded with the respective forms of engagement that came from contributors in the given regions of social location. Greyed out *(secondary)* forms were secondary in proportion, but still significant. Lists are in order of decreasing prominence to reveal the most influential factors first. * refers to also being one of the top two prominent attributes within the social location.
The above summary reveals that each social location involved certain forms of engagement more than others both across forms of engagement and within social locations themselves. While various forms of engagement were evidently available across regions of social location within the 217 tweets explored, there were clear patterns of prominence that contributed to characterizing modes of engagement taken on within neighbour interactions by contributors from the four social locations. In what follows, these characterizations are revealed and exemplified for each social location.

**Low in-degree and low out-degree (LL)**

Contributors who had relatively few followers and who were not following many others within the data set had quite a bit of diversity in their forms of engagement. This may be attributed to the nature of their social location as being not prominently visible and not very well-connected with others in the data set. Although this social location represented the more peripheral positions in the collective, it was incredibly dense with 259 tweets made by 209 LL contributors who existed under the boundaries formed by maximum cut values of 44 followers and 39 following within the set. Because of the density of this region, 60 tweets from the 259 were randomly selected for analysis. Within these 60 tweets, 40% involved contributing a resource or teaching idea, 32% involved revealing practice, and 27% involved sharing accomplishments with 27% also involving signaling identity (as shown in Table 7.2). In relation to the entire data set, these contributors most significantly engaged by sharing accomplishments, revealing practice, and contributing resources or teaching ideas. However, these contributions were generally very diverse and not prominently related to sources of ideational coherence in the collective. In a sense, the contributions in this region were a melting pot of ideas. When taken as a whole, the ideas seemed disparate except for the foundational redundancy of being practice-oriented and related to mathematics teaching.

While many of the tweets made by contributors with low in-degree and low out-degree involved revealing practice, few of them solely revealed practice. Tweets revealing practice commonly involved sharing practice-oriented accomplishments, which were shared in different ways. For instance, through their excitement about improvements in student abilities (e.g., Figure 7.15a), through touting their students’ desires to work on math (e.g., Figure 7.15b), or through sharing their excitement around engaging activities they designed and implemented (e.g., Figure 7.15c).
Tweets in which contributors revealed practice also commonly involved contributing a resource. By offering details about practice, resources or teaching ideas were often naturally included. For instance, both tweets about practice in Figure 7.16 include a resource that others could also use.

The example in Figure 7.16a is slightly more explicit about the resource than in Figure 7.16b. However, both refer to resources that can be Google searched, found, and implemented. While some tweets offered more detail about practice than others, there was a lot of diversity around the kinds of practices and resources revealed, few of which pertained directly to prominent sources of ideational coherence in the collective.

LL contributors also often engaged in sharing accomplishments, and when they did, the accomplishments shared were most significantly oriented around revealing practice (e.g., Figure 7.15 above), and sometimes also contributing a resource or teaching idea (e.g., Figure 7.16 above). However, some accomplishments shared were not revealing of practice. These involved celebrating the accomplishments of other teachers (e.g., Figure 7.17a below); or signaling identity such as around revealing one’s ‘mathiness’ (e.g., Figure 7.17b below).
In these ways, contributors revealed their pride and excitement for things they had achieved and used the MTBoS space as a platform to show those off. It is noteworthy that most of the tweets that evidenced sharing accomplishments in this LL group were celebrating personal successes, primarily around practice.

While many of the tweets in which contributors in this more peripheral social location involved contributing resources or teaching ideas through revealing practice (exemplified earlier), tweets were also made that solely contributed a resource or teaching idea without situating it in practice. Within these tweets that contributed resources or teaching ideas, there was a prominence of pragmatism, particularly towards specific mathematical topics. Take for example the tweets shown in Figure 7.18 below.

In fact, most of the resource contributions by those in this LL social location often involved a specific mathematical topic and some indication of how one could teach that topic. These contributions were sometimes aligned with sources of ideational coherence in the collective, and other times were not. For instance, the concepts of zero or
imaginary roots were not central in the ideational network albeit relevant to mathematics teachers, while the geometry concepts, particularly around perimeter, were more central.

Some resource contributions in this LL social location also seemed to signal an identity. For instance, by sharing a Jo Boaler quote as a poster to hang in the classroom (Figure 7.19a), the contributor was asserting their identity as a progressive and caring educator. And, some contributions also served as endorsements. For instance, in revealing how well Kahoot helps in student success (Figure 7.19b), it endorses the tool.

Interestingly, very few of the resource contributions within this LL group advocated any sort of polarized opinion or stance. Rather, the focus was primarily pragmatic in nature.

In summary, the tweets made by those with low in-degree and low out-degree were primarily practice-oriented and focused on pragmatic approaches to teaching mathematics. Their contributions were generally focused on their own practices and were very ideationally diverse, often misaligned with sources of ideational coherence of the collective. As such, the advantages this group of contributors offered the collective was their capacity for ideational diversity. That is, they held an inert capacity for generating novel material that could help the collective adapt in resilient manners to shifting contexts, but their key barrier was visibility. They were not well seen by others in the collective and as such, their contributions were not feeding into the ideational space due to this social limitation that prevented neighbour interactions. Also, since they were not seeing many of the tweets made by others, they lacked the redundancy that allows for moving beyond that of simply sharing interests around mathematics teaching and into invoking sources of ideational coherence in the collective. It is also likely their tweets might continue to be misaligned with sources of ideational coherence, in turn making them less visible both because of lacking ideational alignment and social visibility.
Low in-degree and high out-degree (LH)

Some contributors, however, were privier to the content publicized by others in the data set because they followed a significant number of others in the data set, evidently taking interest in their endeavours. Their incoming follower counts within the group, however, remained low; perhaps they hadn’t yet been seen by others in the collective. However, they generally seemed to invoke topics and ideas that were relatively aligned with the sources of ideational coherence in the collective and acted in ways that seemed to aim towards provoking attention towards their contributions. Out of the 31 total tweets, these 27 LH contributors most prominently revealed practice (48% of their tweets), signaled identity (42% of their tweets), and shared accomplishments (35% of their tweets). They were also found soliciting advice or resources (26% of their tweets) and endorsing (26% of their tweets). More broadly across all groups, they significantly revealed practice, shared accomplishments, signaled identity, and solicited advice or resources. While revealing practice and sharing accomplishments were also prominent forms of engagement in the tweets made by LL contributors, the nature of these contributions differed slightly. In particular, LH contributors seemed to align more closely with sources of ideational coherence in the collective and generally came across as ‘attention-grabbing’ in their engagements especially when sharing accomplishments, signaling identity, and soliciting advice. However, their focus remained primarily drawn towards personal gains and interests, similar to those in the more peripheral LL social location.

As noted, revealing practice for this group of LH contributors was primarily oriented around sharing accomplishments. However, their tweets were generally less useful than the practice-oriented tweets in the group of LL contributors since they generally did not offer as much detail about the practices to be useful as a resource or teaching idea. Rather, revealing practice often came across as a way to celebrate personal successes instead of sharing for the utility of others. In other words, pragmatism seemed less of a concern. For example, in Figure 7.20a, the contributor touted their success with a parent, and in Figure 7.20b, the contributor indicated how their student continued to work on their math work while waiting in a storm shelter.
A concerned parent emailed me tonight, and far from the usual script she asked how we could ply our “parent/teacher partnership” towards the success of her student.

Swoon.

Student relationships extend past the student.

#teachers #mtbos #kmathed #ITeachMath

We had an actual TORNADO WARNING during school today (first ever in my 20 years of teaching) and while all 1600 students were waiting out the storm in shelter, I see a student of mine working on her math ❤️❤️ #mtbos

**Figure 7.20  Examples of revealing practice through personal successes**

These examples both reveal aspects of practice that indicate the teachers’ accomplishments as mathematics teachers, but do not offer enough detail about how such practice-related accomplishments were achieved. The vagueness around implementation details is also evident in the examples in Figure 7.21 a and b.

**Figure 7.21  Examples of revealing practice with vagueness of implementation**

Both of these examples reveal contributors who shared about practice in alignment with sources of ideational coherence in the collective and who signaled identities as being teachers who use engaging activities for their students. However, rather than detailing what the activity was, how it was designed, and how it helped students learn a concept, the focus remained on publicizing evidence and celebrating the implementation of these non-traditional activities in the classroom. In a way, this signals competence and value around using hands-on learning and engaging activities, both of which are central in the ideational network. This was the case for most of the tweets that involved revealing practice for this group of LH contributors, and points to their characteristic of being focused on signalling personal capabilities rather than being useful for others.

Sharing accomplishments was also a prominent focus for contributors in this LH social location and supported their emergent characteristics of contributors who wanted to be more visible. By sharing accomplishments, they could advertise their capabilities and in
turn, hopefully get noticed. Like those with fewer outgoing following relations, sharing accomplishments almost always occurred through revealing practice, but often occurred with efforts at signaling identity. For instance, the tweet in Figure 7.22a was made with excitement about implementing an engaging review activity while publicizing evidence that they have the capacity to engage students. And in Figure 7.22b, the contributor shared pride for creating a poster that advocated for accessibility to STEM for everyone, while at the same time signaling an identity as someone who values the inclusion of social justice within contexts of mathematics and science. Both examples revealed legitimate ways to share excitement and pride while signaling their identity as aligning with sources of ideational coherence in the collective. The neighbour interactions here seemed to hold not only ideational value, but also social value.

Whether it was around practice or not, sharing accomplishments was a very prominent form of engagement among those in this LH social location and typically involved topics that were more central to the ideational network than those shared by the LL contributors who were not following as many other contributors in the set. Perhaps indicating their capabilities was a manner of trying to get accepted by the collective as insiders, or possibly was merely a natural byproduct of following more users in the set of contributors and in turn feeling more connected to those they followed.

While signaling identity was often manifested through sharing accomplishments around practice (as exemplified above), signaling identity sometimes occurred without revealing practice. Instead, signaling identity also occurred through contributing a resource (e.g., Figure 7.23a), soliciting advice (e.g., Figure 7.23b) or endorsement (e.g., Figure 7.23c).
Endorsement was a relatively prominent secondary way identity could be signaled without revealing practice overall. Through endorsing another user or resource, the contributor could project their familiarity with the ideational coherence of the collective, and therefore signal their identity as a MTBoSer. Again, the intent of such signaling could have been a result or a cause of their social location.

It is also noteworthy that this group of LH contributors were also more prominent than other groups in terms of soliciting advice or resources. Although soliciting advice was found in all categories of contributors, this group seemed to have the most faith that they could ask questions in MTBoS, and that they would get a response. For instance, one contributor asked for suggestions about how to involve dance movements into Algebra 1 topics (Figure 7.24a). Another revealed their vulnerability to publicize that they forgot an approach they had seen for measuring out perfect fractions (Figure 7.24c). And, another indicated they were struggling to help students understand variables as possibilities rather than numbers (Figure 7.24b).
What these examples have in common is that they each reveal contributors who were willing to be vulnerable enough to ask questions indicating their challenges with things within their professions without seeming to be concerned about being viewed as incompetent. This sense of vulnerability in asking questions was not evident in those who did not follow as many others in the set of contributors and may be indicative of how following others in MTBoS, even without being followed back much, can instill a sense of comfort in being vulnerable within the space. It may also reveal the attention-provoking nature of those in this social location where asking questions is an alternative form of attracting attention similarly to that of sharing accomplishments. Asking questions is a direct form of looking for connection, and since these contributors had fewer connections, asking questions may have been one way to seek out connection.

In summary, the tweets made by those LH contributors with low in-degree and high out-degree continued to be mostly practice-oriented like their lower out-degree counterparts. However, this group had a slightly more prominent tendency towards revealing practice to share personal accomplishments and signal an identity of being aligned with sources of ideational coherence in the collective rather than revealing practice and resources merely for pragmatism and utility. That is, fewer details were included in practice-oriented tweets, but more excitement about successes were evident.

This group also came across as more comfortable with revealing struggles with specific issues within teaching mathematics and asking such questions publicly. While asking such questions could also be considered a way to attract attention similarly to that of
sharing accomplishments, it points to this group’s willingness to be vulnerable and evidences their faith in the collective as a place of non-judgmental support. As such, the advantage this group of LH contributors offered was their capacity to invoke sources of ideational coherence to make connections with others through asking questions or sharing accomplishments in practice. They evidently shared redundancy with many other contributors in the collective and had the capacity to engage in meaningful discussions. However, their key barrier was social visibility. Since they were not seen by others in the collective as prominently as they may have liked to be, their contributions were not feeding into the ideational space of possibility and were more likely to not be responded to. As such, they could either gain more followers through their tenacity and perseverance in tweeting content that was aligned with the collective’s ideational coherence, or they could choose to disengage if they did not receive the responses they seemed to be actively seeking. Nonetheless, this group had much to offer the collective since they were focused on personal practices and seemed willing to try ideas in their classrooms. They just needed more socially visibility to reap the benefits of the collective and to contribute their practice-oriented expertise to the collective.

**High in-degree and low out-degree (HL)**

In contrast to those who did not have many followers within the collective, those who had more followers tended to evidence more activity around serving others in the collective rather than remaining primarily oriented around their own needs. While this could be an effect of having more followers, it could also be a reason for why they acquired more followers. Some contributors with high numbers of followers within the set, however, did not follow many others within the set back. Perhaps they simply did not keep up with following others back when they were followed, or they wanted to keep their follower lists small and manageable. Nonetheless, contributing resources or teaching ideas was their primary form of engagement, occurring in 53% of their tweets, which was higher than any of their other forms of engagement. It was also the only form of engagement in which this group was most prominent in overall among all groups. Unlike resource contributions in other social locations, the resources or teaching ideas contributed by this HL group were delivered primarily without revealing practice or sharing practice-related accomplishments, but rather, as ready-to-try activities and lesson plans. These HL contributors also engaged in building community (20% of their tweets), soliciting advice of resources (20% of their tweets), and advocating an opinion or stance (17% of their
tweets); and, they served as secondary contributors in tweets related to these three forms in the overall data. While this was achieved in various ways, the HL contributors primarily seemed oriented around feeding their audiences novel materials and using their social visibility to stimulate engagement among their followers.

As noted, this well-followed group of HL contributors engaged primarily through contributing resources or teaching ideas. However, unlike the group with lower numbers of followers and followings, they did not do so through revealing practice. Rather, the resources and teaching ideas contributed were mostly ready-made resources intentionally put together to be used by others, very rarely did they involve any of the other forms of engagement in their presentation. For instance, in Figure 7.25a, a lesson plan for using a popular real-world context to teach dividing decimals was provided and included downloadable files. And, sometimes such contributions were even more polished with a marketed flavor such as in Figure 7.25b, where the tweet included a link to a webpage with five complete ready-to-implement scientific notation activities.

![Figure 7.25  Examples of polished and complete resource contributions](image)

Many of these resources seemed self-generated or self-curated, and although a few were endorsements of other resources, most contributions by this group were in some way original. For instance, in Figure 7.26a, the contributor designed a ‘broken calculator’ mobile application that could be used for students to try to create a target value with only certain numbers and operations under the guise of a ‘broken calculator’. Not only did the contributor create and provide this application, they also elicited feedback about their creation. So, not only were they comfortable asking, they had created something and then asked for feedback. Another example of this is seen in Figure 7.26b, where the contributor wrote about their new theoretical framing of mathematical modeling through a
metaphor of spies and analysts. The contributor created and published the resource, soliciting advice and feedback all while advocating their opinions about modeling.

Evidently, this group engaged deeply in producing and sharing resources that others could use. However, they also capitalized on their social location by advocating their opinions and sometimes engaging in community building by making attempts at involving others in discussion about the resource or teaching idea they shared.

Community building attempts occurred relatively prominently among this HL group. This may have had to do with their awareness and responsibility towards their respective audiences. Aside from building community through soliciting feedback on resources they created and shared as exemplified in Figure 7.26 above, some HL contributors also used their social locations to advertise their own community initiatives (Figure 7.27a), advertise and endorse community initiatives of others (Figure 7.27b), and publicizing statements of gratitude for community contributions (Figure 7.27c).
In Figure 7.27a, the contributor initiated a community building resource where they wanted to solicit the engagement of other mathematics teachers. Interestingly, this was not a ground-up initiative because MashupMath is a resource company that sells resources and is sponsored by various partners (MashupMath, 2019). While the company is founded by a mathematics teacher and offers free resources that are teacher-created, it is ultimately a business rather than an individual or a group of individuals who volunteer time to create resources. Consequently, it was evident in their tweets and in the replies to their tweets that they were not operating on social relationships with others, but rather, through their capacity to offer free resources. This may be why they were well followed. In contrast, it is evident that the examples in Figure 7.27b and c were more socially connected to those they were endorsing and advertising; in Figure 7.27b, the contributor announced a special guest they were excited about who would lead the chat that week, and in Figure 7.27c, the contributor expressed gratitude for those who engaged in their chat. What all three examples have in common, though, is their attention to bringing others into the space either through resource exchange, information about a chat, or acknowledgement of individuals’ contributions.

While the HL contributors generally involved in leadership-oriented engagement, they tended to solicit advice or resources more than some of the other groups, as partially seen within their attempts at community building. However, unlike the group with low in-degree follower counts and high out-degree counts, these contributors seemed to solicit advice primarily for the purpose of mobilizing ideas that everyone can benefit from rather than towards satisfying personal needs. Just like their efforts at contributing resources...
for use by others, their questions conveyed a general interest in creating space for discussion rather than seeking a single answer. This occurred through provocative questions that invited discussion (Figure 7.28a), requests for feedback about metacognitive thoughts about mathematics teaching (as seen in Figure 7.26b), and by redirecting questions from other users to the broader community (Figure 7.28b).

![Figure 7.28](image)

(a) Examples of soliciting advice as creating space for discussion

In these ways, the HL contributors were using the privileges of their social location to amplify questions that did not necessarily invoke a single answer, but rather, invited conversation and negotiation. So, although their tweets were at times misaligned with the collective’s sources of ideational coherence due to their lack of awareness of those actively participating in the community, they used their privilege of being highly followed to act on behalf of others, to amplify their thoughts, and to motivate negotiation.

HL contributors also used their social location to advocate for opinions or stances that mattered to them or that they may have thought mattered to their followers. Some statements held evident passion and vigour towards promoting and defending the activities of the MTBoS community, such as in Figure 7.29 a and b below.

![Figure 7.29](image)

(a) Examples of passionate advocacy for MTBoS

In Figure 7.29a, the contributor was advocating their passion for connecting with the community and gleaning from resources and ideas that had already been curated and
developed by others, particularly for those pre-service teachers who are just starting out. In Figure 7.29b, however, the contributor was both endorsing and defending the MTBoS community as a collective of quality teachers who implement teaching practices that defy stigmatized traditional views of education. Their comment came as a response to a user who posted that they believe US public schools encourage the idea of kids who get math versus kids who don’t, claiming this attitude rejects otherwise capable students. Clearly, the responding contributor believed that MTBoS users do not make such judgements and they embrace children’s thinking through the activities they implement.

While some of the advocacy of opinions and stances were made as passionate statements of personal beliefs, others in this group seemed to advocate for ideas that were already taken-as-shared and presumed as normative within MTBoS. In some sense, they were reifying these ideational artefacts, making their importance and presence even more explicit. For instance, one contributor tweeted about Jo Boaler’s quote on growth mindset, emphasizing this very central ideational artefact in the ideational network (Figure 7.30a), while another tweeted about why #wodb (which one doesn't belong) could be a valuable activity to implement in classrooms (Figure 7.30b).

![Figure 7.30 Examples of advocating taken-as-shared stances](image)

While both these ideational artefacts were prominent in the ideational network and were generally taken-as-shared in the space at the time, these HL contributors were emphasizing them more explicitly. They were promoting these ideas possibly with hopes that others will find them useful. However, through emphasizing these central ideational artefacts, they were also advocating for these stances, highlighting their importance.

The potential for advocacy in this group paired well with their high followings since their social visibility could be used as a platform for advocating change. However, their lack of following their followers back put them at a higher risk of being ideationally misaligned.
with the collective. For instance, the tweet in Figure 7.30b emphasized the importance of ‘which one doesn’t belong’ (#wodb) activities, which have been popular and prominent in the collective for quite a long time based on my insider view. Posting about #wodb was already somewhat passé during this time frame and was no longer a central node in the ideational network. So, they were ideationally misaligned. Another even more evident example of this was when one organization tweeted, “How can YOU shift the emphasis of math learning away from speed and timing and towards depth and creativity?” These were things many MTBoS users already focused on, so the contributor evidently was not close enough to current conversations to have relevant ideas to advocate for.

Overall, the advocacy revealed by this group continued their key characteristic of serving others, and in turn, signaling their own importance. Rather than trying to radically shift views, their attempts at advocacy were mostly conformist in nature, likely in relation to their lack of awareness of current ideational content and low attention to fostering and maintaining social relationships in the space. In other words, HL contributors who were well seen but were not looking at others tended to advocate for ideas that had already become established in the space. This made them less capable of pushing boundaries.

In summary, tweets made by HL contributors with high in-degree and low out-degree primarily involved serving a utility role for others through contributing resources, directing questions, and building community where possible to allow space for discussion rather than looking for a direct benefit for themselves. They most prominently engaged in contributing resources or teaching ideas in the form of complete lessons or ready-made activities that were full of specificity, but often lacked contextualization in practice. There seemed to be a sense of hope that by sharing these lesson ideas, others would find them useful, would implement them, and would offer feedback or gratitude. However, since they were not following many of the current contributors to MTBoS, they were at times misaligned with the sources of ideational coherence in the collective.

Their key advantage in the collective was their high social visibility, giving them the privilege of being seen, which many of them used to amplify the ideas of others, to publicize resources that could be useful for others, and to advertise important events. However, since they were not following many of the current contributors to MTBoS, their key barrier was a lack of attunement to the current sources of ideational coherence. Since the ideational network is ever shifting and evolving, their distance from those
currently contributing created the risk of their contributions being irrelevant to current issues. While many seemed to overcome this, possibly because they followed enough MTBoS contributors overall that they had a sense of the key interests (or because they were the leaders of these interests), some seemed to be more misaligned ideationally than others. Those who seemed more misaligned were typically organizations that were trying to tweet content to the community but without being socially involved. This points to the importance of both social visibility and social relationality as well as ideational coherence in the neighbour interactions among agents.

**High in-degree and high out-degree (HH)**

Finally, the HH contributors who not only were highly followed but also followed many others in the shared social space evidenced a different character that was particularly oriented towards stimulating neighbour interactions. Not only did they have the privilege of being socially visible, they were also privy to the inner workings of many others in the space. As such, their tweets were generally more ideationally aligned with the current sources of ideational coherence and their advocacy and community building were prominent manners of not only emphasizing taken-as-shared ideas, but also pushing boundaries. In fact, these HH contributors were the most prominent contributors to tweets that involved building community, advocating an opinion or stance, and endorsement. They also notably contributed resources and teaching ideas (44% of their tweets), engaged in signaling identity (35% of their tweets), and involved in endorsing (35% of their tweets). Although only 26% of their tweets involved building community and 23% were advocating an opinion or stance, none of the other groups involved as much in community building and advocacy as did this group. However, the ways in which they involved in these forms of engagement were slightly different than how the other groups involved in them, which is likely related to their privileged position of both their wide view on MTBoS activity and their highly visible social location.

As noted, this group of HH contributors most notably tweeted in ways that served to build community. They also strikingly referred to MTBoS as a community rather than merely as the organizing hashtag that all data was collected with as contributors in other social locations did. They evidently treated MTBoS as a community rather than a place. This was clear in how their engagements in community orienting tweets were primarily social in nature, with much of their community building endeavours involving expressions
of gratitude for MTBoS itself. For instance, in Figure 7.31a, the contributor referred to MTBoS as their family and signaled their identity as being an internal MTBoS member. They shared this sense of connectedness with pride. In Figure 7.31b, the contributor also referred to MTBoS as their family and touched on how grateful they were for the support MTBoS has provided them, particularly in their career. And, in Figure 7.31c, the contributor alluded to how important MTBoS has been for them.

Active efforts at building community were also made by organizing ways to welcome new members (Figure 7.32a), letting them know they can ask for help (Figure 7.32b).

Efforts towards building community in this group also involved advocating opinions for change and stimulating engagement in initiatives that pushed boundaries of dominant structures. In other words, they were more likely to question processes, challenge norms, and stimulate creation of novel material. One example of pushing boundaries was seen earlier in the ‘NCTM’ and ‘conferences’ relation in §6.2 where the contributor provoked thinking about NCTM fees for presenters. Another related example involved a contributor who promoted the presence of MTBoS within NCTM conferences by advocating for a sign-up process to host a MTBoS booth by linking the sign-up sheet and stating, “I will see you there! Please help the #mtbos out by doing this quick thing!”
And, in another case, a new professional learning initiative developed by contributors in MTBoS was advertised and advocated for as exciting to enroll in.

Yeah, I know it feels like Summer 2018 isn’t even over yet, but it’s never too early to think about NEXT Summer! @TableTalkMath Summer Course series is going to be fantastic. Excited to be part of this team.

Overall, building community for this group of HH contributors involved using their privilege of social visibility to support others relationally and to push boundaries within the ideational space. Their position of being privy to the sources of ideational coherence along with their awareness of being socially visible allowed them to harness their privileges to build community, but also to question existing norms.

Further, another prominent form of engagement for this group of HH contributors was around advocating opinions or stances as agents of change. This occurred within their attempts at community building, but also in other situations such as advocating for changes in practice. Although the HL contributors were also prominent with advocating opinions or stances, the HH group expressed their opinions primarily in combination with signaling an identity that they were aware of prominent sources of ideational coherence for the purpose of pushing ideational boundaries. For instance, opinions were advocated for that revolved around social justice (Figure 7.33a), caring for students (Figure 7.33b), and finding ways to focus on conceptual understanding (Figure 7.33c).

Figure 7.33  Examples of advocating stances or opinions as agents of change

The key difference between how this group advocated for opinions or stances and how the HL group did, was that this group involved more of their own personal selves in their comments, thus making themselves more vulnerable and revealing their own positions.
or practices. Their tweets were not so general that they could fit anyone, such as tweeting about growthmindset quotes or why a ‘which one doesn’t belong’ activity structure is useful (such as in Figure 7.30), but rather, they involved their own experiences, emotions, and intentions. As is seen in Figure 7.33a above, the contributor advocated for the importance of the content in the Justice Podcast they were listening to. In Figure 7.33b, the contributor advocated for positive phone calls to parents of students claiming it brings joy, presumably because they personally did it themselves. And, in Figure 7.33c, the contributor directly revealed their personal tactics towards emphasizing they care about why mathematical procedures work, not just how they work. These contributions were more personal and emotionally charged than those advocating stances with fewer out-going followings.

Many also focused on revealing practice, and through revealing practice found a way to advocate an opinion they were passionate about. For instance, in one case, the HH contributor revealed their tactic for communicating to students how they cannot allow them to use ‘tricks’ in class.

My student T was having a s explain how to x 2 binomials. I heard kid say: so I found this trick! // Waited for ST to make a comment & then I literally screamed: Don’t ever use that word in my classroom! // I’m sure she & kid saw their life flash before their eyes! // #MTBoS

Another equally passionate tweet involved the HH contributor expressing their frustrations with having to implement benchmark tests in the third week of classes.

Infuriated-giving a departmentally mandated benchmark the 3rd week of Alg2 which includes logarithms, imaginary numbers, and trigonometric functions in quadrants other than the first. Waste of time, demoralizing for students. Have to wonder who this serves. #mtbos

And, another HH contributor acknowledged the challenges of teaching and passionately advocated that MTBoS is a place for supporting each other in these challenges, a community to find others who share struggles with and to get support.

Teaching is hard, y’all. If you feel like a failure. If you’re discouraged. If you feel like everybody else knows something you don’t. If you’re nowhere close to the teacher you want to be (& feel like you’ll never get there). Just know YOU’RE NOT ALONE. #mtbos #iteachmath

All of these examples advocated for specific opinions that were grounded in personal experiences that these HH contributors presumably desired to illuminate and emphasize.
In doing so, they also signaled their identities as progressive teachers of mathematics who believe mathematics teaching needs to change. That is, to emphasize opportunities for evoking student reasoning, instill positive affect in students learning mathematics, and decrease mandated standardized testing that demoralizes students.

Not only did this group target sources of ideational coherence, they seemed to push their boundaries by questioning presumed norms. The advocacy-oriented tweets made by this group surfaced passions held not only by contributors themselves, but also by the community they felt supported by since their boundary pushing statements were often met with significant response from others who agreed with them. In this way, HH contributors not only pushed ideational boundaries with their advocacy, they also mobilized otherwise inert topics and issues commonly laden with frustration among mathematics teachers. These expressions of advocacy aligned with community values enough that they could be probed, pushed forward, and contextualized as necessary.

Further, while many tweets in all categories contributed resources or teaching ideas, the ways in which this was done across contributors in various regions of social location differed. HH contributors primarily contributed resources that elevated the status of others through endorsement or that elevated their own status through signaling an identity or advocating an opinion. For example, in Figure 7.33a shown earlier, the contributor endorsed the social justice resource contributed by another user, thus elevating that user’s status through the privileges of their social location. Endorsements and hat-tipping like this when sharing resources was common among HH contributions. On the other hand, HH contributors also tended to use sharing resources to signal identity and further establish their social position in the community. For instance, the contributors in Figure 7.34 a and b below both indicate their awareness of the prominent ideational artefacts of ‘howmany’ and ‘unitchat’ as well as ‘real-world’ contexts.
Figure 7.34  Examples of resource contributions that signal identity

Signaling such identity occurred with more robust contributions more directly pertinent to classroom practice as well. For instance, in Figure 7.35a, the contributor revealed their capacity over involving desmos activities by adding in their own custom activity. And, in Figure 7.35b, the contributor described their approaches to including student self-reflections in response to mathematical errors. Not only did they seem aware of what was valued in the community, they are also evidenced their capacity to contribute to the space with ideas that fit sources of ideational coherence in the collective. This could possibly be a way one could become valued for contributions.

Figure 7.35  Examples of ideationally aligned resource contributions

Further, the resource-oriented tweets made by HH contributors also aimed to push boundaries of what was typical to publicize. That is, resources shared were in some way novel in ways that pushed boundaries. For instance, the desmos tasks provided in Figure 7.35b above included a new activity created by the contributor. Another example of such boundary pushing within resource exchange involved a contributor who linked to a fellow user’s blog post about their use of ‘mathographies’. They endorsed it by stating, “@JennSWhite writes about her students’ mathographies!”. Endorsement occurred in other contributions as well, for example when the no pencil test strategy (discussed earlier in §6.2 and §6.3) was endorsed by one contributor who linked to the original post
and stated, “Brilliant strategy from @howie_hua #mtbos.” Such endorsements amplified these ideas in the collective. So, not only did this group of HH contributors seem aware of what was valued, they contributed novel approaches to achieving some of the more negotiable ideational artefacts. In this way, the intentions of these contributors seemed not only self-serving, or community-serving, but also boundary-pushing. In order to push boundaries, an awareness of the boundaries must exist, which seemed to be the case within the material contributed by this group.

Although HH contributors strived to push ideational boundaries, it also seemed important to them to maintain and reinforce their identity as insiders in MTBoS in various ways. As indicated earlier, some did this within resource contributions. However, others also advertised their internal status within community building.

I feel like I should say to the whole #Mtbos that anyone visiting LA area has an open invitation to hang with me!

Hi all! Sorry I couldn’t chat tonight. Soccer Mom life! But WELCOME to the #MTBOS please let me know if you have any q’s. I teach #alg2chat near Clt. Would love to have any visitors anytime!

Alternatively, they signaled identity through opinions they advocated such as in relation to social justice topics or around how they embraced caring for students as learners within the practices they revealed (e.g., Figure 7.33 a and c from earlier). However, it is noteworthy that signaling identity in this HH group hardly came across as intentional. Rather, it seemed to be interwoven in their participation and may be reflective of Twitter culture more broadly, which is often associated with identity formation (e.g., Marwick & boyd, 2010; Veletsianos & Kimmons, 2016). It could also be a natural by-product of being in such a highly inter-connected social location; the privilege of being visible to many of those one follows may make it natural to project a certain status within one’s tweets and to involve ideas in the community, which may very well be aligned with one’s imagined community. As such, the notion of signaling an identity may be a natural consequence of being so connected within a particular social sphere.

In summary, the tweets made by HH contributors with high in-degree and high out-degree were primarily oriented around building community, advocating for opinions they seemed passionate about, and pushing ideational boundaries in the collective. Given the social location of this group as those who were not only well-followed, but followed many other internal contributors, they generally had a strong awareness of what the collective
valued and engaged in. By holding capacity over these redundancies, they were able to harness their own personal diversities and point them towards advocacy for changes they envisioned to be necessary. As such, their key advantage was both social visibility and awareness of sources of ideational coherence in the collective. From this position, they were able to operate at the boundaries by contributing novel teaching ideas, posing thoughtful questions, advocating for necessary organizational changes, and mobilizing the ideas of others in their community. While they offered diversity, it was delivered in manners that grounded them in redundancies familiar to those in the collective, in turn making their contributions familiar, but novel. Familiar because they played on common experiences among mathematics teachers in general, and MTBoS participants in particular, but novel because their personal directives were evident. As such, they were able to stimulate neighbour interactions in ways that emerged further ideational material and mobilized otherwise inert capacities available in MTBoS. Their interests also extended to expressing care for others and being supportive relationally, not only ideationally. As such, they held capacity for operating as a community among themselves, as a subset of the complex collective. However, while their position seems advantageous in all ways for the collective, particularly for ideational emergence, the key barrier for this group is that of stamina. Engaging so deeply can take a lot of energy and can lead to burn-out. Although not in this particular data set, statements indicating burn-out are most definitely encounterable in the space and some users will quit or distance their participation because of the work involved in maintaining relationships and producing novel content. That is, the energy required for full participation should be acknowledged. The density of HH contributors is therefore important for the overall resilience of the collective, and mechanisms for replenishing such internal community members who engage so deeply and influentially need to be considered.

7.4 Modes of engagement: Social responsibility and ideational alignment

Overall, contributors in the various regions of social location varied significantly in terms of forms of engagement used to deliver ideational content, and therefore in how they contributed to neighbour interactions within the ideational space. The analysis of forms of engagement in each social location emerged characterizations of social locations in terms of their overall modes of engagement in the space. That is, their general stances
towards participation in MTBoS. These modes of engagement related to two key factors: social responsibility and ideational alignment.

LL contributors who had low numbers of followers and followings in the set engaged primarily in practice-oriented ways focused on pragmatism and utility. Although tweets made by these contributors were generally quite diverse among each other, the redundancies were that engagements were personally focused and often ideationally misaligned with sources of coherence in the collective. This was typically because of a strong focus on pragmatism that kept tweets focused either on specific content or pedagogy, without making connections between them. They also often lacked referencing popular pedagogical approaches prominent in MTBoS such as ‘noticewonder’ or ‘groupwork’. Instead, tweets remained more generic, living in the broad ideational realm of mathematics teaching. This sort of misalignment with the ideational space of MTBoS was prevalent among most tweets in this group and may reflect their inability to see others in MTBoS and their overall lack of visibility. As such, I refer to LL contributors as newcomers due to their position as the unseen and unseeing. While it’s possible they may not be newcomers in the pure sense of the term, their level of participation in following and being followed by others specifically in MTBoS was numerically low. Over time, their position could change.

LH contributors on the other hand could see others in MTBoS but were not yet prominently seen by others. As such, I refer to LH contributors as observers. Possibly as a result of their awareness of what others in MTBoS were tweeting, they seemed more ideationally aligned with sources of coherence in the collective than newcomers. However, they maintained a predominantly personal orientation in their contributions. Unlike newcomers, observers engaged by tweeting about practice in ways that highlighted their implementation of pedagogical sources of ideational coherence such as ‘instructional routines’, ‘desmos activities’, and ‘noticewonder’, and not only mathematical ones such as ‘geometry’. In such ways, they were projecting their identities as those who belonged to MTBoS, which may be related to how highly they followed others in MTBoS even though they were not as visible by others. Observers were also the most willing out of all social locations to show vulnerability in asking specific questions about practice. Their observational vantage point seemed to offer them the benefit of feeling comfortable to share vulnerabilities without being necessarily worried about maintaining their status. In fact, it seemed as though sharing
vulnerabilities was a way to solicit attention since asking questions and sharing one’s issues can attract others to respond.

HL contributors, however, had more social capacity for influence but less visibility on others to feel this sort of comfort. As such, I refer to them as *influencers* due to their advantage of being well-followed. Influencers similarly engaged in attracting attention as did observers, but they were less open to being personally vulnerable. Rather, their primary form of engagement was around providing resources for others, therefore attracting attention by offering tools for others to use. In doing so, they were less personally focused, and instead more community oriented. Their contributions revealed acknowledgement of their readership, and they came across as interested in serving their followers as well as using their privilege of visibility to endorse others. However, since they did not follow as many MTBoS contributors, their contributions were at times ideationally misaligned and either overly redundant, passé or distanced from currently prevailing issues. They were also less relationally involved and at times remained socially distanced from others. However, with such high followings and a focus on providing resources, they held a capacity for influence on the collective.

HH contributors, however, lived the privilege of both worlds that HL and LH contributors experienced. Having the advantage of being seen and of seeing others made them more likely to involve as leaders who maintained awareness of sources of ideational coherence and had the social capacity to push the boundaries of these ideas, to challenge and question them. As such, I refer to them as *leaders*. They also seemed to acknowledge their followers by tweeting in socially responsible manners. For instance, in tweeting about their experiences in mathematical learning situations, they provided enough detail for others to be able to engage, linked to important resources, and often posed deeper questions about their experiences. In these ways, they found ways to push ideational boundaries and create opportunities for negotiation. They also were the only group who involved in expressions of gratitude for MTBoS, referring to MTBoS as a ‘community’ they were part of and thankful for. Their capacity for leadership, therefore, was supported not only through ideational influence, but through fostering and maintaining relationships with others and the community they felt part of.

Very notably, these categories of social location, from now on referred to as *newcomers* (LL), *observers* (LH), *influencers* (HL), and *leaders* (HH) are dynamic. Contributors have
opportunities to traverse between them over time either naturally through time spent following and being followed, or possibly by playing with various associated modes of engagement. These categories of social location also allow for characterising possible modes of engagement in MTBoS. By looking across the two dimensions of in-degree and out-degree and considering the respective changes in modes of engagement across them, more global patterns became observable. Namely, changes in in-degree (or visibility) proved intertwined with social responsibility. That is, those with lower in-degrees were generally practice-oriented and self-oriented, tweeting from personal perspectives and often missing details that could have been helpful to readers in understanding their tweets. And those with higher in-degrees tweeted in more community-oriented manners, showing awareness of their readership in how they presented tweet content, which evidenced their tendency towards being useful, giving, and motivating for others in the collective. Conversely, changes in out-degree, or the capacity to view content published by others, proved intertwined with ideational alignment in terms of invoking sources of ideational coherence and prevailing discourse in MTBoS. More specifically, those with lower out-degrees were generally pragmatically oriented and remained in the broad ideational space of mathematics teaching rather than invoking topics and pedagogies most prevalently associated with MTBoS. They tended to focus on utility and applicability to the classroom rather than fitting into or pushing the boundaries of MTBoS space. On the other hand, those with higher out-degrees more often invoked prevailing discourses of MTBoS in both content and pedagogy that signalled their belonging to MTBoS and not merely to the realm of mathematics teaching in general. That is, they either associated with prominent sources of ideational coherence or pushed ideational boundaries, evidencing their authority over the ideational network of MTBoS and not just of mathematics teaching. More simply, they seemed aware of what was valued in the community, and either signalled their belonging to it with excitement or in ways that challenged things typically taken-as-shared. As such, increased awareness about the activity in MTBoS proved intertwined with an increased tendency towards ideational alignment. The dynamic nature of these associated dimensions is shown in Figure 7.36 below.

Similarly, Naaman et al. (2010) found that users tweeting in personally-oriented manners (as ‘meformers’) tended to have fewer followers than those tweeting informatively (as ‘informers’).
When the two factors of visibility through in-degree and awareness through out-degree came together, a sort of informed leadership in the collective became evident, as indicated in Figure 7.36 above. In other words, being both followed and following others proved necessary for holding capacity to engage in neighbour interactions that contributed to ideational emergence in the collective. Not only were leaders tweeting in alignment with sources of ideational coherence, they more often seized opportunities to push ideational boundaries amidst their awareness of and responsibility towards viewers. By being aware of the ideational space and having the social capacity to push its boundaries, leaders were able to initiate neighbour interactions in novel ways. They did this by offering resources, opinions, and community building initiatives, through which they also evidenced a sense of social responsibility towards their followers. As such, leaders played an important role in the ideational emergence in MTBoS.

However, those who excelled in only one of these factors seemed to miss aspects of informed leadership; influencers were socially responsible but at times ideationally misaligned, observers were ideationally aligned but at times self-centered, and newcomers remained personally practice-oriented. These various social locations therefore describe a sort of trajectory that defines the space of possibility between
legitimate peripheral participation and informed leadership in the collective. Newcomers begin with self-oriented tweeting and as they begin to gain followers, their sense of responsibility for others seems to increase. This highlights the prominence of ‘meforming’ that is typical of social media settings (Forte et al., 2012; Naaman et al., 2010), and evidently serves an important purpose in the trajectory of establishing one’s social position in the collective.

Overall, the variables of in-degree followers and out-degree followings associated with the two key characteristics: social responsibility and ideational alignment. Combinations of these characteristics defined possibilities for modes of engagement as manifested by contributors in each social location. Social responsibility increased as social visibility increased. The tweets of contributors with higher in-degrees evidenced their desires to be useful to others, to build others up, and to mobilize ideas for their followers. Although this mode of engagement through social responsibility may have had various underlying motives (e.g., pleasing others, being seen as popular, being seen as useful, seeking connection, etc.), the effects of such tweets showed care for others in the space. Ideational alignment on the other hand, increased as contributors’ awareness on the activities in the space increased. The tweets of contributors with higher out-degrees evidenced their attempts at fitting into sources of ideational coherence and ‘meforming’ to reveal their capacity for invoking shared discourse. Ideational alignment involved keeping a pulse on trends and values to build from in contributions and engage in informed leadership. The combination of social responsibility and ideational alignment as modes of engagement contributed to informed leadership, which evidenced capacity over shared discourse, relationships, and neighbour interactions. Such capacity evidently contributed to the ideational space in MTBoS, and possible instances of emergence of novel material.

However, although the capacity for eliciting ideational emergence in the space was stronger for those with higher visibility and awareness, and therefore social responsibility and ideational alignment, they were not sufficient in and of themselves. Ideational emergence was evident among the content produced by leaders but did not always occur. As such, further determining conditions were necessary to explore. Namely, the feedback mechanisms could also point to when and why instances of ideational emergence occur, as is explored in the following chapter.
Chapter 8  Feedback mechanisms

As it turns out, however, mechanisms to amplify small perturbations are essential to the viability of living and learning systems . . . complex unities must incorporate positive feedback mechanisms that will permit certain perturbations to be amplified so that seemingly small events can come to matter greatly. (Davis & Sumara, 2006, p. 102)

Activity on Twitter is fundamentally driven by positive feedback loops\(^74\). At the most basic level, users contribute ideational content in the form of tweets, which in turn are seen by other users. The mere awareness contributors have of imagined audiences\(^75\) that can view their publicized content can provoke them to continue tweeting. However, there are also feedback mechanisms built within the Twitter platform that further amplify certain tweets more than others, and in different ways. Namely, tweets can be ‘liked’, ‘retweeted’, or ‘replied to’. ‘Likes’ are typically used to either save tweets for future viewing or to show the contributor of the tweets support or comradery. The receiver can then feel validated that their content has been seen and considered valued by others. ‘Retweets’ are typically used to push content to more followers but are sometimes also treated as a way to save content for one’s own future viewing. The receiver can then feel validated their content was not only valued by others, but worthy of them associating with it. And, ‘replies’ are typically used to interact with content, engaging in private conversations in public. Comments made can range widely from supportive commentary to inquisitive or even challenging statements directed publicly to the user who posted the content. The receiver of the replies can then further interact with those who have replied, possibly leading to negotiation. While these are general presumptions, they are supported by my insider observations of user activity and from ad hoc conversations I have had with users. However, it evidently remains unclear why certain content becomes amplified in different ways through such positive feedback loops, and how these amplifications contribute to ideational emergence in the collective.

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\(^74\) Positive feedback loops are self-sustaining cause and effect loops in which both factors become both cause and effect, together stimulating each other to create a perturbation. Negative feedback loops in contrast involve processes that counteract the activity rather than that amplify it. Twitter does not involve negative feedback because attempts at counteracting activity are limited to controlling harassment (i.e. blocking malicious content), and not typical of the activity in the MTBoS.

\(^75\) Imagined audiences (or communities) refer to the perceived publics individuals direct their communication to, particularly in spaces such as Twitter (e.g., Gruzd, Wellman, & Takhteyev, 2011; Litt, 2012; Marwick & boyd, 2011).
In terms of opportunities for amplification, I am suggesting that each of the different feedback mechanisms afford a different form of amplification. ‘Retweets’ have the most obvious amplificatory effect since they directly increase the frequency of content by bringing it back into the public arena and making it more likely to gain subsequent feedback. Figure 8.1 models this sort of temporal frequency afforded by ‘retweets’ by showing the times (on the x-axis) at which each retweeted tweet in the complete data set was retweeted, where each unique value on the y-axis represents a different tweet.

![Figure 8.1  Retweeted tweets with #MTBoS over time during Sept. 21-29, 2018](image)

It can be observed in such a visualization that certain tweets endured stronger patterns of positive feedback through ‘retweets’ than others, creating a chain effect of ‘retweets’ for particular content. Evidently, the tweets that attracted such amplificatory behaviour through chains of ‘retweets’ held some sort of quality that triggered such response. And, the effects compound since once a tweet is retweeted based on its quality, it is more likely to be retweeted again (as modelled in the causal loop diagram in Figure 8.2 below). Therefore, ‘retweets’ offer a positive feedback loop that amplifies content through increasing its frequency, or diffusion\(^{76}\), over time in the public space.

![Figure 8.2  Causal loop model for retweets](image)

\(^{76}\) As conceptualized by various authors, information can be diffused through Twitter networks through retweets (e.g., Hermida, 2010; Rieder, 2012).
‘Replies’ have a similar amplificatory effect because each time a tweet is ‘replied’ to, it is brought back into the public space and those tagged in the tweet receive notifications. However, it is not merely diffusing the content directly to others as ‘retweets’ do, it is also refracting\(^{77}\) it because replies can change the nature of how the tweet is perceived and interacted with in subsequent iterations of feedback. Therefore, a positive feedback loop exists through ‘replies’ that amplifies the content through increasing both its diffusion and refraction in the public space, and through stimulating further ‘replies’ to those made. As such, a model similar to that presented in Figure 8.2 above could be made for ‘replies’.

‘Likes’, however, have less of a diffusive or refractive effect because they do not increase the frequency of the content in the public realm over time or alter perspectives on it through comments. Instead, they endorse the content by signalling to contributors that their content has been seen and considered valuable. As such, I refer to this as an endorsive amplification. This sort of amplification adds value to the content, making it more likely for the contributor (and anyone tagged in the ‘liked’ tweet who also receives notifications of the tweet being ‘liked’) to continue the activities they are awarded for, producing further such content given the feedback it has received. Albeit different in nature, ‘likes’ also provide the opportunity for a positive feedback loop, one that shows content has been endorsed. Therefore, ‘likes’ can also be considered as involving a causal feedback loop similar to the one shown in Figure 8.2 above.

As such, each of the three feedback mechanisms embedded in the Twitter platform offer opportunities for positive feedback loops that can steer the production of subsequent content and privilege certain content to more visibility than other content, thus shaping the ideational space of MTBoS. While these mechanisms are fundamentally inherent to the technological structure of the environment, they are also user determined in terms of how they are used and the meanings they take on through such usage. Given these aspects have not yet been attended to in this way in prior literature, I introduce language with which to refer to these forms of amplification. Namely, I suggest that ‘likes’ offer endorsive amplification, ‘retweets’ offer diffusive amplification, and ‘replies’ offer refractive amplification. These are illustrated in Figure 8.3 below.

\(^{77}\) I borrow the language of diffusion and refraction from Rieder (2012).
However, it is yet unclear what sorts of qualities in tweets provoke each of these forms of amplification and how they steer ideational activity. As such, in what follows, I pursue a deeper investigation into the redundancies and diversities among tweets attracting different forms of amplification in the core data set.

### 8.1 Identifying and examining amplification mechanisms

Towards investigating the core data set with respect to feedback mechanisms and forms of amplification, counts for each of the three feedback mechanisms were tabulated for each of the 444 core data set tweet clusters. If a tweet was not the dominant tweet in a tweet cluster, the counts from the dominant tweet in the tweet cluster were taken. This choice was made in order to attribute the mechanisms primarily to the ideational contributions of the tweet clusters rather than specific contributors. This means that for each of the 444 tweet clusters, there were three values: one for the maximum number of 'likes', one for the maximum number of 'retweets', and one for the maximum number of 'replies' the ideas in the tweet cluster received. The maximum number of replies was counted to include both the number of threads and their depths (i.e., all possible responses stemming from the initial tweet in the tweet cluster). This means that some tweet clusters with a high number of replies could consist of many single responses while others could consist of a few longer threads with back and forth replies.

When all data was considered according to these three variables, it created eight options for combinations among the three variables. As such, a Venn diagram was used to organize the options. Initially, the Venn diagram was used to include tweet clusters in a
region as long as they had at least one count in that region’s variable. This resulted in a diagram as in Figure 8.4, which included a majority of tweets in the overlapping central regions between ‘likes’ and ‘retweets’, meaning most tweet clusters in the core data set solicited at least one ‘like’, one ‘retweet’, and possibly one ‘reply’. However, such a Venn diagram did not emphasize the key features involved in the forms of amplification offered by each feedback mechanism because the criteria included a tweet in a region given it had at least one count in that form of feedback. For example, a tweet with one ‘like’ was placed in the ‘likes’ region. But, ‘likes’ can range into the hundreds and it is common to get at least one ‘like’, so this parsing did not account for the differentiation between a significant number of ‘likes’ (or other forms of response) and an insignificant one.

Figure 8.4 Core data set tweets parsed according to feedback received

For instance, a tweet with seventeen ‘likes’, two ‘retweets’, and one ‘reply’ would be included in the central region, which aims to represent tweets receiving all forms of amplification. However, such a tweet should only be considered well amplified by ‘likes’, and not by ‘retweets’ or ‘replies’. Since the goal of this investigation was to explore features associated with each form of amplification, further emphases were necessary.

As such, the distributions of value ranges for each feedback mechanism were considered, and ‘cut-off’ values were determined to establish ranges that would permit tweets to be included in a region given they satisfied a significant amount of that region’s
feedback mechanism. That is, a new Venn diagram was created which now restricted inclusion in a region to only tweets with values greater than or equal to the pre-determined cut-off value for that region. These cut-off values were determined by arranging the distributions of all values for each feedback mechanism in decreasing order and taking the top 20% of them. These distributions are shown in Figure 8.5.

![Figure 8.5 Distributions of values for each feedback mechanism](image)

Notably, they each approximately follow long-tail power law distributions, and therefore, taking the top 20% of possible values (indicated in blue in Figure 8.5 above) adequately determined tweets that should be considered as having a significantly high value in each of the feedback mechanism dimensions. This was more appropriate for this dataset than for instance choosing an arbitrary cut-off value that was the same for each dimension. Choosing the same cut-off value for all dimensions would ignore the differences in range in each. From this process, it was determined that a tweet that was significantly ‘liked’, ‘retweeted’, or ‘replied to’ received at least 12 likes, 5 retweets, or 4 replies, respectively. Boundary cases were tweets that had values equal to the above values. These were included in the region but were marked as boundary cases within analysis.

However, by using the above criteria in the construction of the new Venn diagram, only 164 of the 444 tweet clusters were included within any of the regions, leaving 280 of the 444 in the low response category under all cut-off values. While the internal regions of the Venn diagram were reasonable for analysis, the external region of tweet clusters with low amounts of response was disproportionately large in comparison with the other regions. As such, a random sampling approach was taken to select a more reasonable number of tweets for analysis that would be relatively representative of this category. To emphasize the low response qualities in this category, the sample space for the region was restricted to include only those tweets that were at least two counts lower than the
cut-off values for each feedback mechanism, which left 196 tweets to sample from. 20% of these were randomly selected, resulting in 40 tweet clusters in this category. This sample size was adequate given it was within the higher end of the range of sizes between the smallest and largest categories in the Venn diagram. Random sampling was chosen over further restricting criteria in the region to allow for broader representation in this category and to mimic the nature of randomly encountering such content. The final selections of tweets are indicated in Figure 8.6, where each tweet cluster is indicated with its tweet cluster ID number from the core data set.

![Figure 8.6](image)

This view defined the possibilities for amplification available among combinations of available feedback mechanisms. Focusing on high values in each dimension of feedback afforded a view that helped identify the qualities involved in provoking each form of feedback, and in turn, the opportunities for amplification in the space.

Towards a more comprehensive investigation of each of these regions, each of the sets of tweet clusters included in Figure 8.6 above were analyzed qualitatively. As may be evident, various opportunities for investigation of these tweets was available. Namely,
the prominent ideational artefacts in each region, the social locations and modes of
engagement of contributors in each region, and the qualities within how content was
presented in contributions within each region. While the first two of these were explored,
they did not on their own yield evident insights into why the amplification occurred or
what it was responding to (other than the basic trends of social visibility affecting
opportunities for feedback). This is perhaps because ideational and social factors were
not on their own stimulating amplification, but rather, acted as a gateway for it to occur
(as further detailed in Chapter 9). However, by attending to the redundancies and
diversities among tweets within each region, qualities that provoked each form of
amplification became evident.

To this end, a process of iterative coding was undertaken by attending to redundancies
and diversities among tweet clusters in each identified category until themes emerged
that could characterize each form of amplification. Although analysis was primarily
oriented around the content presented in each tweet cluster, it also included
consideration of contributor attributes particularly when looking at the low-response
region. The coding process was initially undertaken on the tweet clusters within each of
the regions that were strongly responded to with only one form of amplification. These
themes were then used to further explore those with two significant forms of
amplification, then with all forms of amplification, and finally, with no forms of
amplification. This order was chosen since it was presumed that some repetition of
features would become evident in composite regions, which they did. In what follows, the
characteristics of each of the singular forms of amplification are discussed and
exemplified, followed by characteristics found in composite regions as well those with
little response. Finally, key implications for amplification are discussed.

8.2 Exploring forms of amplification

In the examination of contents within tweet clusters parsed into each of the regions
identified in Figure 8.6, unique characteristics were identified as being redundant within
tweet clusters in each region. Through this window, qualities in tweets that related with
them being more ‘liked’, ‘retweeted’, or ‘replied to’ offered implications about the nature
of stimuli these feedback mechanisms amplified in tweets through forms of amplification.
It also afforded insight into the *modes of amplification*\textsuperscript{78} that occurred within the system that ultimately fed back into the ideational fabric of the MTBoS collective.

**Highly liked**

Tweet clusters that evidenced significantly high numbers of ‘likes’ were most often celebratory in tone, often involving excitement and pride about personal accomplishments and other indicators of success. They did not typically invoke a negative tone or involve controversy. Instead, they were evidenced excitement about successes not only that were worth celebrating, but also that fit into sources of ideational coherence in the collective. For example, one contributor illustrated their student’s success in developing ‘growthmindset’ thinking within mathematics (Figure 8.7 below).

![Figure 8.7  Example of highly liked growthmindset tweet](image)

Not only did this contributor signal their identity as a teacher capable of fostering ‘growthmindset’ thinking in mathematics, a central source of ideational coherence in the collective, they also evidenced pride in the accomplishments of their student and to some extent, themselves. Other contributors celebrated their own accomplishments even more explicitly. For instance, as measured through their students’ grades.

*Proud Teacher* Every one of my S’s in contemporary math has a C or better. This is HUGE for two reasons.1) Most of these students have not felt much success in math previously. 2) I hold all my students to a high standard- these kids have earned their grades. #GVUSDShares #MTBoS

Alternatively, personal successes in mathematics teaching were also measured with self-reflections revealing the tenacity endured to persevere through student pushback for the sake of reform-oriented mathematics teaching, believing it to be worth the effort.

\textsuperscript{78} Again, using the distinction between *forms* and *modes* from Remillard (2012), but now in terms of *forms* and *modes of amplification*. That is, *forms of amplification* involve available possibilities for *amplification* and *modes of amplification* involve general intentions behind these possibilities.
When you try new learning strategies with your students, ones that are TOTALLY different than any way that they’ve ever been taught, questions and resistance is expected. This year, I will fight for education reform, one student at a time. #MTBoS #mathchat #octmchat

There was also a prevailing redundancy among highly ‘liked’ tweets around sharing accomplishments of overcoming teaching- and community-related struggles in the aim of upholding values of the MTBoS collective. For instance, one contributor tweeted, “I just wrote my first blogpost!” with an image of a flying school-aged superhero and a blog post link about helping students change their mindsets. In this case, the contributor invoked both celebration of contributing to the community through a blogpost, which is valued, and their capacity to foster ‘growthmindset’ behaviours in students through the adoption of specific strategies. This was worth celebrating because the contributor experienced personal accomplishment as a teacher and blogger, and in their implementation of valued practices in the collective. Not only were highly ‘liked’ tweets typically celebrating successes, the successes were valued ones in the collective, such as that of student success, student perseverance, growthmindset, and advocacy for education reform. Evidencing a personal capacity to foster such successes was in turn celebrated. As such, the positive feedback loop involved in the feedback mechanism offered by ‘likes’ is shown in Figure 8.8 below, in which the amplified qualities in the tweets that get highly ‘liked’ involve pride and accomplishment in successes related to ideational coherence in the collective, which in turn are celebrated and endorsively amplified.

**Figure 8.8  Amplification through 'likes'**

Although ‘likes’ are often a gateway feedback mechanism since they take the least amount of effort and risk to make, there were some cases in which ‘likes’ remained low even though either ‘replies’ or ‘retweets’ were significant. Exploring such non-examples further supported the tendency of highly ‘liked’ content to be celebratory around accomplishments made by contributors because tweets that were highly ‘retweeted’ and ‘replied to’ but not highly ‘liked’ involved expressions of struggle and requests for help.
For instance, one contributor asked for help with a geometry problem they claimed they could not even get started on (Figure 6.25b in §6.2) and received few ‘likes’ amidst many ‘replies’ and ‘retweets’. Another contributor revealed their inability to remember how to solve a complicated algebra problem (Figure 8.9a) with a similar response. Along this vein, another contributor indicated their eight-grade student had “an incredibly shaky foundation in fractions” asking for help with how to “do triage” with them (Figure 8.9b) was similarly met with an insignificant number of ‘likes’ but with an incredibly high number of ‘replies’. The content did not lend itself to be celebrated, but rather to soliciting suggestions for how to help the student. Revealing inadequacies signalled a need for help rather than for celebration.

This also applied to content that was controversial in nature. For instance, tweets involving social justice, such as those challenging white privilege within mathematics teaching, did not attract a significant number of ‘likes’.

Calling white folks! Great post by @sophgermain Being called out is really not that bad abrandnewline.wordpress.com/2018/09/24/bei... #MTBoS #iteachmath #iteachCS

This is presumably given the controversial and provocative nature of the content, which challenged systemic racism. By ‘liking’ the content, a participant also associates themselves with the content, showing support for it. Some may wish not to associate themselves with seemingly controversial content, nor is it something to celebrate.

As such, in examining tweets that received significant numbers of ‘likes’ and contrasting them with those that did not, ‘likes’ came across as most prominently being driven by celebration, with contributions primarily involving accomplishments. Many of these involved a sense of pride around accomplishments that involved sources of ideational coherence in the collective. That is, successes were often not only personal
accomplishments, but also accomplishments of the community. As such, ‘likes’ endorsively amplified qualities of pride and accomplishment in successes related to sources of ideational coherence, driven by underlying intentions of celebration.

**Highly retweeted**

In turn, tweet clusters that involved highly ‘retweeted’ tweets were primarily oriented around things that were relevant for others in terms of holding utility and novelty. For instance, information about upcoming events and opportunities were prone to being highly ‘retweeted’ for their utility. This commonly included information about ‘chats’, which involved users gathering on Twitter under a particular hashtag to actively converse with each other synchronously during a set time frame.

Resources for teaching mathematics that were perceived as useful for others were also highly ‘retweeted’, and also typically involved sources of ideational coherence such as ‘geometry’ or ‘which one doesn’t belong’. In addition, most of the ‘highly retweeted’ resources had a certain quality of novelty in them. For instance, in Figure 8.9 below, the contributor created a ‘which one doesn’t belong’ task for ‘absolute value equations’. Their contribution could evidently be important to mathematics teachers in general since solving ‘absolute value equations’ is a common expectation of mathematics teachers. But it was also important to the mathematics teachers in MTBoS since ‘which one doesn’t belong’ activities invoke ‘noticewonder’ practices and more generally, ‘instructional routines’, which are prominently used in the space. As such, this contribution not only fit the ideational sources of coherence, it offered both utility and novelty, making it relevant for mathematics teachers in MTBoS. ‘Retweeting’ such a task could not only allow a user to save the task for their own future reference, but also give their contributions currency since it may be useful for other mathematics teachers.
Another example that similarly offered both relevance and novelty is shown in Figure 8.10, where the contributor posed a ‘geometry’ problem involving the midpoints of edges joined by the vertex on a cube along with a 3d visualization of the task. While the task itself may not have been novel within mathematics teaching, it served as a novel task in the ideational network of MTBoS because of its pairing with the particular visual representation constructed as a prompt. The prompt provided a concrete representation of the problem, which could be used directly with students as a provocation for either ‘noticing and wondering’ or ‘problem solving’.

![Figure 8.10 Example of a highly retweeted which one doesn't belong tweet]

Proving others with tasks such as this through retweets can in turn increase the value a retweeter provides the network. By associating themselves with something with high currency that is being diffusively amplified in the network, they can associate themselves with something they observe as valuable to both themselves and others in the network. So, association is then a driving mode of amplification for diffusive amplification through retweets, which are provoked by qualities of utility and novelty.

While the above two examples evidenced some novelty in the sense of offering unique pairings within the existent ideational network, other ‘highly retweeted’ examples involved more novelty. For instance, Figure 8.11 shows a tweet in which the contributor revealed a silent video showing the process of creating ‘islamic geometry’ ‘mathart’.

![Figure 8.11 Example of a highly retweeted geometry task tweet]
Figure 8.12  Example of highly retweeted Islamic geometry video

This not only fit with the sources of ideational coherence such as ‘geometry’ and ‘mathart’, but also offered a novel resource that pushed ideational boundaries. In a sense, it has an aspect of diversity in it that contributed to making the content novel while maintaining key sources of redundancy that kept it relevant in the network.

However, content that was highly ‘retweeted’ was not always so unique, and sometimes remained closer to the ideational sources of coherence in the collective. For instance, a tweet of a ‘growthmindset’ quote from Jo Boaler claiming, “Everybody can grow and change their brains and learn any level of math”, as seen earlier in Figure 7.30 in §7.3, was highly ‘retweeted’. This was likely because it was relevant for others, and those ‘retweeting’ it felt it was worth being associated with. This can be inferred since ‘growthmindset’ was such a strong source of ideational coherence in the collective and singlehandedly garnered the most amount of feedback in the system out of many other sources of ideational coherence. As such, it held value in the collective, and was therefore relevant, useful, and worth associating with. By ‘retweeting’ it, a user could also increase their social value because of the ideational currency of the content.

Therefore, not only was ‘highly retweeted’ content novel in nature, it was also useful for others in the ideational realm of MTBoS and was driven by the underlying desires for association with content valued by others in the collective. This is shown in Figure 8.12.

Figure 8.13  Amplification through 'retweets'
In contrast, contributions that did not receive significant amounts of ‘retweets’ lacked the sort of currency afforded by the utility seen across all tweets that were highly ‘retweeted’. Often, more personally oriented tweets fell in this category, where they received significant response in terms of ‘replies’ and ‘likes’, but not with ‘retweets’. For example, in one case, a contributor indicated they were going to speak to pre-service teachers about MTBoS and wanted some ideas for what to share.

Speaking to pre-service teachers on Tuesday about using Twitter and MTBoS for professional learning. Whose got something I can glean from? #MTBoS #ITeachMath and apparently #Ialsoteachtwitter

In another case, without any sort of request for help, a contributor indicated they love the thread about emphasizing failure as part of the learning process. This was still personally oriented. In fact, all of the tweets that had significant amounts of ‘replies’ and ‘likes’ but not ‘retweets’ were written from a very personal perspective that indicated either something they had tried, something they were going to try, or general expressions of gratitude for MTBoS. Perhaps these sorts of posts were too redundant or even irrelevant to ‘retweet’ to others. Or, possibly, they were not seen as things necessary or desirable to share with others. In some cases, this included more controversial topics such as the debate about NCTM fees for speakers explained in more detail earlier in §6.2. That tweet received an incredible amount of response with ‘replies’ and quite a few ‘likes’, but very few ‘retweets’. Considering that ‘retweets’ are most strongly associated with content that gives ‘retweeters’ social currency through its relevance and overall ideational value, a controversial tweet about NCTM fees may not have been something many wanted to associate with publicly even they took interest in ‘replying’ or ‘liking’ the content. This is in part because ‘retweets’ very publicly amplify content, while ‘replying’ and ‘liking’ amplify less publicly.

As such, ‘retweets’ were most significantly responsive to content that was relevant to others through utility and novelty, and that held social currency by eliciting validated ideas in the sources of ideational coherence in the collective. They also more significantly amplified tweets that invoked informative discourse indicating utility for others rather than when content was presented more personally or reflectively. And, ‘retweets’ amplified more novel content that involved some diversity in the ideational network without being overly redundant, therefore amplifying content that pushed
ideational boundaries. The diffusive amplification by ‘retweets’ was also driven by desires for association with content that could be valued by others in the network.

**Highly replied**

In contrast, tweet clusters that included many ‘replies’ primarily involved content that was in some way provocative, and therefore, worth interacting with through discussion. Provocation of ‘replies’ was most commonly stimulated with contributions that were revealing in that they exposed vulnerability and evoked curiosity. For instance, when contributors asked questions, often admitting their own shortcomings, and provided enough information for others to attempt to explain their approaches, this typically attracted ‘replies’. While requesting help came in many forms, the requests that elicited the most feedback in the form of ‘replies’ typically exposed the contributor’s vulnerability and stimulated respondents’ curiosities through asking questions that seemed familiar, yet in some way remained diverse or even controversial. One form of this was when contributors asked for help with mathematical problems, making themselves vulnerable to public scrutiny for not knowing their mathematics as mathematics teachers, but somehow feeling comfortable to do this because their questions were framed as curiosities rather than inadequacies. Examples of this were shared earlier in Figure 8.8. However, this was also seen to some extent within tweets that expressed curiosity in how others teach certain mathematical concepts.

Any suggestions for a fun game involving basic subtraction facts? @ElemMathChat #MTBoS

#alg1chat #mtbos #iteachmath How far do you go in domain and range for algebra 1? Do you cover interval notation? Do you talk about domain and range for discontinuous functions? If so, what about union and intersection?

They also sometimes requested specific resources they knew others would be able to help them find.

Somewhere, there's a graphic with the prime numbers that says something like "up to 100, check 2, 3, 5, 7, for divisibility" and then up to "new boundary," check (these primes). Anyone know the visual I'm talking about?? #MTBOS help me #Iteachmath

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79 However, not all direct requests were responded to with replies. In fact, many direct requests went unanswered, as further detailed in Larsen (2017b).
And, sometimes the provocation was not made in the form of a question, but rather a
detailed account around an experience, such as through a blog post describing an
unexpected turn that occurred during mathematics teaching (e.g., Figure 8.13).

Figure 8.14  Example of highly replied to tweet with reflective blog post
So, sharing detailed personal reflective accounts that evaluate one’s experiences
honestly and with a curiosity for growth also provoked ‘replies’, and even negotiation.
Perhaps there was simply enough material that others could connect with and relate to,
creating space suitable for negotiation.

Interestingly, although vulnerability was prevalent among all tweets that were highly
‘replied to’, there was a lack of shame in sharing the vulnerabilities. Instead, the
vulnerabilities were framed with a sense of curiosity and wonder, or a thirst for learning
from the perspectives of others who also presumably struggle with mathematics
teaching. Inadequacies were embraced and presented as curiosities, allowing for
opportunities for learning. At times, these curiosities were even controversial in nature.

I’m in need of a bit of brainstorming here, #mtbos. I had a student ask
today if we could watch the Kavanaugh hearings rather than do math,
“isn’t that more important than this?” The cynical may think this S
wanted out of math class, but I know her, she was sincere.

Open questions such as this did not necessarily have an answer, but nonetheless,
stimulated debate about how to help resolve situations presented by contributors who
stimulated ‘replies’, particularly when the ‘replies’ were in fact negotiations. While
negotiation, in terms of the back and forth dialogue between users, did not occur often in
the data set, vulnerability and curiosity were important qualities evident among all tweets
that spurred negotiation. So, content evidencing contributor vulnerability and curiosity
provoked ‘replies’, allowing for refractive amplification, as driven by desires for
interaction beyond merely ‘liking’ or ‘retweeting’. This is modelled in Figure 8.14 below.
However, not all content was highly ‘replied to’. In fact, few tweets (96/444) in the core data set had significant amounts of ‘replies’, and even fewer of these involved back and forth conversations, or negotiation (47/96). So, only about 10% spurred negotiation. Although social visibility could easily be a have been a contributing factor (explored further in Chapter 9), looking at tweets that garnered both high numbers of ‘likes’ and ‘retweets’ but not ‘replies’ shed more light on why many tweets did not solicit ‘replies’.

Among tweets that received other forms of feedback but not ‘replies’, an evident redundancy of alignment with sources of highly endorsed sources of ideational coherence was seen. There was a lack of controversy among these tweets, with no instances of pushing on boundaries. Instead, positively couched contributions were made evidencing excitement about accomplishments and lauds about resources that ‘worked’ in the classroom. For instance, Figure 8.15 shows a couple examples of tweets that did not receive many ‘replies’, but that did receive other forms of feedback.

Figure 8.15 Amplification through ‘replies’

3 days later and I am still catching kids working on this during their free time and “stealing” dry erase boards from my room to work it out. It’s definitely a keeper! #mtbos #iteachmath @ninchmaths

Solemn Alabouz @Alabouz - Sep 26, 2018

We are doing a Difference Sudoku from @ninchmaths for this week’s problem solver. I love this twist. #ITEachMath #MTBoS #Teach160 #TeachAlgebra #VTeD (ninchmaths.org/content/id/579...)

Difference Sudoku

If you’re a fan of the problems on @openmiddle (openmiddle.com) or you’ve ever considered making a problem of your own, you’ll want to download this file to help you make the images so much more quickly. bit.ly/2OdekDG #MTBoS #iteachmath

Figure 8.16 Examples of resources shared that did not receive many replies

Both pride and utility are evident in these. Figure 8.15a invokes pride and utility because it indicates the contributor had success with the activity they link to their tweet. However, they do not create much controversy that called for any sort of resolution, it was already
resolved. This is also seen in the tweet in Figure 8.15b, which is useful, relevant, and worth celebrating or sharing with others, but is not controversial because ‘open middle’ problems are well endorsed in the ideational network. What was most striking among these tweets that received other forms of feedback but did not garner many ‘replies’ was that they did not create a need for ‘replies’. Given they received other forms of feedback, they were evidently visible to others, but their contents were presented as already resolved, without much room for negotiation. Vulnerabilities were not exposed, and curiosities were not stimulated.

In summary, ‘replies’ offered diverse opportunities for feedback that provoked various forms of interaction from others ranging from supportive comradery to inquisitive negotiation. Although back and forth negotiation was rare within ‘replies’, it was a possibility unlike with other forms of amplification, and replies could also colour subsequent interpretations of a tweet. Therefore, ‘replies’ offered a refractive amplification since comments tailored what was being amplified more accurately to certain aspects of the ideas presented in the original tweet rather than to the whole tweet. ‘Replies’ offered a space of possibility for a more diverse array of activity, including the same kinds of engagement as original tweets offered as well as that ‘likes’ and ‘retweets’ offered. That is, a ‘reply’ could be a simple supportive comment similar to a ‘like’, it could be an associative comment that links to other resources similar to a ‘retweet’, or it could take on its own form through any of the previously identified forms of engagement for tweets. However, with only about one quarter of the data evidencing a significant number of ‘replies’, receiving them was rarer than may be expected in a space that offers possibilities for such interaction. Upon examination of both tweets with significant ‘replies’ and those that had other significant forms of feedback except for ‘replies’, ‘replies’ did not occur significantly unless original contributors exposed vulnerabilities, such as showing inadequacies or asking for help, with tones of curiosity for learning from others (rather than with shame). That is, the tweets that provoked refractive amplification through ‘replies’ generally presented something that was either controversial that challenged the existing ideational boundaries, or it was incomplete in some way, creating space for others to respond. Although many of the highly ‘replied’ to tweets invoked some sources of ideational coherence, some also challenged them, offering diversity in perspectives, and sometimes stimulating negotiation.
Composite cases

When tweet clusters involved two or more significant forms of feedback, they tended to involve qualities identified with those forms of feedback in a composite way. That is, rather than evidencing new qualities, they involved combinations of qualities found in tweets with only one significant form of feedback. Some of these examples have already been shown in the contrasting non-examples in each of the above three categories. However, a few more examples are shown here to reveal how these compositions of qualities were made and what forms of amplification they resulted in.

Tweets that were highly ‘liked’ and ‘retweeted’ involved qualities found in both highly ‘liked’ tweets and in highly ‘retweeted’ tweets. So, they evidenced pride or accomplishment as well as aspects of utility or novelty. For example, in Figure 8.16 below, the contributor stated excitedly that they “talked about ducks today!” This revealed their sense of pride and accomplishment for implementing a ‘fun activity’. However, they also thanked @saravdwerf, a professional development consultant who has a blog filled with resources. This implied that they gleaned the idea from her.

Figure 8.17 Example of highly liked and retweeted tweet about ducks

Further, not only did they present a sense of pride and accomplishment in implementing the task, the images included were revealing enough that they could be useful for others who might want to implement such an activity. The activity slide and remnants of student thinking was shown in the images, giving enough information for others to be able to also implement the activity. While the task itself was not necessarily novel, the idea of using ducks in the image tied with evidence of student success with the task made the tweet have more utility and novelty than if it were just an image of the task. The aesthetic appeal of the tweet is also notable, as ducks can engender a sort of humour or jest both for readers and potential students. Albeit having some novelty, the task itself would have been rather familiar to others on MTBoS since it was ideationally close with sources of
coherence, particularly around noticing and wondering, fun activities, visual patterns, and counting. So, this task was really a variation on a theme, contributing to it being endorsed with ‘likes’ and diffused through the network through ‘retweets’. It’s familiarity yet novel and useful nature made it valuable for users to ‘retweet’ and to therefore associate with it, to be considered valuable by their followers. However, the tweet did not receive much feedback in terms of ‘replies’. This is perhaps because there was not much to resolve. It was presented as a resolution already. There were no vulnerabilities or curiosities posed and did not invite active conversation. Rather, its purpose was to share success that others may experience if they try the activity shared and to reveal a light-hearted approach to mathematics.

Another example of a highly ‘liked’ and ‘retweeted’ tweet is shown in Figure 8.17 below, where the contributor shares an Desmos activity about optimizing a path across snow and asphalt that they designed. This activity may also be seen as a variation on a theme because it is a rendition of a past activity shared by Dan Meyer, the Chief Administrative Officer of Desmos, who had in the past created a similar activity about optimizing a path to get to a taco cart across sand and asphalt pathway. While some differences exist, the activity is familiar not only because of its relevance for mathematics teachers because of the content around optimization, but also its relevance for MTBoS participants, who have a shared history around optimization problems in Desmos.

Hey #MTBoS! A @Desmos activity I made a while ago for you to share.

Who loves optimisation problems as much as I do? Especially ones that could save your life.

Quick, bust out the pen and paper, its a matter of life and death! teacher.desmos.com/activitybuilder...

#iteachmath @mrchowmath

![Image](image.png)

Figure 8.18  Example of highly liked and retweeted tweet of a Desmos activity
So, the activity offers utility as a plug-and-play activity for optimization problems that others could use directly in their classrooms as well as some novelty in terms of context. This makes it very shareable and was therefore highly 'retweeted'. ‘Retweeting’ such an activity would add to one’s social value since it is a contribution that others could use directly in their practice if they are teaching optimization. The phrasing in the tweet also indicates pride in accomplishing the design of an activity in Desmos since the contributor indicates excitedly, “Hey #MTBoS! A @Desmos activity I made a while ago for you to share.” They are not claiming they made it for themselves, they are explicitly stating they made it for the community so that it can be shared. Adding value to the community is something to be proud of, and it is an accomplishment met with 'likes’. As such, it is amplified not only through diffusion through the network, but also endorsesively through being ‘liked’ and appreciated. However, as with the previous example, it is a polished product that leaves little room for discussion. It is posed as a ready-made activity that is meant to provoke engaging with the mathematics in the activity with a “pen and paper”. There are no vulnerabilities or curiosities posed that question or ask for help. Rather, it is presented as already resolved and ready to use. As such, it did not provoke many ‘replies’, but it did solicit many ‘likes’ and ‘retweets’.

On the other hand, tweets that were highly ‘liked’ and ‘replied to’, but not highly ‘retweeted’ lacked this sort of outward utility for others and the ready-to-use and resolved shareable nature. Instead, they involved a sense of tentativeness, vulnerability, and curiosity, as well as pride or accomplishment. Such tweets were not necessarily useful for others and often were not something others would have wanted to associate themselves with. In fact, they were often controversial in nature, inviting conversation and refraction rather than mere diffusion. For example, one contributor posed a challenge to the community about tweeting practices themselves particularly for when others share teaching strategies in the space.

As much as I appreciate tweets about strategies that “worked,“ I also love tweets about strategies that didn’t work so well, & what might be tried next time. This would be a much more nuanced & helpful conversation for me. Is there a hashtag for this? #iteachmath #mtbos

This contributor came across as proud to say they “love tweets about strategies that didn’t work so well” because they were yearning to learn from others and wanting the space to be open to this sort of activity. In making this statement, they were making themselves vulnerable to criticism because they were questioning the norms in MTBoS.
around ways users typically post. That is, in positive ways that show successes rather than failures or things to improve on. The contributor was also aiming to normalize and create space for being vulnerable in curious ways, in terms of thinking about “what might be tried next time”. Given the provocative nature of such a statement that exposed both vulnerability and curiosity, it solicited quite a few ‘replies’ with support for the idea, even relating it to student learning as parallel to teacher learning.

I love this thread. Anything that emphasizes that failure is part of the learning process, especially for educators.

Some even directly shared their failures in teaching in response to the provocation.

I’ve got one - after doing SO much work with non-rectangular prisms in 6th, I asked my new 7th Ss “what is volume?” They said l*w*h. Such a hard thing to un-teach (both that the defn isn’t a formula AND that formula only works sometimes).

Others suggested various hashtags for the purpose of sharing incomplete and work-in-progress ideas for teaching mathematics based on classroom experiences. Although there were not too many back-and-forth conversations, a sense of negotiation arose and a passion for shifting norms in the community were evident. The sense of pride in making such an observation and statement was met with ‘likes’, endorsively amplifying the idea by giving it fuel of acknowledgment. And, the provocative nature stimulated ‘replies’, refractively amplifying it by giving it new meanings and potentialities. However, the controversial nature of the tweet made it less prone to being ‘retweeted’ and diffusively amplified through the network. Perhaps it was not something others wanted to associate themselves with and did not see it as something they wanted to pass on to their followers. Perhaps it did not carry enough utility or novelty to share widely even though it was evidently visible based on the high ‘replies’ and ‘likes’ it received.

Another example of a tweet that was very highly ‘replied to’ and ‘liked’ but not ‘retweeted’ very much was the provocation referred to earlier about challenging the fees associated with speaking at NCTM conferences. This tweet is shown completely in Figure 8.18 below to emphasize the controversial and provocative nature of the tweet as a whole.
It represents something that was important to discuss, but that evidently was not desirable as something others wanted to associate with publicly in their personal feeds. As such, it evoked many ‘replies’, filled with various perspective and negotiations about how things should be or how the conferences are worth it. And, it also received many ‘likes’, showing the provocation was acknowledged and endorsed by others. However, ‘retweets’ remained below what would be expected for a post that gained so much traction otherwise. This is likely due to its controversial nature that does not contribute to a retweeter’s social value and is therefore not seen as something to associate oneself with even if one agrees with it or participates in negotiation around it. In fact, the negotiations stemming from this tweet were often significant in terms of not only number of replies, but also conversation depth and quality. Links were shared about NCTM processes, points were made about the value NCTM provides within their conferences, and sentiments about social justice, equity and inclusion surfaced. Below is an example of a threaded conversation, showing how three users other than the initial contributor replied to each other after being provoked by the initial contributor.

**User 1:** Yep, that is steep. I find it hard to swallow that speakers pay anything at all. The nonrefundable bit seems like adding insult to injury. Also, it seems to me that the real issue here is equity and inclusion. I would certainly feel excluded and marginalized.

**User 2:** #NCTMregionals have complimentary speaker registration, but no reimbursement for handouts/materials. Annual conference charges speakers but reimburses workshop materials (not sessions). Why the difference? Which is preferable/equitable/encourages access & participation?
User 3: Handouts/materials are *much* less expensive than registration costs. Most sessions I go to have shifted to digital handouts, so there wouldn’t be any reimbursement cost.

The provocation also presented an issue that has no easy solution, exists in an unresolved state, and is highly meaningful to many MTBoS participants since it is a form of professional development they clearly want to engage in given their attention for participating in MTBoS, but that they do not feel they have access to. Although familiar and within the realm of ideational coherence in MTBoS, the initial tweet presented both vulnerability and curiosity as well as a tentative and unresolved issue, which created space for conversation and refractive amplification through ‘replies’. However, it is something others agreed with and supported, and was therefore endorsively amplified through many ‘likes’. That is, although the content was not necessarily positive, the advocacy for reducing fees was being celebrated.

In contrast, tweets that were highly ‘replied to but not highly ‘liked’ and instead highly ‘retweeted’ continued to maintain the tentative and unresolved tone that presented vulnerability and curiosity around issues, but that did not carry messages that were prone to celebration. Instead, they offered curiosities that were useful for others but not appropriate to be celebrated. And, tweets that were highly ‘retweeted’ and ‘replied to’ but not ‘liked’ as much involved both vulnerability or curiosity and utility or novelty but not necessarily as celebratory a tone. For instance, one contributor posed a question about whether secondary mathematics teachers use manipulatives or not and which ones.

Secondary maths teachers...do you make use of manipulatives in your classroom and if so, which ones? TIA please RT #maths #MTBoS

This tweet is not posed with pride or accomplishment and is therefore not prominently responded to with ‘likes’ that show celebration. Rather, it solicits ‘replies’ filled with various links to resources and ideas for teaching secondary mathematics with manipulatives, and even explicitly asks to “please RT”. ‘Retweeting’ occurs relatively significantly here, and this is likely due to the utility in the tweet as well as the ‘replies’ that have refractively amplified it. That is, there is not only utility in the initial tweet, there is utility in many of its replies. In fact, this tweet solicited 39 replies, which is quite significant, and many of which involved negotiation. For instance, many respondents shared specifics about various resources they use in secondary mathematics, such as “multilink cubes, algebra tiles, bead strings, mini whiteboards, fraction walls, fraction
shapes, 3D shapes” and many more. Some even linked full blog posts to outlines of all their resources and how they use them. The original contributor often kept the conversations going by asking questions such as, “What is it about them that makes them work?” Her tenacity in further questioning and showing curiosity opened the space of negotiation around the idea. Interestingly, some respondents were surprised that others use manipulatives in secondary mathematics classrooms.

User 1: We have packs on the tables with Dienes, multilink, bead strings, geo board, whiteboard pen and whiteboard, dice and also laminated hundred square and Gattegno chart.

User 2: In secondary? Do you not spend all day picking them up off the floor?! Jealous.

One user even responded to the original contributor with a simple “Nope”, to which the original contributor replied, “Any particular reason why not?” The challenger then claimed, “More trouble than it’s worth in my opinion.” Clearly, this topic was familiar enough to many, but controversial enough to solicit ‘replies’. It was useful enough to solicit ‘retweets’ but was not as prominently ‘liked’ as other tweets that receive this much attention otherwise, pointing to the possibility of it not being celebratory enough for endorsive amplification. However, a space for negotiation was opened, and illuminated the various stances available around the notion of manipulatives in secondary mathematics classrooms. With the help of the original contributor provoking respondents in replies, this reveals an example of refractive amplification. The idea is amplified in a refractive way, revealing various interpretations that can arise from such a provocation.

Finally, tweets that were highly ‘retweeted’, ‘likes’, and ‘replied to’, generally involved features identified in all of the forms of feedback. That is, receiving high response in all manners involved pride, accomplishment, utility, novelty, vulnerability, and curiosity. An example of this can be seen in one of the most highly ‘liked’, ‘retweeted’, and ‘replied to’ tweets in the data set, which revealed an attempt at building a multiplication fluency activity by combining various approaches commonly encountered in MTBoS. This tweet is shown in Figure 8.19 below.
Trying to build multiplication fluency across multiple grades using an activity that combines the search for patterns and relationships (like @joicooler’s visual number chart) with lined up equations (like @mburnsmath does). Thoughts?
drive.google.com/open?id=1mLM0O... #mtbos #iteachmath

**Figure 8.20  Example of highly amplified tweet on building fluency activity**

This tweet received 570 ‘likes’, 196 ‘retweets’, and 56 ‘replies’ that ranged from various resources being shared, to negotiation about what makes a better activity and learning setting for improving students’ fluency in multiplication. Considering how the tweet was initiated, it is evident that the contributor revealed not only their unresolved issue, but also attempts at resolving the issue along with links to documents they developed and an image of student work resulting from the activity. They also involved several sources of ideational coherence in MTBoS such as visual patterns, number strings, and noticing patterns, which made the tweet relevant and familiar not only because multiplication fluency is an interest of all mathematics teachers, but also because such activities are generally valued in MTBoS itself. The tweet was also both personally and publicly oriented, with an awareness of the community, while simultaneously reflecting on their own progress in developing the activity they shared. Not only was it ideationally coherent
with MTBoS, it had elements of novelty and boundary pushing without being overly controversial, but rather, filled with curiosity for learning with others.

Other tweets that received all forms of feedback significantly similarly involved both personal and public elements, both familiar ideational coherences and unresolved curiosities, and evidence that they have put in the work to attempt resolution but that they invite others to join in their learning. For instance, one of these included the ant on the box problem (see earlier in Figure 6.25c) that revealed the contributor tried the problem with students, didn’t know the answer, but then figured it out, and now was curious how others would have approached it. The sense of vulnerability, wonder, and contribution was amplified endorsively, diffusively, refractively, evoking celebration, association, and provocation for interaction. However, such tweets were incredibly rare, and seemed to also be the result of a perfect storm of random events that coordinated to bring people and ideas together.

**Low responses**

In contrast, there were many tweets that exhibited many of the traits involved in soliciting feedback, but that did not attract much of it. This points to how the themes developed in association with the feedback mechanisms were necessary but not sufficient in evoking each form of feedback. Since feedback mechanisms on Twitter are positive feedback loops, a lack of positive feedback results in extinguishment of certain ideas that end up not being amplified. As Davis and Sumara write, “positive feedback serves to amplify (or to extinguish) some dynamic aspect of a system” (p. 102). As such, it is interesting to consider the non-examples, the tweets that did not provoke positive feedback loops and in turn, were not amplified.

Extinguished tweets often held many of the qualities involved in amplified tweets. For instance, one contributor shared, “Love this for a Notice-Wonder activity! Which batteries last longest?” and included the image shown in Figure 8.21 below.
Figure 8.21  Example of noticewonder activity about battery life

This contribution has utility for others, shows excitement about the activity that is ready-to-use and involves a prominent source of coherence around noticing and wondering as provoked by images from real life examples. Although it does not present an unresolved issue, it should have at least received more ‘likes’ or ‘retweets’. Perhaps it is too redundant in the network and the idea was not novel enough, or the contributor did not reveal that they tried it with students to show how it went and how they would improve the activity. However, it should also be noted that the contributor was indeed in the newcomer category, and simply, may not have been seen.

Another contributor who received little response was directly asking for help around a prominent issue in MTBoS: classroom culture. They stated, “Help! Need to develop a thinking/learning culture ASAP, 8 week sub job. What are “must do’s” and “let it go’s”?”. This tweet received no feedback, with only one ‘like’ by a bot that retweets anything with the MTBoS hashtag in it. It was also made by a newcomer with very few followers.

However, it also lacked revealing any attempts at resolving their issue, as seen in the highly ‘replied to’ tweets, or indications of pride as seen in the highly ‘liked’ tweets, or utility for others as seen in the highly ‘retweeted’ tweets. Not only was this contributor unseen, their tweet also did not fit into the normative behavior in the activity in MTBoS that receives amplification. Their request was also a very large issue that may not have lent itself to direct ‘replies’. Rather, learning about setting classroom culture can take time. So, while it is not an irrelevant request, it is one that has been heavily treaded in the MTBoS space and could require much effort to communicate through tweets.

A similar tweet to this was when a contributor asked, “Can someone compare Khan Academy and Delta Math for me? Strengths, purpose, usefulness. Why do you use the
one you use?” This is also a big topic that may not have been easy to reply to, and more importantly, lives outside of the predominant ideational space of MTBoS because MTBoS tends to orient around meaning making activities rather than pre-made practice platforms such as Khan Academy. Although the teachers likely used such resources, using them was not a focus in the ideational space. As such, this tweet received only one reply, which included the perspective of one other user who tweeted they use both and explained the benefits of each platform.

Many of the extinguished tweets were even more irrelevant to the ideational space of MTBoS and in different ways. In one case, the mathematical content was likely beyond what most users in MTBoS focus on as they asked, “Anyone have a Desmos activity for #parametric functions to share?” In another case, a contributor posted a photo of their ‘math t-shirt’ (seen in Figure 8.22), stating, “Worn to the gym, today.”

![Figure 8.22 Example of math t-shirt worn to the gym](image)

And, sometimes there was simply not enough novelty. For instance, tweets about kids correcting their own quizzes or examples of 3-act-task fodder was either too redundant in mathematics teacher contexts, or in MTBoS contexts.

However, some tweets fit all the qualities for being a well-responded to tweet, but that did not receive the feedback it would be expected to receive. Take for example the tweet shown in Figure 8.23 below. It involves vulnerability, curiosity, utility, pride, and even some novelty as well as fitting into sources of ideational coherence such as involving contexts in mathematical activities and using Illustrative Mathematics resources. However, the contributor was also in the newcomer social location, and therefore was simply unseen. In fact, 80% of low response tweets were made by contributors in the newcomer position (explored further in Chapter 9). Not only did they often lack qualities associated with provoking feedback, they were also invisible to the others in the space.
As such, extinguished ideas involved a combination of lacking qualities associated with forms of amplification and social visibility. The restrictions of social location on opportunity for amplification will be explored further in Chapter 9. However, the fact that many of the poorly responded to tweets lacked several of the qualities identified as associated with soliciting amplification indicates the validity of these qualities as aspects that provoke amplification. It also alludes to the possibility of a symbiotic relationship between how users present ideational content in tweets and their social location.

### 8.3 Modes of amplification: Celebration, association, and interaction

Overall, positive feedback loops in the MTBoS environment allow for a variety of ways that content can be amplified. By examining tweets that evidenced significant amounts of response through each of the three available feedback mechanisms ('likes', 'retweets', and 'replies') as well as their non-examples and compositions, qualities redundant within each category became evident. Namely, highly ‘liked’ content involved language evidencing pride and accomplishment in the tweets and an overall positive tone. Highly ‘retweeted’ content involved content that held utility and often novelty while remaining within sources of ideational coherence. And, highly ‘replied to’ content was provocative often through direct questions but also through detailed statements about classroom occurrences or general musings, most evidently exposing contributor vulnerabilities through ways that revealed a sense of curiosity for learning from others. Finally, composite cases revealed that these qualities composed with each other. So, tweets that received a significant amount of feedback in all three ways evidenced a sense of pride or accomplishment, vulnerability or curiosity, and utility or novelty. These are modelled in Figure 8.24 below.
Figure 8.24 Amplified qualities by each feedback mechanism

These identified qualities provoked each related feedback mechanism to amplify content in different ways. Namely, ‘likes’ amplified content through endorsement, in that, they did not increase the frequency of that content in the public realm directly, but they gave feedback to the contributor that their contributions were valued. In turn, users who receive ‘likes’ for content may choose to continue tweeting in such ways. And, users who see a tweet has received many ‘likes’ can validate that others have also endorsed the content and considered it valuable enough to ‘like’. ‘Retweets’ on the other hand amplified content through diffusion through the network. That is, by increasing the frequency of the content in the public realm directly. ‘Retweets’ made the content more likely to be seen by more users, and therefore, more likely to be further ‘retweeted’. It also had a bit of an endorsement effect as well since just as with ‘likes’, ‘retweets’ indicated both to the contributor and viewers that the content is endorsed and valued enough to push it to more viewers. Finally, ‘replies’ amplified content through refraction in the network. Although it also had effects of endorsement and diffusion as ‘replies’ could contain endorsements and being replied to makes the content more visible to others, they also refracted the meanings in the original tweet. That is, with new ‘replies’ attached to the content, the idea space was altered by the comments made, thus holding the power for it to be negotiated and re-shaped in the collective.

Furthermore, not only were there particular qualities in tweets that provoked feedback mechanisms to amplify content through endorsive, diffusive, or refractive forms of amplification, these were each associated with modes of amplification that underpinned their usage. Namely, ‘likes’ inferred a desire for celebration by those making the feedback towards the contributors who indicated their accomplishments and things they were proud of. There was an evident ‘cheering on’ effect that ‘likes’ created based on the
nature of the content they most prominently amplified through endorsement. Content indicating any sort of struggle or failure, unless couched within a success story, was not existent in the highly ‘liked’ categories. On the other hand, ‘retweets’ inferred a desire for association. This is because any highly ‘retweeted’ content was typically useful for others and novel in nature, making it valuable for others to associate with by passing it onto their followers. While it is possible that those ‘retweeting’ content were not consciously intending to associate with the content, the residual effect of their ‘retweet’ communicated an associative valuation of the content. That is, they were willing to be seen as associated with that content given that ‘retweets’ imply a desire to share things that others may value. And finally, ‘replies’ inferred a desire for interaction, a form of human connection. By ‘replying’, one makes themselves open to conversation and further interactivity. Even those who reply with a very endorive or celebratory comment that could have been made as a ‘like’, they are creating space for further connection with those to which they are replying. ‘Replies’ could also be associative in nature since they can link to other resources or tweets to show their value as a resource provider, but in making a reply, they open space for further interaction. Overall, ‘replies’ varied greatly, and some even garnered prolonged negotiation between users around ideas they were able to share different perspectives on. While prolonged negotiation within ‘replies’ was generally rare, the space created by ‘replies’ for such negotiation was an opportunity for interaction and human connection.

Taken together, each of the three feedback mechanisms respond to certain qualities in tweets, which provoke those qualities to continue being produced and tweets without those qualities to become extinguished. Resulting from each of these feedback mechanisms, three different opportunities for forms of amplification in the system are afforded: endorsement that is directed at those producing content, diffusion that is directed at those consuming content, and refraction that is directed at those who actively participate with the content. And, each of these forms of amplification are driven by modes of amplification, which direct users who engage in participating in using feedback mechanisms in choosing when and why to use those feedback mechanisms rather than others. Namely, users using feedback mechanisms are broadly driven by desires for celebrating, associating, or interacting. Each of these imply a yearning for human connection, revealing how the social network underpins all ideational activity. All of these abovementioned features are presented in visual form in Figure 8.25 below.
Twitter is fundamentally driven by positive feedback mechanisms that amplify content in different ways and selectively extinguish content that does not exhibit certain qualities. Positive feedback mechanisms are most useful in creating far-from-equilibrium situations in which small perturbations become important in the system. This has important implications for ideational emergence in complex systems such as the MTBoS collective. Not only are tweets that carry ideational artefacts and relations made public through reification and negotiation, emerging the ideational space, these publications are then selectively amplified or extinguished by the system through the available opportunities for amplification. This means that not only does ideational content emerge, it also has the possibility of becoming resilient. Content that becomes resilient is upheld and amplified if it invokes desires for celebration, association, and interaction in viewers as provoked by qualities in content that reveal pride, accomplishment, utility, novelty, vulnerability, and curiosity. Through such conditions, amplification occurs endorsively, diffusively, and refractively, creating opportunities for continuing and adapting to changing conditions in the ideational network. Failing such amplification, ideas become extinguished, and other ideas flourish instead. As such, the modes and forms of amplification have great implications for emergence in the ideational network of MTBoS because the quality of content is determined by the system itself. And, they are necessarily interwoven with both features in the social network and the ideational network, the inter-relations of which are further explored in the next chapter.
Chapter 9  Decentralized control

Within a structure-determined complex system, external authorities cannot impose, but merely condition or occasion possibilities. The system itself ‘decides’ what is and is not acceptable. (Davis & Sumara, 2006, p. 145)

Decentralized control is not simply a lack of controlling mechanisms in the organization of a system, but rather, that the control is distributed. Davis and Sumara emphasize that “just as ‘neighbors’ is used to refer to ideas, ‘control’ has to do with emergent conceptual possibilities” (p. 144). Since the focus of decentralized control is around the possibilities for shared knowing in the collective rather than interpersonal relationships, control can be conceived of as a form of authority, in the sense of authorship, or a capacity to act in the consensual domain, rather than dominance.

Instead of seeing it as an external and monological imposition, authority might be described in terms of capacities to invoke prevailing discourses – or, in complexity terms, to act within a consensual domain. (p. 145)

In a decentralized network such as MTBoS, with no external authority, the capacity to act is largely defined by feedback mechanisms that control what is amplified in the ideational space, and therefore, what is deemed acceptable and valued. Although amplification is identified as primarily responsive to how tweet content is presented (as described in Chapter 8), amplifying tweet content also amplifies contributors. And, since contributors live in various social locations, their views on the network can affect how they choose to tweet, in turn affecting what sort of amplification their ideas receive. Tweet visibility is determined by social locations (as defined in Chapter 7), and therefore offers a fundamental form of amplification: social amplification. Social amplification allows content to be visible through both being published and then being seen by the followers of the contributor. After content is socially amplified, it can then be endorsively, diffusively, or refractively amplified, which further determines the value of the ideas contributed, and in turn, the value of contributors themselves. Thus, there is a co-activity between social amplification and content-specific amplification, which implies their co-implied capacities for invoking authority in the network. As such, ideational content, social locations of contributors, and forms of amplification are necessarily interwoven and inter-related. Therefore, it proved necessary to more deeply explore these inter-relationships between results stemming from investigations in Chapter 6, Chapter 7, and
Chapter 8 with aims of examining how they direct authority, and therefore, how they steer ideational emergence. To this end, the following inter-relationships were explored:

a. ideational artefact frequencies in each social location
b. ideational artefact frequencies within each form of amplification
c. forms of amplification frequencies in each social location
d. social location frequencies in each form of amplification

While investigations into ideational artefact frequencies in social locations and forms of amplification (a & b above) did not yield observable implications initially (as noted in 7.1 and 8.1), they proved important after considering the results from examining relationships between social locations and forms of amplification (which are revealed in 9.2). In considering social locations and forms of amplification (c & d above), two prominent findings arose that highlighted privileges for authority available to agents who had more social amplification and content that attracted all other forms of amplification. These privileges were identified as two forms of capital: social capital and content capital. Taken together, these drive ideational authority, which steers what comes to matter more in the ideational network, as further detailed and described in what follows.

9.1 Social capital and content capital

Although social capital may be conceived of in different ways (e.g., Lin, Cook, & Burt, 2001), it is used in the context of this research as the capacity for accessing feedback mechanisms, and therefore, amplification in the collective. More simply, social capital involves the capacity an agent has over soliciting forms of amplification, and therefore the ability to foster neighbour interactions. It is necessarily limited, but not defined, by an agent’s social amplification. On the other hand, content capital involves the capacity for the ideas themselves have to attract content-related forms of amplification. As such, while social capital concerns the privileges that agents have in accessing amplification through feedback mechanisms in the collective, content capital concerns the qualities in ideas that attract amplification by feedback mechanisms in the collective. Since feedback mechanisms directly define what is amplified, content capital is therefore defined by the ability of content to attract all forms of amplification. That is, not only to be seen, but also to be highly ‘liked’, ‘retweeted’, and ‘replied to’. Attracting all of these reveals a higher reach and influence for the content in the ideational network, and
therefore invokes authority. As such, both social capital and content capital are determined by the system through forms of amplification. Because social capital and content capital are both defined by a capacity to invoke amplification, they are necessarily co-related and co-emerging, living in a symbiotic relationship with each other. This is because without accessing amplification, ideas contributed by agents cannot attract amplification. And, without ideas that hold qualities valued by the collective as indicated through feedback they attract, agents cannot improve their access to amplification. Therefore, amplification plays an important role in shaping authority.

Towards investigating how amplification shapes authority in the collective more deeply, it is important to first consider the initial conditions of what is being amplified, and how it is given authority. Fundamentally, ideas are published by agents in ways that offer neighbour interactions among ideas through reification and negotiation, as modelled in Figure 9.1 below.

**Figure 9.1  Visual model of relationship between agents and ideas**

In Chapter 6, an ideational network was constructed from all such interactions that were available in the core data set. However, it became evident in Chapter 7 that some agents were privier to the contributions of others in the system, and that they could be seen by others in the system to varying degrees. By considering these opportunities for social visibility and awareness that privileged agents in different ways, social locations were defined through in-degrees and out-degrees of following relationships within the data set (revealed in Figure 7.36 in Chapter 7). Given that leaders (those with high in-degree and high out-degree) held the highest potential for both being seen and seeing others within the data set, this group had the most social amplification since their contributions had a higher capacity for being seen by others, and to be ideationally aligned with sources of ideational coherence in the collective. Social amplification, which responds to visibility and awareness, is therefore a precursor to other content-related forms of amplification. And, social amplification makes it possible for endorsive, diffusive, and refractive amplification to respond to qualities in tweets such as pride, accomplishment, utility, novelty, vulnerability, and curiosity. This integrates findings from Chapters 6-8, and is depicted visually in Figure 9.2 below.
However, the relationship between social amplification and endorsive, diffusive, and refractive amplifications remains vague and not yet fully supported except for the basic tenet that visibility allows access to feedback. As such, a deeper consideration of the inter-relations between social locations and forms of amplification was pursued.

To this end, I first tabulated the frequencies of forms of amplification found in each social location, and vice versa. For reference, I also tabulated the frequencies of each of these across all data. In examining frequencies of amplification, I used the visual structure of the Venn diagram that was initially constructed to reveal tweet clusters falling into highly ‘liked’, ‘retweeted’, and ‘replied to’ categories of feedback mechanisms (as revealed in Figure 8.6). That initial diagram indicated the number of tweet clusters out of the complete core data set of 444 tweet clusters that were found in each region. For purposes of comparison, these values were converted into percentages and are revealed in Figure 9.3 below. As indicated, 63% of the tweet clusters received little amplification, about 20% were either highly ‘liked’, ‘retweeted’, or ‘replied to’, and only 6% attracted high values in all ways.

Treating this as a baseline to compare with, further parsing was pursued to explore these same frequencies across various social locations. To this end, the tweets in each
of the four social locations were identified according to the forms of content-related amplification they received. These results are shown in Figure 9.4 below.

Interestingly, the percentages of tweets being either highly ‘liked’ or ‘retweeted’ within social locations were about equal, with tweets highly ‘replied to’ being slightly higher than those either ‘liked’ or ‘retweeted’ within each social location. However, the overall averages of the percentages of being highly ‘liked’, ‘retweeted’, and ‘replied to’ increased gradually in the order of LL (newcomers), LH (observers), HL (influencers), and then HH (leaders). Most notably, not receiving any significant form of amplification decreased gradually in this order, with 75% for newcomers and 41% for leaders, a prominent shift from the 63% average. And, receiving a significant amount of all forms of amplification increased in the same order, with 2% for newcomers and 15% for leaders, as compared to 6% overall. Evidently, leaders held a much higher capacity for receiving amplification, with influencers closely behind them, followed by observers and newcomers with less such capacity. Given that capacity to access amplification constitutes social capital, leaders who were defined so by both their high in-degrees and out-degrees in following relationships evidenced the most social capital out of all contributors in the data set since they could provoke more amplification.

Figure 9.4  Amplification frequencies in each social location
To further explore the opportunities for amplification as related to in-degrees and out-degrees in following relationships, I also parsed the data into low and high values in each variable (in-degrees and out-degrees). In each parsing, the same process was undertaken to identify frequencies in regions. Figure 9.5 below reveals these results.

Figure 9.5  Amplification frequencies across in-degree and out-degree

Although similar increasing patterns prevailed, parsing by in-degree gave a stronger effect on receiving no significant forms of amplification (75% for low in-degrees and 48% for high in-degrees) than across out-degrees (65% for low out-degrees and 52% for high out-degrees). This was also the case in comparing the percentage values of receiving significant amounts of all forms of amplification across in-degrees, with only 3% for those with low in-degree versus 14% for those with high in-degree. The differences were not nearly as significant across out-degrees as they were across in-degrees. As such, in-degree seemed to be a stronger indicator of agent contributions being amplified than was out-degree. However, out-degree still had increasing effects. Therefore, those with both high in-degree and high out-degree, and therefore with higher social amplification, held the most social capital in the network, and presumably had greater capacity over making their ideas more important in the ideational network. Notably, in-degree is less controllable by users than out-degree since gaining followers is out of the hands of a contributor. However, out-degree, which is more controllable by agents, also improved their capacity for receiving content-related amplification.
To further confirm these results, an investigation into the prominence of each social location among forms of content-related amplification was also pursued. That is, to consider the social locations of contributors among the contributions that attracted each form of content-related amplification. To this end, overall averages were first identified to indicate the percentages of contributions made by each social location out of the 444 tweet clusters in the core data. These values are shown in Figure 9.6 below, indicating that 58% of tweets were contributed by newcomers, 7% by observers, 14% by influencers, and 15% by leaders, with 6% in the boundary region.

Figure 9.6  Tweet distribution among social locations

This served as a comparison for the same calculations within tweets that significantly attracted each form of amplification, which is revealed in Figure 9.7 below. To identify these frequencies, the tweets in each region were examined and tabulated based on which social locations their contributors were found in. The highest values in each parsing are highlighted in yellow, with secondary values in grey.

Figure 9.7  Frequencies of social locations within forms of amplification
Evidently, in almost all regions of amplification, contributions were mostly made by newcomers, although most often less than their 58% overall average. Newcomers contributed less than average in all cases except for their contributions in either the highly 'liked', highly 'retweeted', highly 'liked' and 'retweeted' or 'none' regions. In other words, newcomers were more apt to receiving either endorsive or diffusive amplification than refractive amplification and had less capacity over provoking 'replies'. This is important given that their contributions often involved soliciting advice, implying many requests for advice went unanswered. Even more jarringly, newcomers took up 80% of the contributions that received very little amplification. Therefore, newcomers had very little capacity for accessing amplification, and therefore had much less social capital than others, and in turn, their contributions also held little content capital. This further supports the presumption that social amplification is necessary for garnering further content-related amplification in the network.

Observers, on the other hand, who simply followed more users within the data set, had a much stronger capacity of receiving at least one other significant form of amplification. In particular, they had a higher than average contribution within highly ‘retweeted’ content, and also within content that was both highly 'liked' and 'replied to'. They were also never only highly 'liked', but additionally invoked other forms of amplification. This may have had to do with their vigilance for aligning ideationally and revealing accomplishments in practice. This same sort of pattern occurred for influencers, with their content being significantly ‘retweeted’ and often also highly ‘liked’ and ‘replied to’. This may have related to their attention to contributing ideas that others could use in practice, and therefore content others wanted to associate with.

In contrast, leaders were most likely to make contributions that received either a significant amount of ‘likes’ and ‘retweets’, ‘likes’ and ‘replies’, or all forms of amplification. Leaders were also more likely to attract significant numbers of ‘replies’ than other social locations, so their contributions were more likely to be refractively amplified. Not only did they take up the most significant portion of regions involving high ‘replies’, they also took up more than their average amount of contribution in those regions. This may have related to their attention to building community and advocating opinions or stances, which in turn, invoked conversations as well as other more celebratory and associative forms of support. As such, leaders seemed to have more control over their capacity to evoke amplification in various ways, perhaps intentionally.
And, all of their contributions received at least one significant form of amplification. This further supports their social capital in terms of their authority to invoke amplification, which typically not only invoked ‘likes’ and ‘retweets’, but also ‘replies’. However, it is noteworthy that their social capital did not directly relate with the content capital in their contributions. That is, while their social capital gave them access to amplification, not all of their content attracted all forms of amplification. This is perhaps because of their often far-from-equilibrium nature of pushing boundaries, particularly when advocating certain controversial ideas. Such attempts at harnessing their social capital for social change may have at times resulted in not receiving as many ‘likes’, but instead, provoking ‘replies’ and ‘retweets’, as is somewhat evident in the above data.

These results highlight not only the presence and authority of agents with social capital and ideas with content capital, but also the relationship between social capital and content capital. Social capital allows contributors to harness their privilege and push boundaries, which is not necessarily accepted or taken on by the system in all manners. On the other hand, content capital signals to contributors what the system accepts in various ways. That is, what others endorse, associate with, and interact with gains more resilience than when only some of these are evoked. However, boundary pushing is an important aspect of maintaining resilience in the collective since it allows for randomness to surface and the space of possibility to expand. Without trying out a new idea, however rogue it may be, it is impossible to know how the system will take it up. As such, social capital and content capital work together to direct and steer ideational activity that lives in the relation of agents and ideas in the collective. This is modelled in Figure 9.8 below, which extends Figure 9.2 by including social capital and content capital, as mediated by amplification of agents and ideas, respectively, along with associated processes.
However, the effects of social and content capital need to be considered not only distinctly, but also in relation to each other as well as in their combined effect on the ideational network as a whole, which is explored in what follows.

9.2 A skewed network

Since social capital and content capital direct ideational activity, it is interesting to consider exactly how they did so in the snapshot of data collected for this thesis. The ideational network developed in Chapter 6 evidenced certain ideational artefacts that had higher frequencies than others. However, these frequencies were taken from all possible contributions without considering who contributed them, and how they were taken up by the system. As such, my inquiry turned to examining the content produced only by contributors with high social location, and only with all forms of amplification. These were at first considered independently, and then together.

First, to consider the impact of social capital on the ideational network of MTBoS, I began with the top ten ideational artefacts based on frequency in the whole ideational network (as initially shown in Table 6.1). I then took only contributions that were made by leaders (a total of 66 tweet clusters) and recalculated frequencies of ideational artefacts appearing in this new set of tweet clusters. This resulted in a list of frequencies for ideational artefacts. Taking the top ten ideational artefacts according to frequency
among all content versus only content contributed to by leaders, who hold social capital, resulted in evident differences. These results are revealed in Table 9.1 below.

**Table 9.1 Influence of social capital on top ten artefacts by frequency**

<table>
<thead>
<tr>
<th></th>
<th>Top ten in whole network</th>
<th>Top ten with social capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>↓ engaging students</td>
<td>mtbos</td>
<td></td>
</tr>
<tr>
<td>↓ fun activities</td>
<td>real life examples</td>
<td></td>
</tr>
<tr>
<td>↑ mtbos</td>
<td>algebra</td>
<td></td>
</tr>
<tr>
<td>↓ geometry</td>
<td>geometry</td>
<td></td>
</tr>
<tr>
<td>↓ games and puzzles</td>
<td>social justice</td>
<td></td>
</tr>
<tr>
<td>↓ using technology</td>
<td>visualizing math concepts</td>
<td></td>
</tr>
<tr>
<td>↑ visualizing math concepts</td>
<td>hands on learning</td>
<td></td>
</tr>
<tr>
<td>↑ hands on learning</td>
<td>noticewonder</td>
<td></td>
</tr>
<tr>
<td>↑ real life examples</td>
<td>engaging students</td>
<td></td>
</tr>
<tr>
<td>↓ using manipulatives</td>
<td>fun activities</td>
<td></td>
</tr>
</tbody>
</table>

Note: Arrows indicate change in rank from whole network to only the one with social capital considered. Shaded artefacts indicate differences between lists.

By applying the influence of social capital on the top ideational artefacts in the network, certain artefacts evidently became more important. Namely, ‘algebra’, ‘social justice’ and ‘noticewonder’ were significantly more prominent in the contributions of those with more social capital than when considering the overall ideational network with all contributions. Conversely, ‘games and puzzles’, ‘using technology’, and ‘using manipulatives’ decreased in order of frequency. And, other artefacts shifted in importance between the two lists in response to applying the influence of social capital, as indicated by the arrows in the above table. Most of the artefacts in the social capital list had increased in importance, except for the most notable downwards shift of ‘engaging students’ and ‘fun activities’. These moved from being most frequent to least out of the top ten. Instead, ‘mtbos’ and ‘real life examples’ became most prominent. Such shifts indicate the topics for which leaders advocated. What this shows is that although ‘engaging students’ and ‘fun activities’ were found most prominent when broadly considering all contributions in the network, these were not necessarily being advocated for or even seen in the network as widely. Instead, topics that were either more controversial (e.g., ‘social justice’) or that involved community building (e.g., ‘mtbos) were elevated through social capital.

Further, in considering the impact of content capital on the ideational network of MTBoS, I began with the same top ten artefacts from the whole network, but now considered the effects of content capital by taking only contributions that attracted all forms of content-related amplification. Within the contents of these contributions (a total of 28 tweet
clusters), I recalculated the list of frequencies and took the top ten for comparison with the original network. These results are revealed in Table 9.2 below.

**Table 9.2  Influence of content capital on top ten artefacts by frequency**

<table>
<thead>
<tr>
<th></th>
<th>Top ten in whole network</th>
<th>Top ten with content capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>↓</td>
<td>engaging students</td>
<td>visualizing math concepts</td>
</tr>
<tr>
<td>↓</td>
<td>fun activities</td>
<td>assessment</td>
</tr>
<tr>
<td>↓</td>
<td>mtbos</td>
<td>engaging students</td>
</tr>
<tr>
<td>↓</td>
<td>geometry</td>
<td>geometry</td>
</tr>
<tr>
<td>↓</td>
<td>games and puzzles</td>
<td>reasoning</td>
</tr>
<tr>
<td>↓</td>
<td>using technology</td>
<td>students discussing and debating</td>
</tr>
<tr>
<td>↑</td>
<td>visualizing math concepts</td>
<td>using manipulatives</td>
</tr>
<tr>
<td>↓</td>
<td>hands on learning</td>
<td>using technology</td>
</tr>
<tr>
<td>↓</td>
<td>real life examples</td>
<td>conceptual understanding</td>
</tr>
<tr>
<td>↑</td>
<td>using manipulatives</td>
<td>problem solving</td>
</tr>
</tbody>
</table>

Note: Arrows indicate change in rank from whole network to only the one with content capital considered. Shaded artefacts indicate differences between lists.

As may be observed in the above table, different artefacts became more prominent when considering the effects of content capital. Namely, ‘assessment’, ‘reasoning’, ‘students discussing and debating’, ‘conceptual understanding’, and ‘problem solving’ became increasingly prominent based on how the system as a whole amplified these. In turn, ‘fun activities’, ‘mtbos’, ‘games and puzzles’, ‘hands on learning’, and ‘real life examples’ did not get amplified as prominently. Many of the artefacts with more content capital increased in importance as a result of amplification, with only ‘engaging students’ and ‘using technology’ decreasing slightly in rank. However, most of the top artefacts in the overall network decreased in importance. What is interesting about this result is that amplification evidently plays an important role in privileging certain content over other content, and points to the power in the collective to determine which ideas have authority. Once the ideas are visible enough to the collective, its decentralized organization allows for feedback mechanisms to determine authority and resilience. In this case, there is a clear indication that topics that are generally laden with value within mathematics teaching contexts but that pose challenges around implementation (e.g., ‘assessment’, ‘reasoning’, ‘problem solving’) received more response in the system than those that evoked replication of similar material (e.g., ‘fun activities’, ‘hands on learning’).

Evidently, the effects of social capital were slightly different than those of content capital in privileging artefacts in the network. Social capital gave more authority to ideas advocated for by those with high social locations while content capital gave more
authority to ideas that impacted users in the system. In considering how these influences compared with each other, I put both the social capital list and the content capital list side-by-side to compare similarities and differences. This is shown in Table 9.3 below.

Table 9.3 Influence of social capital compared with content capital

<table>
<thead>
<tr>
<th>Top ten with social capital</th>
<th>Top ten with content capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>mtbos</td>
<td>visualizing math concepts</td>
</tr>
<tr>
<td>real life examples</td>
<td>assessment</td>
</tr>
<tr>
<td>algebra</td>
<td>engaging students</td>
</tr>
<tr>
<td>geometry</td>
<td>geometry</td>
</tr>
<tr>
<td>social justice</td>
<td>reasoning</td>
</tr>
<tr>
<td>visualizing math concepts</td>
<td>students discussing and debating</td>
</tr>
<tr>
<td>hands on learning</td>
<td>using manipulatives</td>
</tr>
<tr>
<td>noticewonder</td>
<td>using technology</td>
</tr>
<tr>
<td>engaging students</td>
<td>conceptual understanding</td>
</tr>
<tr>
<td>fun activities</td>
<td>problem solving</td>
</tr>
</tbody>
</table>

Note: Shaded artefacts indicate differences between lists.

Only three artefacts remained in common between these two lists: ‘geometry’, ‘visualizing math concepts’, and ‘engaging students’. This means contributions involving these three artefacts were not only amplified socially but also endorsesively, diffusively, and refractively. Therefore, they were upheld by both social capital and content capital. These three artefacts were also found in the top ten artefacts for the ideational network without any influences of social capital or content capital applied, albeit with different ranks. Therefore, these three artefacts evidently prevailed as strong sources of ideational coherence in the collective that defined the space. Their overall stability prevailed regardless of various influences pertaining to social location and feedback mechanisms. As such, they could be considered part of the identity of MTBoS.

Conversely, other artefacts shifted significantly in importance depending on influences applied. In comparing artefacts valued more highly by social capital with those more with content capital, the duality and interplay between these influences is further highlighted. Namely, social capital revealed an advocacy by leaders for boundary-pushing topics such as ‘social justice’, resource-creating topics such as ‘real-life examples’ and ‘noticewonder’, and community-building topics such as ‘mtbos’. On the other hand, content capital revealed values of the collective around contentious practice-oriented topics such as ‘assessment’, ‘students discussing and debating’, and ‘problem solving’, and highly celebrated and useful practice-oriented strategies such as ‘using manipulatives’ and ‘using technology’. Although specific ideational artefacts are always
shifting in prominence over time, these dominant features of boundary-pushing, resource-creating, and community-building aspects being given authority through social capital, and contentious and celebrated practice-oriented strategies being given authority through content capital indicate the defining living tension within the collective. That is, the tension between practice and community.

Taken together, these influences steer the ideational network towards sources of ideational authority in the system, which in turn, can drive further activity. Not only is there an ideational network created by all contributions made in the space, there are processes that determine which ideational material lives, and which dies. As Davis and Sumara indicate, “positive feedback serves to amplify (or to extinguish) some dynamic aspect of a system” (p. 102). As such, certain ideational content gains authority, while other content loses it. Therefore, towards identifying what social capital and content capital influenced in terms of ideational authority in the system, I combined their influences by taking all contributions that were either made by leaders or were highly amplified in all ways and recalculated the frequencies. A new list was created by taking the top ten artefacts by frequency as emerging from content that was given authority by both social capital and content capital and comparing it to the original list of top ten artefacts from the uninfluenced network. These are shown in Table 9.4 below.

Table 9.4 Influence of content capital with social capital on top ten artefacts by frequency

<table>
<thead>
<tr>
<th></th>
<th>Top ten in whole network</th>
<th>Top ten in network with authority from social or content capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>↓ engaging students</td>
<td>mtbos</td>
<td></td>
</tr>
<tr>
<td>↓ fun activities</td>
<td>mtbos</td>
<td>visualizing math concepts</td>
</tr>
<tr>
<td>↑ mtbos</td>
<td>geometry</td>
<td></td>
</tr>
<tr>
<td>↑ geometry</td>
<td>real life examples</td>
<td></td>
</tr>
<tr>
<td>↓ games and puzzles</td>
<td>engaging students</td>
<td></td>
</tr>
<tr>
<td>↓ using technology</td>
<td>using manipulatives</td>
<td></td>
</tr>
<tr>
<td>↑ visualizing math concepts</td>
<td>assessment</td>
<td></td>
</tr>
<tr>
<td>↑ hands on learning</td>
<td>noticewonder</td>
<td></td>
</tr>
<tr>
<td>↑ real life examples</td>
<td>students discussing and debating</td>
<td></td>
</tr>
<tr>
<td>↑ using manipulatives</td>
<td>using technology</td>
<td></td>
</tr>
</tbody>
</table>

Note: Arrows indicate change in rank from whole network to only the one with content capital considered. Shaded artefacts indicate differences between lists.

Although such calculations could have been made in various ways, combining the content that was promoted either by social capital or content capital and calculating the frequencies of artefacts among these proved to be a viable manner with which to reveal
the compounded effect on the ideational network from both influences. Evidently, most of the artefacts listed on the right side of the above table were promoted by the effects of both social capital and content capital, except for ‘engaging students’ and ‘using technology’, which fell in importance overall. ‘Fun activities, ‘games and puzzles’, and ‘hands on learning’ also became less important overall. In the data, these were identified as highly reifiable artefacts, which often were contributed to with content that came across as overly redundant. Perhaps the redundancy decreased their attractiveness for provoking feedback. Or, they were simply ways for newcomers to engage in the space.

On the other hand, ‘mtbos’ rose to the top of the list, and ‘assessment’, ‘noticewonder’, and ‘students discussing and debating’ moved up into the top ten list from less prominent positions. It is noteworthy, that these artefacts were overall highly representative of deeply held interests in MTBoS, as evidenced both in the data and in my views and fieldnotes as an insider. They were also highly negotiated within the data as well as in my personal encounters in the space. Perhaps this is not surprising in the case of ‘assessment’ since ‘assessment’ is typically tension-laden and something many teachers struggle with in the general context of mathematics teaching, particularly when they take interest in reform approaches to teaching. This is in part due to the competing demands between desires, professional expectations, and mandates that mathematics teachers navigate (Rouleau & Liljedahl, 2017). In the data, it attracted interest from various users including both newcomers and leaders and evidently overlaps between the interests of mathematics teachers in general and the MTBoS community. This was also the case for contributions in the ‘students discussing and debating’ since the idea of having students discuss and debate in mathematics classrooms is a value projected by reform approaches to teaching. One that is often communicated implicitly by phrases such as ‘be more student-centered’, ‘avoid being the sage on the stage’, and ‘get the kids to do the talking’ commonly heard among reform-oriented educators. In this data, contributions to ‘students discussing and debating’ also attracted various social locations and serves as an example of a general topic that is primarily rooted in classroom practice that has moved into the online space.

In contrast, ‘noticewonder’ is more of an internally curated idea. Although it is advocated for by various resource collections and professional development consultants, it is an example of resource-generating structure that is adaptable to curricular needs while providing guidance for implementation. As such, much of the curation of resources for
‘noticewonder’ activities is developed in the MTBoS space and brought into classroom spaces. This sort of adaptable structure is therefore highly capable of influencing classroom practice, as evidenced by various tweets revealing mathematics teachers who took ‘noticewonder’ resource examples from MTBoS and implemented them in practice with students. The ideas behind ‘noticewonder’ are also in line with other values such as those relating to notions around ‘growthmindset’. And, not only does it provide a template for how resources for teaching mathematical ideas can be designed and implemented, it also provides a template for how to engage in the MTBoS space by offering such resources.

It is noteworthy to add that this sort of practice-oriented resource-creation structure has been upheld in other ways in my years of experience as an insider in MTBoS. In the past, similar structures dominated the discourse and offered this same sort of function. Namely, ‘3 act tasks’ initiated and curated by Dan Meyer, ‘fraction talks’ initiated and curated by Nat Banting, ‘visual patterns’ initiated and curated by Fawn Nguyen, ‘wodb’ curated by Mary Bourassa, and many others. However, over time, as their resource collections grew, often initiating their own dedicated websites, their redundancy made them less central in the ideational network, and therefore less amplified. However, these sorts of structures seem to prevail in this space. More recently, in mid-2019, the idea of ‘menu tasks’, initiated and curated by Nat Banting, have gained such traction. As such, resource-generating activity structures seem to prevail within MTBoS and contribute to defining its identity as a collective. They serve as anchor points for both contribution to the collective and bridging the gap between the online space and classroom practice and are evidently determined by both social and content capital.

Since the specific artefacts themselves are constantly in-flux depending on the activity occurring in the network during various points in time, my interests in this investigation were more concerned with the general attributes valued and the processes involved in this valuation than the specific artefacts per se. Although specific artefacts that gained authority in this particular time frame may not be always so authority in the network, that other artefacts that involve such features may gain authority and resilience during other time frames. Although it is possible that content with less authority continues being produced in the network even without receiving amplification, the lack of energy provided by amplification can make susceptible to being extinguished over time. However, it is also possible that some artefacts that evidence less authority may simply be ways for
newcomers to begin to engage in the space and to initiate their journey along the trajectory of participation in the collective. Also, some content becomes overly familiar if too redundant over time and may go dormant, living deep within the shared histories of participants. However, given appropriate provocations fueled by new diversities brought in by agents, it may be re-awakened in the system. For instance, ‘3 act tasks’ is an example of this. It used to be extremely popular in MTBoS, but it did not evidence such prevalence in the data set collected in this thesis. However, there are times when a new contribution towards a ‘3 act task’ is made, and its familiarity serves as a launching point for others to engage in negotiating how to tinker with what already exists. That is, both redundancy and diversity are necessary for amplification to occur, and the diversity cannot become useful without adequate redundancy (Larsen & Liljedahl, 2017). This sort of attention ultimately involves both social capital and content capital, but also the nature of the artefacts themselves. Overall, in considering the features evident in artefacts that increased in frequency through the effects of social and content capital, it is evident that practice-related, resource-related, and community-related artefacts were most valued and had the most capacity for authority in the network. That is, to attract neighbour interactions beyond that of simply what becomes visible, but also that which is endorsed, diffused, and refracted. The energy provided to ideas by such amplification gives them resilience and is ultimately informed by the contexts experienced by their contributors.

9.3 Ideational authority: Social capital and content capital

In summary, social amplification gave leaders more content capital, which predicated their privilege to accessing forms of content-related amplification, which in turn, amplified various qualities in content either endorsively, diffusively, or refractively. Content that attracted all forms of amplification held more content capital because it had more ways in which ideas were amplified. Therefore, ideational authority is determined not only by who contributes the content, but also how the content is amplified and taken up by the system. As such, ideational authority is determined by both social capital and content capital, which in turn, inform each other. Taking these both into consideration, along with modes of engagement and amplification that drive forms of engagement and amplification as well as the qualities involved in provoking these, a synthesized conceptualization of these processes is presented in Figure 9.9 below.
The diagram above builds on the agents and ideas relation first shown in Figure 9.1, to which forms and modes of amplification were layered onto in Figure 9.2, and to which social and content capital and their associated qualities were then appended to in Figure 9.8. The key addition here is that of ideational authority, which is directed by all the previously diagramed processes. Taken as a whole, the perspective offered by this concluding diagram shown in Figure 9.9 reveals the synthesized results of this research, which highlight and nuance the affordances of a network that involves decentralized control, high agential density, diversities and redundancies among agents, and the capacity to allow for ideational neighbour interactions to occur easily. Ultimately, decentralized control shifts the locus of authority to emergent processes in the network that drive social capital and content capital, which in turn, direct what comes to matter in the system. In MTBoS specifically, as based on the segment of time examined in this thesis, agents gain social amplification based on their visibility and awareness of the collective as driven by their activity in engaging in social responsibility and ideational alignment. This social amplification privileges certain users more than others, giving them more social capital to make certain ideas more important in the network than others. In turn, feedback mechanisms determine which content receives which other form of amplification, with a limiting condition of being socially amplified. Based on
certain qualities such as pride and accomplishment, which prompt endorsive amplification through ‘likes’, utility and novelty, which prompt diffusive amplification through ‘retweets’, and vulnerability and curiosity, which prompt refractive amplification through ‘replies, content capital is established by the system. Endorsive amplification is driven by desires for celebration of content, diffusive amplification is driven by desires for association with content, and refractive amplification is driven by desires for interaction with content. Through these forms and modes of amplification, the system determines what content receives more capital in terms of capacity for resilience in the system. Therefore, the co-action of social capital and content capital steers ideational authority, and in turn, contributes not only to further ideational emergence, but also to ideational resilience. I only differentiate between these because there are ideas that have longevity after they have emerged. And, in the observations made of data in MTBoS, both social capital and content capital are necessary aspects to promoting the longevity and resilience of ideational artefacts that emerge in the complex collective. As such, these processes model the intertwined relationship between individuals and the collective, pointing to the necessary connective tissue between levels of observation.

Davis and Sumara indicate that complex adaptive systems depend on positive feedback loops, and that as a result, they necessarily tend towards states that remain far-from-equilibrium. Such far-from-equilibrium behaviour is directed by forms of amplification and allows the collective to adapt to changing conditions.

One of the most deeply entrenched assumptions of analytic science is that dynamic systems tend toward equilibrium, toward a steady state. More specifically, the assumption has been that systems in motion must be governed by negative feedback mechanisms, by which extreme variations in activity are somehow pushed toward and held in acceptable ranges. Negative feedback is a means of maintaining an internal equilibrium, even when external conditions might be volatile and uncontrollable. The opposite of negative feedback – predictably known as positive feedback – can be profoundly troublesome to a system that seeks equilibrium. (p. 102)

The effectiveness of the MTBoS space at offering processes that allow for such far-from-equilibrium adaptive activity challenges traditional approaches to professional development and questions their adaptive capacity. Namely, in what other settings can teachers of mathematics and mathematics education leaders congregate with such density and have naturally emergent opportunities to build ideas that gain authority and resilience as determined by their own feedback rather than by governing bodies?
Chapter 10  Conclusions and implications

Taken together, the inquiries and conceptualizations presented in this thesis have far-reaching implications and provide a multi-faceted view on the nature of professional activity around mathematics teaching. While the setting studied is in some sense opportunistic, it offers a window into the naturally emergent professional activity around mathematics teaching that is uninfluenced by any external administrative bodies or researchers. Having the ability to observe such a setting without aiming to control it in any governing way has afforded deeper insight into the driving features that seem to propel and sustain activity in the setting. These findings have important implications for those who wish to design and provide professional development opportunities for teachers of mathematics. While the context studied in this thesis may not be replicable due to its size, density, and natural emergence, findings from this research reveal features that drive professional activity around mathematics teaching as found flourishing within the Twitter medium. Although the medium itself creates unique opportunities for communication, the effects of which may need to be explored in further research, it is interesting to consider some of the findings as relevant for more traditional, in-person contexts. For instance, how do the features found in this social media context occur within in-person contexts within which they may not have yet been explored? Given the evident challenges around professional development for mathematics teaching, it behooves the field of mathematics education to take into consideration the naturally emergent professional activity around mathematics teaching that happens largely without influence of researchers, consultants, or governing bodies.

The structure of this thesis allowed for a three-dimensional view on MTBoS data, with foci on the ideational network (Chapter 6), the underlying social network (Chapter 7), and the feedback mechanisms (Chapter 8). At the end of each of these chapters, concluding remarks and theoretical contributions were revealed. In Chapter 9, these findings were woven together to consider their inter-related implications towards theoretical contributions around the driving forces behind ideational activity in MTBoS. However, further discussion as to the meaning and implications of these results is warranted. As such, at this point, it is worth revisiting the research questions that were initially presented at the end of Chapter 3:
RQ1: What is the nature of ‘MTBoS’ and how does it emerge from ‘the people of MTBoS’?

RQ2: How and why does ‘MTBoS’ invoke a sustainable form of professional activity around mathematics teaching?

These two questions pertain primarily to learning about MTBoS in terms of what it is, how it emerges, and how it sustains. While these questions were mostly addressed by the synthesis revealed in Chapter 9, I now refine these results by presenting them through a set of themes that together attend to both of these questions. I then address each question specifically with reference to these themes. Additionally, since my initial more global interest in this study was around what can be learned from MTBoS activity towards highlighting the nature of professional activity around mathematics teaching, I also subsequently include a section on learning from MTBoS. In this section, I consider what sorts of implications the results from pursuing the abovementioned two research questions have for various stakeholders such as professional development leaders, mathematics education researchers, and even MTBoS itself. Therefore, this chapter reveals both conclusions and implications for the field.

10.1 Learning about MTBoS

By considering the synthesis of findings from each of the three core perspectives that were taken on the data and then laying these against the key considerations highlighted within the related literature in Chapter 2, I first draw out the most significant learnings that have become evident through this study about the nature of MTBoS.

An ideational network around mathematics teaching

One of the most prominent findings of this research lies in the ideational network developed from the data collected, which identifies the needs and interests of mathematics teachers who engage in professional activity around mathematics teaching in an unconstrained and unmandated setting. Given that one of the key findings from research on professional development for mathematics teachers indicates that activities need to reflect and be driven by teacher needs and interests (Lerman & Zehetmeier, 2008), this ideational network can serve as a window into these needs and interests without researcher influence. By systematically coding ideational artefacts as well as the ideational relations between them, and then attending to their frequencies of occurrence,
the space of possibility of unmandated communication about mathematics teaching is revealed. Although the findings in this thesis are representative of only a short snapshot of time in the complex life of MTBoS, given the random selection out of a large quantity of data, it is nonetheless worthy of consideration. And, by attending to the meta-attributes of these prominent artefacts and relations, implications may be offered about the nature of needs and interests in discourse around mathematics teaching in general.

In the case of this research, it became evident that some of the most prominently communicated ideational artefacts during the week of September 21-28, 2018 included 'engaging students', ‘fun activities’, ‘mtbos’, ‘geometry’, ‘games and puzzles’, ‘using technology’, ‘visualizing math concepts’, ‘hands-on learning’, and ‘real life examples’ (as per Table 6.2), as well as relations between these (as per Table 6.1). In light of the vast literature that highlights the various aspects of what mathematics teachers ought to ‘know’ to be effective practitioners (e.g., Ball et al., 2008; Davis & Simmt, 2006; Selter, 2001), these prominent ideational artefacts reveal the kinds of ‘knowing’ that are of most interest to those communicating about mathematics teaching. Not only does the ideational network reveal the most prominent artefacts, it also indicates the inter-woven connections between them and the relations that hold them together. While the intent of this research is not to emerge a new view on mathematics teacher knowledge or knowing, a by-product of developing an ideational network around mathematics teaching from MTBoS activity may offer potential insights for this body of work. That is, the ideational network may provide a view into the kinds of ‘knowing’ that seem to be supportive of mathematics teaching practice, especially given the high number of self-reports of improvements in teaching practices attributed to engagement in MTBoS.

As such, in examining the ideational artefacts and relations in terms of the meta-implications they may offer, various categorizations may be considered. For instance, one of the most commonly referred to categorizations of teacher knowledge is suggested by Ball et al. (2008), who propose that mathematical knowledge for teaching consists of subject matter knowledge and pedagogical content knowledge, each of which are further parsed into finer-grained categories. Loosely, this offers a sort of content-practice bifurcation that can be observed among artefacts in the ideational network. For instance, ‘geometry’ pertains to subject matter knowledge whereas ‘hands-on learning’ involves more pedagogical content knowledge. The ideational network also reveals these are being related with each other in various ways. For example, the prominent
ideational relation between ‘geometry’ and ‘visualizing math concepts’ bridges subject matter knowledge and pedagogical content knowledge. However, many of the most prominent relations (shown in Table 6.1) do not attend to subject matter knowledge, but instead, live within pedagogical content knowledge (e.g., in the ‘hands-on learning’ and ‘using manipulatives’ relation). This is perhaps because content is often a vehicle for communicating pedagogical strategies within MTBoS contributions.

Another parsing that may be considered is that proposed by Selter (2001), who claims that in order to be effective, mathematics teachers need to develop an awareness of practical, didactical, psychological, and mathematical components of mathematics teaching. The practical involves general teaching strategies for any subject matter, the didactical involves sequencing subject-matter for delivery, the psychological involves an awareness of student thinking and learning, and the mathematical involves mathematical awareness on its own. Selter (2001) characterizes these components as coherent, integrated, and difficult to pry apart; the components simply offer various foci a teacher can attend to. In considering these, they are all observable in the ideational network of MTBoS. For instance, ‘students on whiteboards’ is a practical component, but relates with ‘visible student thinking’, which attends to a more psychological component. Or, ‘creating new activity resources’ is didactical in nature, and is often paired with ‘real-life examples’, which involves more of a mathematical focus. Through thousands of these relations, all four of these components are being brought together in different ways, and in turn, the resulting meanings are enriched and co-implicated. Overall, this contributes to a more integrated representation of discourse around mathematics teaching. Although all four of the components are evidently being addressed by the hundreds of different ideational artefacts, in looking at the top fifteen ideational artefacts shown in Table 6.2, most of them are practical or didactical in nature, with only a few alluding to psychological or mathematical foci. This is interesting since it further characterizes MTBoS as a space for focusing on the pragmatics of mathematics teaching.

Another view on knowledge and knowing for mathematics teaching that is more attentive to the complex co-implicated and dynamic nature of such phenomena is proposed by Davis and Simmt (2006). They propose mathematics-for-teaching as involving more stable categories of knowledge, including mathematical objects and curriculum structures, and more dynamic aspects of knowing, including classroom collectivity and subjective understanding. These aspects are organized as ‘branches’ of the same tree.
While the level of nuance offered by this framing may not be so evidently observable in the ideational network, a loose attempt at identifying them within the ideational network reveals a strong orientation in MTBoS towards non-mathematical discourse. More specifically, in looking at the top ideational relations in Table 6.1 and the top ideational artefacts in Table 6.2, most of these artefacts pertain to structures developed for delivering content in ways that make mathematics enjoyable for students, help teachers develop relationships with students, and help teachers connect with each other. While there are instances of in-depth exploration of mathematical objects or curriculum structures, these are often working as vehicles for the more prevalent inter-personal objectives of being seen by their imagined audience as ‘good mathematics teachers’.

For instance, the most commonly occurring ideational relation is between ‘engaging students’ and ‘fun activities’. While there are diverse mathematical ideas within the tweets that make up this relation (such as geometric properties in puzzles, flexible reasoning around long division, or logical deduction within games), the prevailing intentions behind these posts come across as generally vectored towards revealing one’s competence at engaging students and effectiveness as a reform-oriented teacher. Also, in sharing such activities, there is a sense of connecting with other professionals in MTBoS and becoming part of the collective through such posts. While there are foci on developing subjective understanding in instances of mathematical problem solving (particularly in those in which negotiation became active), these are not as commonplace as the more general ideas around pragmatic approaches to practice. And, while classroom collectivity is attended to in some discourse around groupwork, it is also not so frequently occurring, and such instances often appear to be a means for connecting with other educators who also implement groupwork. Therefore, the nature of professional activity around mathematics teaching in MTBoS seems to most prominently involve contextualized resource exchange that highlights pragmatic approaches to teaching and serves as a means for connecting like-minded professionals.

By attempting to observe Davis and Simmt’s (2006) aspects of mathematics-for-teaching in the ideational network of MTBoS within the timeframe analyzed, it becomes evident that the loci of discourse in MTBoS is not centered around mathematical experiences, but rather, is more concerned with bridging mathematical teaching experiences with the professional network of MTBoS. That is, not only are MTBoS participants engaging in discourse that seems to help them feel better at mathematics teaching, they also
evidently work at improving as participants in MTBoS through fitting into the collective interpersonally. While Davis and Simmt developed their four aspects of mathematics-for-teaching from working with a cohort of in-service teachers who met every few months to discuss strategies for teaching mathematics, the context in MTBoS is quite different and therefore can offer new insights. MTBoSers autonomously navigate a digital context without any pre-determined purpose or researcher guidance. This difference may account for the shift in discursive foci. Although MTBoS participants ground their activity in personal teaching experiences, as is the case in other instances of in-service teacher professional activity, they are also simultaneously navigating the digital realm and the social structures that support it. This points to the necessity of considering the social relations within such collectives, and how they influence ideational activity.

Taken together, none of the abovementioned theorizations around mathematics teacher knowledge or knowing suggest connecting with a professional collective and learning how to network amongst each other as a component. This may be because such organizational aspects are often decided on by professional developers, or possibly, because the timescale of these studies focuses on activity occurring after such social navigation has taken place and participants have found each other. But in MTBoS, activity can occur without reciprocated social interaction, and finding others is more demanding. By investigating MTBoS, this study highlights how in natural conversations around mathematics teaching, discussion about how to connect with each other and acknowledgement of each others presence, is prevalent. And, social relations are intertwined with ideational activity, and play an important role in occasioning emergence.

Consequently, in the ideational landscape of MTBoS, artefacts may be considered very broadly as content-related, practice-related, or connection-related. These themes are evidently intertwined and corelated within the ideational discourse and are suggested here as a loose parsing for the purpose of summarizing the results and for contrasting the findings from the ideational network constructed in this study with prior literature. Through this parsing, the ideational network reveals a space of possibility that moves beyond activity strictly oriented around content- and practice- related aspects of mathematics teaching, but that also includes the interpersonal realm. The presence of such aspects among contributions reveals that beyond the practice-related and content-related ‘knowing’ that mathematics teachers are expected to have capacity over, perhaps more attention is needed in incorporating connection-related capabilities. In
MTBoS, connection-related aspects are evident not only in how resources are provided and how classroom practices are revealed, but also through direct statements about MTBoS as a welcoming place for sharing about mathematics teaching experiences. Such connection-related skills seem to propel and motivate sustainable activity around mathematics teaching and may be more necessary than we think in fostering improvements to mathematics teaching practice. While literature on professional development points to the necessity of collaboration, literature on mathematics teacher knowledge does not focus on this aspect of building strategies for collaborating with colleagues. Perhaps more attention to this inter-personal bridge is therefore warranted.

Further, it is important to highlight the various ways in which ideational relations can be made beyond what they are since the focus of this research is on the processes involved in sustaining activity around mathematics teaching. As discussed in Chapter 6, agents produce statements that communicate certain ideational artefacts, which are related with each other in different ways. Through being made either explicitly or implicitly, and revealing either ideational convergence or divergence, ideational relations can be made through active or inert reification or negotiation, with reification emerging from convergence, and negotiation emerging from divergence. That is, not only are ideas related, they may either be positioned in tension with each other, or in an accepted and unchallenged agreement. Interestingly, most of the relations in the ideational network involved reification, and only some instances involved negotiation. The implications of this are that communication around mathematics teaching in MTBoS is often agreeable in nature and public confrontation or diversity of opinions are a rare occurrence. This may relate to findings generally characteristic of the Twitter platform indicating it is either an ‘echo chamber’ (Sunstein, 2001) or sometimes, a ‘refraction chamber’ (Rieder, 2012), where ideas are often in convergence with slight divergences that can shift the public ‘sphere’ in certain directions. What this research adds to this, however, is that by examining the ideational relations for divergence and convergence, it is possible to identify when there is more potentiality for negotiation. Namely, if ideational artefacts are being related with inert negotiation, where differences in meaning exist that are not addressed, there is an opportunity for negotiation to occur given a ‘perfect storm’ of conditions that precipitate it. The implications of this for MTBoS itself is that more attention should be placed on questioning meanings in order to foster ideational growth.
Overall, the various possibilities for relations create a space of possibility for artefacts to live along a continuum between negotiation and reification. Artefacts with more negotiated relations around them reveal more negotiability, and those that have more reified relations around them evidence reificability. By attending to the relations around an artefact, it can illuminate the ideational directions towards which the ideas can be either negotiated or reified. The states of reification and negotiation also both serve important but different purposes in the overall ideational network. Reification provides stability, while negotiation offers dynamism. Interestingly, the ideational space in MTBoS seems to be primarily oriented around reification, more so than negotiation. That is, many prominently recurring artefacts revealed reification, while only some evidenced negotiation. While this may have to do with the specific time frame, it is possible that it is reflective of the whole space and that it is what contributes to its sustainability over time. For instance, in this data set, artefacts such as ‘hands-on learning’, ‘real-life examples’, ‘fun activities’, ‘games and puzzles’, and ‘engaging students’ were either strongly reified or evidently reifiable. These artefacts were taken-as-shared in the collective and continued to be unquestioned. This is not to say that they should be questioned, as they are clearly important aspects of mathematics teaching practice that are focused on helping students develop capacities for mathematical thinking, but the fact that they were still prominently tweeted about while being so taken-as-shared remains intriguing. Perhaps their taken-as-sharedness provides a means for participation that is likely to be accepted and unchallenged, creating a comfortable option for engagement that is more prone to attracting affirmation and validation. And, if teachers have a desire for affirmation, which is a prominent feature of social media environments overall due to the strong ‘like-culture’ that prevails in such settings (Burrow & Rainone, 2017), perhaps tweeting about practice in ways that could lead to confrontation becomes undesirable.

However, some instances of negotiation in the network revealed that not all activity in MTBoS is replicative and taken-as-shared in nature. In particular, ‘problem solving’, ‘geometry’, ‘using manipulatives’, ‘visualizing math concepts’, and ‘noticewonder’ were actively negotiated. These negotiations were typically generative of a span of approaches towards these ideas, and particularly revealed the interests of MTBoS to offer diverse perspectives around implementing inquiry-based practices in mathematics teaching. Some of this negotiation was about implementation strategies, such as with ‘visualizing math concepts’, while others were in fact negotiations that were
mathematical in nature, such as with ‘geometry’. Although negotiations were not framed as oppositional arguments (except in one case in which there was debate about costs of NCTM conferences), they typically involved a give-and-take of various ideas offered in efforts to help ‘brainstorm’ around the topic. Most interestingly, even during engagements around a mathematical problem, conversations quickly turned from mathematical content-related discourse to practice-related discourse in which contributors applied the mathematical strategies evoked through engaging in problem solving to imagined student-related scenarios. That is, when engaging in mathematics themselves, participants tended to steer discussions back to how such problem solving could be fostered in their students within their respective classroom settings. This is an important result since professional development initiatives sometimes aim for including opportunities for mathematics teachers to engage in doing mathematics as learners before unpacking experiences as teachers (e.g., Davis & Simmt, 2006; Liljedahl, 2016).

As such, the emergence of an ideational network around mathematics teaching not only reveals how ideas form in the MTBoS space along the continuum between negotiation and reification, but also leaves by-products that inform the kinds of topics that mathematics teachers take interest in and the activities they are willing to entertain. To this end, the results of this thesis reveal that teachers of mathematics who engage in unmandated professional activity take interest in finding practical implementation strategies for inquiry-based practices (such as those that involve noticing and wondering or problem solving), connecting with each other socially, and acting as problem solvers of mathematics towards honing their own problem-solving abilities. Another implication is that through engaging in problem-solving, conversations among teachers may naturally draw towards implementation strategies and teaching practices. This may be particularly useful to know for those wishing to design professional learning environments, and aligns with some of the efforts of organizations such as PCMI (Park City Mathematics Institute), which offers grants for mathematics teachers to join a multi-week daily program that engages them in problem solving activities throughout the summer. While this sort of in-depth setting may not be suitable for everyone, the tendency of aiming towards practical strategies for implementation of practices that facilitate inquiry-based mathematical activity in classrooms is evidently a need and interest among teachers of mathematics who seek unmandated professional learning opportunities.
Moreover, in terms of the nature of mathematics observed within activity in MTBoS, it is evident that mathematics is treated as a space of possibility and that there is interest in examining not only single options for solutions, but multiple pathways, perhaps as is necessary for guiding and supporting students’ mathematical thinking. This attention to exploring multiple pathways is evident in episodes of mathematical problem solving, where contributors do not stop at a single solution, but continue by exploring various approaches and visualizations that could help enhance the solution space. It is also evident in the heavy attention on ‘noticing and wondering’ within activities proposed for use in mathematics classrooms as well as in the strategies for practice that are suggested in comments. And, within the activities that are designed and shared online, particularly those that were well-attended to (such as the broken calculator example that specifically emphasizes multiple solutions, or various Desmos activities that likewise aim to solicit various solutions), a general attention to revealing a space of possibility is taken. Lastly, the focus of attending to multiple pathways seems also to be a means for making and supporting relationships with students. Many contributions emphasized the importance of listening to student thinking, and valuing and attending to their emergent ideas, as a way to connect with students. Although considering the nature of mathematics in MTBoS is not a primary aim in this research, it has emerged as a consequence of exploring communication about mathematics teaching in MTBoS. It is possible, as well, that these emphases on inquiry-based strategies in designing mathematical learning situations are heavily driven by the mandates of the Common Core State Standards (National Governors Association Center for Best Practices, 2010) that many teachers of mathematics in the U.S. are working to abide by and aim towards. Perhaps the presence of this document even brought them to reaching out to a more global community towards exploring the ways in which other teachers adapt to these standards. Nonetheless, a prominent feature of MTBoS as indicated by the ideational network is that of embracing diverse approaches to mathematical thinking. This is important in considering efforts and designs of professional learning initiatives.

Finally, while it is not clear exactly how practice is impacted by engagement in MTBoS, many of the self-reports and the tweets that reveal practice through media indicate at minimum a greater willingness to try activities that others have shared about. In some sense, the residual effects of living within the constantly evolving personalized public of MTBoS creates a sort of ambient awareness and affiliation with the frequently
reoccurring artefacts in the network. For example, if ‘engaging students’ in ‘fun activities’ is frequently seen, it may be more likely a teacher will begin to view doing this as normal and accepted even if their local contexts communicate otherwise. The opportunities for recurrence of ideational artefacts and relations within this environment highlight the unique capacity of social media for mobilizing ideas. This capacity fundamentally allows MTBoS to be a complex collective driven by self-organized bottom-up activity and holds opportunities for emergence of novel ideas within activity around mathematics teaching.

**Feedback loops as governing structures**

In any self-sustaining complex adaptive system, feedback allows for it to adapt to emergent conditions and maintain its ongoing evolution. To this end, it is interesting to consider the ‘walking tree’ metaphor illustrated by Resnick (2003), who describes the processes that allow for the ‘walking tree’, which is said to grow in Costa Rica, to ‘move’;

> How does the tree move? The roots act as a type of evaluation system, searching for good soil for the tree. If there is good soil on the north side of the tree, the roots on that side dig in deeply and hold firmly. If the soil on the south side isn’t as good, the roots on that side remain shallow and weak. As the roots on the north side become stronger and deeper, the whole tree gradually shifts toward the north, pulled by the strong roots in that direction. As the tree moves, new roots grow around the new location, some of them extending even further to the north. (p. 43)

Resnick uses this metaphor to highlight what he refers to as ‘ecological strategies’, which allow for systems to be responsive to local conditions and adaptive to changing conditions. He notes, “ecological strategies might seem inefficient and indirect, but they tend to be simple, flexible, and robust” (p. 44). He also highlights that such strategies oppose metaphors of Newtonian physics, and instead, embrace phenomena as complex systems that are evolutionary in nature. Through the metaphor of a walking tree, he proposes ‘thinking like a tree’ as a way to conceive of thinking and learning as complex phenomena rather than centralized endeavours dictated by leaders. The essence of this metaphor is feedback. The roots gain positive feedback in places where they can thrive, and in turn, the system shifts towards those sources of vitality. Davis and Sumara (2006) also emphasize the important nature of positive feedback loops, which amplify certain features in a system and allow for small perturbations to matter greatly. Feedback is therefore essential for a system to be resilient and adaptable to changing conditions. However, literature on professional development settings does not focus heavily on
feedback mechanisms. This may be because much of the professional development literature attends to contexts that are designed and implemented by researchers who often embed soliciting feedback in the design of their contexts by offering space for collaboration or processes such as journaling and reflecting. While such activities are generally found effective in stimulating thinking about teaching practices among participants, they necessarily require an organizer to provoke engagement. In contrast, there are no evident organizers in MTBoS since authority is distributed. Instead, the feedback processes are embedded in the communication tool and serve to control and direct ongoing activity. Upon closer investigation of the implications of these processes, this research reveals just how crucial feedback loops are in propelling and sustaining activity around mathematics teaching. While it is not possible to see what the system would be like without these forms of feedback, their presence cannot be ignored.

More specifically, the positive feedback loops embedded in the Twitter platform create different forms of amplification that have come to be used in certain ways by users. Taken together, this research reveals how these amplification mechanisms are pivotal in establishing ideational authority, directing subsequent activity, and in turn, maintaining the collective’s sustainability and resilience (as discussed in Chapter 9). All of this has to do with the ways in which tweets are made and the responses these certain qualities trigger, which then feed back to contributors and the collective simultaneously. Namely, as revealed in Figure 8.25, tweets that evidence pride and accomplishment trigger ‘likes’, which are a form of endorsive amplification made through a mode of celebration. Tweets that evidence utility and novelty trigger ‘retweets’, which are a form of diffusive amplification and are driven by a mode of association. And, tweets that evidence vulnerability and curiosity trigger ‘replies’, which are a form of refractive amplification and are driven by a mode of interaction. These may combine to create compounded effects, driving ideational resilience. Therefore, the positive feedback loops such as those produced through features offered by Twitter (i.e., ‘likes’, ‘retweets’, and ‘replies’), can act as governing structures within a bottom-up emergent collectives such as MTBoS. While the uses of these tools have become socially constructed within the Twitter medium, which has evolved to support the ways in which users have chosen to use the medium (Dorsey, 2019), they are also in some ways very specific to the nature of the collective of MTBoS in its focus on mathematics teaching. Therefore, the results around feedback loops in MTBoS are interesting to consider in contrast to what may occur in a
more physical professional learning setting focused around mathematics teaching or within professional publication settings that have different opportunities for feedback.

In traditional professional development settings, there is typically a central organizer who holds status as a governing entity. Even if they aim to become part of the collective and to distribute their power among agents, they continue to be viewed as an authority. In MTBoS, there are other ways in which authority may be established; however, no one person is appointed as being in control of the setting. Instead, in MTBoS, control is distributed and determined by forms and modes of amplification. The forms of amplification available, which endorse, diffuse, or refract ideas in the network, were found in this research to be driven by modes of amplification, which involve celebration, association, and interaction. It is worthwhile noting that not only were there three different ways in which ideas could be amplified, there were three corresponding driving needs and interests that supported their use. Namely, the driving factors behind amplification were that of celebration, association, and interaction. In considering these findings amidst the literature on professional development around mathematics teaching, it is intriguing to consider how new contexts for professional learning may embrace the needs of mathematics teachers to celebrate, associate, and interact with other teachers’ ideas. That is, it is worth entertaining how systemic constraints can be implemented in professional development settings as feedback mechanisms that mobilize ideas so that they may be engaged with in these various ways. And, in how these constraints allow for endorsement of ideas, diffusion of ideas, and refraction of ideas.

One particular professional development and classroom practice design that offers such features that suggests constraints for mobilizing ideas in the setting, as suggested by these aforementioned forms of amplification, is found in Liljedahl’s (2016) work on building thinking classrooms. In particular, he suggests the use of vertical non-permanent surfaces, on which visibly random groups of three work to develop their mathematical thinking around certain prompts given by the organizer. While there is an organizer in this setting, the aim of defronting the room and other features proposed in his framework allow for a more de-centralized organization within such settings. Having experienced these sorts of settings in both secondary mathematics classroom and mathematics teacher professional development contexts, I have been privy to observing the opportunities for not only the diffusion of ideas through the publicly visible but locally oriented whiteboard sites, but also the endorsement and refraction that can occur.
because these ideas are visible and decentralized. The visibly random groups also aim
to support random interactions among participants, which in turn, allow for celebration,
association, and interaction. When used with teachers, this sort of setting allows for a
group to not only experience mathematics as learners, but also to move discourse
towards mathematics teaching after experiencing mathematics as learners. And, this
discourse can be mobilized through these various forms and modes of amplification.
This is merely one example and many others may exist, but I share it with an intent of
alluding to the potential transferability of these more detailed findings from MTBoS into
other sites of professional learning that embrace collective co-activity. That being said,
certain features available within Twitter are not replicable within a physical setting, such
as the distorted sense of temporality and the density of ideas that are public and
permanent, yet relatively difficult to encounter through intentional searches. Therefore,
while some features such as those pertaining to knowledge mobility, feedback loops,
and bottom-up organization are informing for physical settings, Twitter still fundamentally
offers a different space of possibility for such activity around mathematics teaching.

It is worth also noting the contrast between the MTBoS context and its embedded
feedback loops with that found in the professional scholarship of mathematics education
researchers, who publish ideas, and sometimes interact around them. Although such a
setting is quite different since in professional scholarship, publication moves much more
slowly than within the Twitter context, considering the forms and modes of amplification
supported by sites that collate researcher publications, such as ResearchGate, can
prove as a fruitful avenue for subsequent investigation. The key difference that becomes
evident between professional scholarship and micro-publishing sites is that within
professional scholarship, the feedback loops prevent many ideas from being visible
since reviewers and editors dictate what becomes mobilized in the public sphere.
However, sites such as ResearchGate or Academia are involving more social media
networking tools that aim towards supporting endorsement, diffusion, and interaction
among scholars. Although many publications continue being protected behind paywalls,
there is a growing interest in making publications more openly available. Nonetheless,
the genres differ quite significantly in terms of not only the pace at which publication and
feedback is made, but also in length and depth of each publication unit. Research
papers involve more depth and nuance as well as attention to political, ontological and
epistemological assumptions, whereas this sort of discourse is most often omitted within
the Twitter genre. This means that much of Twitter content either avoids nuance or hides it within hyperlinks, making information more focused on being presentable and consumable. In some sense, then, the feedback afforded within Twitter is attending to the presentation of content as well as the content presented.

‘Viral’ content around mathematics teaching

The possibility for positive feedback loops that amplify certain ideas in significant ways also breeds the possibility for certain content to become ‘viral’. In literature pertaining to social media in general, it is evident that social media settings have the power for triggering collective activity, such as in political movements and grassroots organizations. These movements are often stimulated by content that goes viral. Findings suggest that viral content typically involves high-arousal emotions (Berger, 2011, 2013; Berger & Milkman, 2012) and generally makes people look good if they share it, holds practical value, stimulates curiosity, involves stories, links to readers’ daily lives, and are public so others can imitate (Berger, 2013). Also, it needs to be seeded to a small initial population diverse enough to push content into different well-connected social subgroups (Kaplan & Haenlein, 2011). While such findings are generally useful for broad-ranging topics and popular media, findings from this study suggest that some of these aspects are more nuanced within discourse around mathematics teaching.

Namely, in the case of MTBoS data, traits such as pride, accomplishment, utility, novelty, vulnerability, and curiosity triggered different forms of amplification, which were essentially indicators of virality. Therefore, the kinds of triggers involved in viral content within a knowledge-oriented collective such as MTBoS can involve tempered versions of Berger’s (2013) findings. For instance, while utility is not on its own triggering of a high-arousal emotion, it may be emotionally arousing for someone who finds joy in acquiring resources for mathematics teaching. Or, maybe a certain vulnerability that an educator reveals (e.g., sharing a failed attempt at helping a student) may evoke an emotional arousal in a reader since they may resonate with the associated feelings expressed. Further, it is evident from this research that MTBoS content becomes amplified if it is useful and stimulates curiosity. In the case of this collective centered around mathematics teaching, resources that were easily implementable and were presented with enough detail about how students responded to it were more prone to being amplified. Not only were these highly amplified tweets emotionally provocative, they also
involved specificity, novelty, and in some way, stimulated curiosity. Therefore, by sharing it, a user could 'look good' in the collective. Perhaps this explains the prominence of mimicking in tweet structure and content among tweets in the data since mimicking can give a sense of fitting into what one’s imagined audience seems to desire.

The notions around initial seeding populations are also interesting to consider within MTBoS context since once a tweet is retweeted a certain number of times, it is more likely to continue being retweeted within local networks. However, if everyone who retweets the tweet is in the same social subgroup, it may not continue to be retweeted much further. Therefore, while Kaplan and Haenlein's (2011) findings are useful in marketing contexts, they prevail differently within a tight-knit collective of educators focused on similar ideas. The notion of an initial seeding population among several different social subgroups translates to a MTBoS participant being seen by many different other MTBoS users who are part of their own respective tight-knit networks. Therefore, viral content involves not only the message, but also the messenger and their social location in the network. This reveals the need for both social capital and content capital (as discussed in Chapter 9), which together steer ideational authority. In some sense, it is this ideational authority that can direct an idea to go ‘viral’.

While viral messages were found to be overall rare when considering the sheer number of tweets that received no amplification, they were nonetheless powerful in their effects on the ideational network. For instance, in the data analyzed in this thesis, one particularly viral idea was around ‘growth mindset’ thinking. Not only was this idea highly retweeted, liked, and replied to, it was also supported with high social status since it was associated with Jo Boaler. As such, it made users look good by sharing it, or versions of it, as implemented in their contexts. It also created a sense of need in contributors for attending to the ways in which they implement growth mindset practices with their students. That is, by being a viral idea, it imposed a sort of expectation for subsequent engagements. Therefore, there was a sort of ripple effect in the network, resulting in a sort of ‘fragmented news’ type structure. Haphazardly, messages would involve ‘growth mindset’ ideas either explicitly or implicitly, and overall, they were generally well-received and applauded. A similar effect also occurred in response to a more specific teaching strategy for assessment in mathematics that valued learner discourse, referred to as the ‘no pencil test’ approach (Figure 10.1), which received a significant amount of attention.
In this approach, students are asked to discuss their mathematics tests with each other without any writing utensils before writing them. As evident in Figure 10.1 above, this idea garnered much attention, and included an outstanding amount of negotiation among respondents, attracting both expressions of excitement about it and diversities of opinion as the idea challenged norms around assessment practice. However, it was also an example of a practice that emphasized ‘growthmindset’ approaches to teaching, moving the popular theoretical framing into a pragmatic one. This implicit relation with ‘growthmindset’ thinking seemed to propel its popularity. And the user who proposed it was also identified as a leader in the collective. Therefore, the idea had both social and content capital. Consequently, it continued to consistently appear not only throughout the data collected but throughout many months following the week of data collection. By tracing the idea over time through searches within Twitter, it became evident that this idea was not only viral during the data set, but also incredibly resilient over time. Contributions around it not only responded to the initial tweet, but came up as independent tweets, with many mathematics teachers indicating proof that they implemented the strategy with their students (see Figure 10.2 below).

I tried @howie_hua’s idea of allowing Ss to stand w/ their test & no pencils & talk strategy before they sat down to work. I like that it’s not as much assistance as a partner test, but involved great math conversations & made students feel more confident in their knowledge.

I saw an idea of a 5 minute test talk. I tried it before geometry students took their test on proofs. Great conversations! #thevikelife

(a) October 16, 2018
(b) October 20, 2018
After I handed out the Ss assessment, I let them chat with their peers about things they noticed and wondered to see if they could work through a few things. No pencils allowed. I saw this from someone on the #MTBoS! I heard great math conversation from Ss! Thanks for the idea!

Giving students time to collaborate before an evaluation - no pencils, no calculators. Honouring the #thinkingclassroom collaboration we do in class each day.

Figure 10.2  Howie Hua's 'no pencil test' strategy as continuous and resilient

This idea became so viral that it received attention from Edutopia, which solicited the originator to write an article about the strategy (Hua, 2019), in which Hua attributes the initial source as coming from an in-person co-teaching experience with late, Diana Herrington, who used the strategy with pre-service elementary school teachers to help relax them before exams. Interestingly, attribution of the idea to an originator becomes skewed, as seen in the tweets shown in Figure 10.2, with only the tweet in (a) indicating attribution to @howie_hua, and the others indicating they gleaned it from somewhere in the #MTBoS. This further supports the notion that on Twitter, ideas are privileged over people, simply with how content is presented. However, who posts the idea matters to the extent that the idea is visible and possible to mobilize. Without being carried by such social capital, even ideas with content capital may not catch on. Also, this particular idea satisfies all of the traits identified as related with content that gets amplified. It challenges institutional norms, making it reveal the contributor’s vulnerability, it is framed in a way that stimulates curiosity and taps into emotive aspects of teaching, and it signals novelty and utility in how specific enough it is to be implemented into practice. Therefore, carried by both social and content capital, as well as the serendipitous sequence of events, this idea became incredibly resilient and consequently influential on teaching practice.

However, before moving on, the converse case of virality should also be highlighted. While not all ideas catch on in this way, some do catch on but then die-out. Evidently, over time, they either become swallowed into the historical vortex of MTBoS, sometimes being brought back when conditions prove necessary, or they simply cease receiving attention all together. Perhaps it is because they are no longer novel or necessary in the
Taking into consideration Resnick’s (2003) ‘walking tree’ analogy, it is as if the roots begin amplifying other directions instead of the ones that give it no more nutrients. One example of this sort of ideational die-off is that of the hashtag wcydwt, which stands for ‘what could you do with this’. The hashtag used to be popular with many posts being made with ‘real-life examples’ that could be transformed into mathematics teaching activities. However, over time, by receiving little attention, contributors began abandoning the hashtag. While tweets like this continue to prevail in other ways, the idea in the hashtag has in some ways ‘died-off’, or possibly, transformed into other patterns of activity in MTBoS.

@nomad_penguin Definitely a candidate for #wcydwt. #MTBoS
@DavidKButlerUoA What is #wcydwt?
@nomad_penguin What can you do with this? It used to be common in the MTBoS. Less so, now.
@AlexOverwijk Used to post photos with this hashtag but never got much. #wcydwt
@mathhombre I’m still a fan. And talk about a great race...

Given the potentialities for influence over mathematics teaching discourse and classroom practice, it is crucial for mathematics education research to attend to how ideas can become viral among mathematics teachers, leaders, and educators. The power of viral messages should not be ignored, and this research suggests lessons we may learn in mathematics education from other fields such as marketing and political science. Namely, that viral content requires features that contribute to social and content capital, as paired with mobilization through happenstance encounters.

Underlying social structures

Further, while much interest in mathematics education focuses on the ideational possibilities stemming from interactions, this research highlights the social structures that support ideational interactions and emergence. That is, not only are the messages important, the messengers and their social locations within the emergent social landscape of MTBoS also matter. These social structures are necessarily different in nature than physical settings because of the possibilities afforded by the platform that allow for asynchronicity, publicity, and unreciprocated relationships. That is, physical and temporal constraints do not bind activity, allowing for the emergence of social structures that are fundamentally different than in physical settings. Within physical professional
development initiatives, strategies for involving all participants present are often used, and social norms develop that dictate certain forms of activity. However, in MTBoS, hundreds, if not thousands, of participants can engage in publicizing or reading publicized musings asynchronously, which means that MTBoS reveals a very different kind of setting. As such, it is interesting to consider the novel social structures that can emerge and contribute to new social norms that can in turn direct ideational activity.

In particular, the unreciprocated following relationships can provide insight into social locations. In this research, I identified four social locations based on internal incoming and outgoing following relations. Rather than defining a user’s social location by their total number of followers and followings, I focused on the incoming and outgoing following relationships only within the set of users examined. These results indicated a strong core of users who followed each other back, indicating the potential for a community. Guided by the key concepts from communities of practice (Wenger, 1998), and more specifically, the notion of legitimate peripheral participation (Lave & Wenger, 1991), I aimed to uncover the various levels of peripherality and centrality in the social network that supports ideational activity in MTBoS. However, within these notions of peripherality and centrality, there is an implication that there is one periphery and one core. This generally works in a physical setting since there are positions in which participants may observe the community’s activities within apprenticeship roles of learning to participate, or they may act more centrally within the activities of the community. While physical settings allow for this, MTBoS seems to offer a different social structure, one that nuances these views of peripherality and centrality.

By attending to the natural constraints that indicate social location, which involve two dimensions, one around visibility (or how much one is seen by others in the collective) and the other around awareness (or how much one can see of others in the collective), I defined four social locations (described further in Chapter 7): newcomers, observers, influencers, and leaders. In general, newcomers and observers have lower capacities for visibility than influencers or leaders since they may not be seen by others in the collective. However, they have varying capacities for awareness of others since newcomers have not yet followed many others within the collective, while observers do. Likewise, influencers and leaders have higher capacities for visibility since they are well followed by others within the collective. However, they have varying capacities for awareness of others in the collective since influencers are not following many of their
followers back, while leaders are. In this way, the ‘periphery’ becomes bifurcated into newcomers and observers, and the ‘core’ becomes bifurcated into influencers and leaders. In the case of newcomers and observers, even though neither of these groups are very visible in the collective, observers have more access to the more central activities of others in the collective than do newcomers. This was also reflected in the ways in which these groups participated: newcomers tweeted very diversely and with a position closely focused on mathematics teaching itself, while observers tweeted more with a position that revealed their awareness of sources of ideational coherence in the collective. Therefore, even though observers did not have a strong capacity for accessing social interactions (as revealed in Chapter 9), their ideational content was signalling an identity of being part of MTBoS. That is, they did not have as much social capital, but they were aiming for improving this social capital by tweeting in ways that seemed to have more content capital in general. While this may not have been a conscious effort, it could have been a residual effect of following others in MTBoS.

Newcomers, on the other hand, had no capacity for either social or content capital and their musings were largely ignored. Therefore, the periphery can be considered nuanced by differences in level of engagement towards finding and following those within the core of the collective. This same difference can be seen within the more highly visible influencers and leaders. Leaders keep up with maintaining an awareness of the ongoing developments in the collective through following incoming members, while influencers remain settled in their followings and therefore being less prone to attune to ongoing developments within the evolving sources of ideational coherence in the collective.

In light of social media literature on lurking (Lai & Chen, 2014; Nonnecke & Preece, 1999; Preece et al., 2004) and around ideas pertaining to a legitimate peripheral form of participation (Lave & Wenger, 1991), this nuanced bifurcation of the periphery and the core contests the often-presumed singular nature of these social locations. Perhaps this added level of nuance can help explain how ‘lurkers’ can become ‘posters’ and the intermediary phases involved in such a trajectory. While literature on lurking in social media settings emphasizes the necessity and affordances for those who lurk (even though the term was seen more negatively in the past), it still treats lurking as a single category in which users read and glean from public musings for their own individual purposes without giving back. And, although some research suggests various factors involved in both forms of activity (i.e., Lai & Chen, 2014), it does not offer an
intermediary phase. However, the research presented in this thesis, as most specifically revealed in Chapter 7, suggests that there is a continuum of awareness that an agent may have about the more internal occurrences in the collective, which seems to nuance the possibilities for peripheral and core membership. This is possible because of the inherent publicity of all activities, but also because in order to participate in MTBoS, one needs to first find it, which involves finding the people who engage in it. Taken into a physical setting, this illuminates that participants may come into an established community as newcomers or observers. When they are newcomers, they may not be aware of the inner activities of the collective since they may not know the discourse or norms. As they experience the collective, they begin to gain access into the language and activities, making them more likely to participate in a way that mimics what they see. This sort of mimicking behaviour was most evident in the observer group, through their attempts at forming an identity of being part of MTBoS. It seems that this may be a necessary stage for participants, to tweet in ways that seem to gain amplification for others in efforts to also receive such a response.

Interestingly, in a personal interview with Lisa Henry, one of the founding members of the Twitter Math Camp organization, she indicated that newcomers come to MTBoS and often end up forming social pockets among themselves. This suggests that as participants engage in the social collective of MTBoS, they seem to find each other and create relationships that contribute to the overall collective as made up of various pockets of social networks, which further contradicts the notion that there might be a single core. In fact, in looking at the ideational content of artefacts that leaders tweeted about, even though they were very highly connected socially with each other, the ideas were rather diverse. Therefore, even though social ties indicate a more central social location, there is also the possibility that the social landscape is not only determined by following relationships, but also by general trends of interest. In earlier more preliminary work in which I examined hashtag relations, I suggested that various pockets of interest may determine certain user groups (Larsen, 2017a). While this claim would need further empirical study, it loosely points to how MTBoS allows for diversity in ideas to prevail and to attract various users. For instance, certain hashtags, such as #MicrowaveMath, #lessonclose, #clotheslinemath, #smudgedmath, or #vnps, have emerged over the years of my participation in MTBoS and have attracted very specific groups of users who engage deeply in producing new ideas within the constraints offered by the hashtags.
So, not only are social relations important, they are innately intertwined with the ideational content the users contribute and take interest in. This also relates to the notions suggested by social media literature that indicate tendencies towards homophily in public spheres such as those living within social media settings (Colleoni et al., 2014), and is supported by Mcpherson et al.’s (2001) statement, that “similarity breeds connection” (p. 415). That is, when users congregate and self-organize around certain pockets of interest, they develop enough redundancy to understand each other, while hopefully offering enough diversity to keep the ideas developing. The presence of diversity allows for sustainability and allows for such spheres to ‘refract’ (Rieder, 2012) over time, and therefore, to occasion emergence.

Nonetheless, the social locations offered in this thesis aim to offer a broad categorization of social positions that are possible to have within MTBoS, and how these related with the participation the users in the groups revealed in contributions. As noted, newcomers had high diversity and closeness only to the practice of mathematics teaching without an awareness of the sources of coherence in the collective and observers tended to signal more of this sort of awareness, indicating they are part of MTBoS and their contributions are worthy of attention. Further, the distinction between influencers and leaders offers a nuanced perspective of participation in the core of a collective. While both influencers and leaders are visible to others in the collective, they may attend either more or less to the work of others in the collective. Influencers, who have many followers but who do not follow as many of them back have high social capacity for being responded to, and therefore to influence others, but with the lack of awareness of the current activities in the collective, their contributions were found to be less close with sources of ideational coherence in the collective. Often, their contributions were broad and sometimes almost passé. Leaders, on the other hand, were putting in the active work involved with maintaining closeness with current developments in the collective and were more attuned to the emergent issues and needs of the collective.

Overall, as users gained visibility by others in the collective, they evidenced more social responsibility, and as they gained awareness of others in the collective, they evidenced more ideational alignment. These are not suggested as being in a cause and effect relationship, but rather, as living in a correlative relationship. That is, users may be well-followed because they are socially responsible, or they may be socially responsible because they feel a duty towards their followers. Likewise, users may be following others.
because they want to be ideationally aligned, or they may be ideationally aligned because of their awareness. Nonetheless, through the tweet data, it was evident that those who had both high visibility and awareness within the collective were able to access more amplification and their tweets evidenced more relationality, boundary pushing, and community orientation. That is, their contributions tended to reveal aspects of leadership more so than those who did not have such access and remained in more of a peripheral state. Therefore, not only do core members need to be seen by others, they also need to work at seeing and acknowledging the work of others. This takes constant work and may be why it is common for central members in MTBoS to express burn out. As such, an implication for MTBoS itself is that more users need to consider taking these sorts of leadership roles on so that it is not great a burden on others who do it. It also highlights the need for ‘critical foxes’ (Kilpatrick, 2013) in professional collectives, who push boundaries, and have the social capacity to be visible in doing so.

Beyond negotiation

Taken together, when both content capital and social capital are brought together, as discussed in Chapter 9, an opportunity for ideational resilience is created. Through social capital, users are visible and aware of the prevailing discourses and sources of ideational coherence in the collective, which makes the content they publicize have more capacity for invoking amplification. But clearly, social capital on its own does not condition a response, the message also matters. Therefore, in order for it to be resilient in the collective, content needs to have qualities that attract amplification. Content capital was defined by traits associated with content that attracted high values of ‘likes’, ‘retweets’, and ‘replies’, and involved traits evidencing pride, accomplishment, utility, novelty, vulnerability, and curiosity in how content was presented. Of course, this is in conjunction with being ideationally aligned and socially responsible, as found within the contents most prominently provided by those with social capital.

However, even with all of these traits, while they were associated with ideational resilience in the collective, cannot guarantee the content to occasion such prominence. That is, while a tweet may involve many of these traits, it may not solicit amplification. There are evidently many other factors, which have to do with randomness and the serendipity of encounters involved in created a ‘perfect storm’ environment around a certain idea. Researchers have chased the question of what makes content viral, and
while there are several features that make it viral, it is not always possible to ensure virality. Perhaps some element of serendipity is necessary for such phenomena to occur. However, by pinning down a set of traits associated with the different kinds of amplification, general findings pertaining to viral content are nuanced by this research that pertain more specifically to professional collectives of educators. Nonetheless, even though these various traits have been revealed, the portion of tweets receiving high values of all forms of amplification, in this case being at around 6% of the total (28/444), is very low. This means that many tweets go on with little response from either being ‘liked’, ‘retweeted’, or ‘replied to’. However, if we consider the contrary perspective, the portion of tweets in the core data set of this thesis that received at least one ‘like’, ‘retweet’, or ‘reply’ was 93% (413/444). This means that 93% of those who tweeted with the hashtag #MTBoS received at least one instance of response.

While my initial intent in investigating the activity found in MTBoS was to explore instances of negotiation, I found that while instances of negotiation do occur, when compared with all activity in the data set, they were rare. Only 96/444 core data set tweets were highly replied to, and out of these, only 47/96 involved back and forth conversations. For example, in one case, a user initiated a discussion by indicating a student of theirs asked about whether they could watch the Kavanaugh hearings in math class rather than doing math since it seemed more important to them. By sharing such an experience, others connected with the user by indicating their positions on the issue, evidencing empathy, as well as pragmatic approaches to options that could be considered in response to such a request. Evidently, there was enough redundancy in views and experiences among contributors to carry the conversation, but some diversity that allowed for the conversation to be productive of possible solutions. The issue presented also revealed a teacher tension between morale, school culture, and pragmatics. In another case, a more mathematical focus was taken in negotiating different ways in which a diverse array of ideas for activities designed to help secondary school students develop a stronger grasp on area and perimeter concepts without relying on formulae. Again, contributors had enough redundancy around teaching area, and around problem-based approaches to teaching, that novel ideas could be proposed and built upon. This resulted in new problems being created for the purposes of responding within the threads. While ideas revealed novelty, utility, pride, accomplishment, curiosity, and even vulnerability, they also required visibility and
ideational alignment. As such, these instances of productive negotiation involved all of the features identified as associated with both social capital and content capital.

In general, when users were more central in the social landscape and the ideas shared were made in ways that were familiar yet unique and intriguing, particularly with some emotive value, it seemed more likely for negotiation to arise. However, while this only happened to 47/444 tweets, this means that 397/444 tweets did not engender such negotiation. Given the prominent focus in literature on mathematics teacher professional development on the need for creating and supporting opportunities for collaboration among teachers (Lerman & Zehetmeier, 2008), this result is baffling. How could a setting that leads so many mathematics teachers to claim that it is the best professional development they have experienced involve such infrequent negotiation around issues pertaining to mathematics teaching? And, if there is a significant amount of engagement, why is so little of it breeding negotiation? Or, more simply, what else is happening other than negotiation that propels and sustains the collective?

Evidently, the results of this research reveal the significant interest in publicizing musings that are ideationally aligned with the collective, and that in turn, are likely to receive some form of amplification, even if it is one ‘like’, ‘retweet’, or ‘reply’. That is, the possibility of being responded to in some (however meagre) way seems to keep activity flowing. And the most evident aspect of receiving a ‘like’, ‘retweet’, or ‘reply’, is an affirmation, which acts as an indication that someone ‘out there’ has heard this comment, and that the contributor is therefore ‘not alone’ with their musings. While the prominent attention around amplification in this research was around characterizing the traits that associate with high amounts of amplification, it is interesting to consider the implications of a small amount of amplification. Treating amplification as a tool, the activity it is mediating is that of a contributor working towards being heard. In this sense, perhaps affirmation, being heard, and simply, not feeling alone, is at the heart of activity in MTBoS. In hindsight, considering the typical activities that occur within the professional life of a mathematics teacher and the professional development initiatives they may encounter, how much opportunity do mathematics teachers get for being affirmed in their day to day musings around mathematics teaching? This perspective has significant implications for professional learning initiatives because it challenges the often-centralized top-down facilitator-driven structures that are so common within such settings. However, perhaps mathematics teachers also need room to be prompted in
ways that mobilize their ideas so that they can receive various forms of feedback around
their thoughts from each other, and so that they can be affirmed and validated by peers.

Further, taking into consideration that not all participation in MTBoS involves making a
contribution, and instead, often involves reading the musings of others, it is also
interesting to entertain the implications of the sort of ambient affiliation (Hermida, 2010)
that prevails in such social media collectives as in MTBoS. Part of this has to do with the
necessarily public nature of the space and the high density of users, which increase the
opportunity for encounters with a variety of ideational content. That is, ideas are
mobilized among users, and through participation over time, users build an imagined
history of ideas shared as well as a sense for what matters in the collective. Although in
this research, the ideational network was constructed from the perspective of the
collective, an individual user traverses the ideational network in some sense
haphazardly. Through such activity, they presumably attain the capacity for ‘knowing’
which ideational artefacts are more likely to be amplified, which are reified or taken-as-
shared, and which are more prone to being negotiated. For instance, through my own
insider perspective, I have developed the sensitivity that topics around ‘assessment’ are
more likely to attract negotiation since it is both a topic of interest and one that often
solicits diversity of perspectives. However, many ideational artefacts in the collective are
in fact either reified or reifiable, meaning that they present similar ideas in slightly
different ways, and their meanings have become strongly taken-as-shared among
frequently participating members. This is evidenced by the tendency for reproduction of
such reifiable ideas (e.g., noticewonder prompts), and these kinds of participations seem
to offer an entry for observers to become part of the collective. As such, it is interesting
to consider this aspect of ambient reproduction of similar content recurrently as a form of
participation, and the normalization of certain ideas as a consequence. Research on
professional development initiatives in mathematics education does not seem to address
this idea of frequency of encountering ideas within professional settings. In Larsen and
Parrish (in press), one of our interviewed participants referred to this sort of constant
flow of similar information over time as a ‘slow burn’, indicating, “it’s not a moment of
inspiration so to speak, it's more of a slow burn overtime” (p. 17). However, traditional
professional development settings aim to deliver information in a single event, with little
attention at coming back to ideas and letting them ‘simmer’. This has implications not
only for professional development structures, but also for leaders to listen to ideas that
surface recurrently among participants, and to have ways to continue to surface and mobilize the ideas available within the participants. Perhaps one of the most significant results of this research is that mathematics teachers have a vast amount of experience and insight to offer, and opportunities for surfacing their ideas are needed.

**Growth of the collective**

Although the limitations of this study do not allow for viewing the growth of the collective over time, some of the results around processes that engender ideational resilience in the collective may offer insight into how the collective grows and sustains over time. While social media settings are generally attributed with homophily (Mcherson et al., 2001), in which ideational self-segregation occurs, reducing the opportunity for diverse ideas to enter certain social circles, MTBoS seems to combat this. Users not only engage in diffusive amplification, they also endorsively and refractively amplify ideas. This means that refraction is possible, and therefore, growth of the collective can occur over time. However, what this requires is negotiation among members around ideas. While reification as well as endorsive and diffusive amplification create stability in the collective around certain ideas that may go viral (such as ‘noticewonder’ or ‘growthmindset’), negotiation and refractive amplification allow for diversity to be mobilized, and therefore to not only cohere ideationally, but also to involve ideational randomness. Without novel material and provocations that aim to challenge prevailing ideas, progress cannot occur. However, in order for diverse ideas to be heard and to be negotiated productively, they need to be carried with social and content capital. That is, a minimum level of redundancy among members and visibility is required. Therefore, in order for MTBoS to continue growing, contributors need to continue actively welcoming new members, creating space for their contributions to be heard and amplified, and engaging in provoking taken-as-shared meanings. Also, given that many questions go unanswered, particularly when posed by newcomers, processes that allow for answering the unanswered may help the collective grow. However, a danger of burnout exists within this strategy and therefore may involve more users who engage in seeking opportunities for negotiation and continuing to be brave and vulnerable with idea sharing. Another danger of growing the collective is also that it may disintegrate into smaller components or reorganize in new ways. Perhaps there is simply a threshold to how many users it can handle as limited by the number of those who continue engaging
as leaders. However, the collective cannot grow ideationally without negotiation. Therefore, negotiation is an important aspect of activity, and should not be undermined.

As others who have studied MTBoS have revealed, MTBoS offers a variety of desirable features such as including cognitively demanding tasks, mentorship, sense of community, a place to continue in-person conversations, a place for negotiating ideas and generating activities, and a tight-knit community. However, what this research adds to the results of these investigations, is a snapshot of all of these opportunities as embedded within a cross-section of activity in MTBoS during a single period of time within its natural state without in-person initiatives to enhance its activity. What such a perspective offers, is insight into the processes involved in sustaining the collective as well as the frequencies of occurrence of the aforementioned desirable results. As this research illuminates, while there is possibility for a tight-knit community within those who highly follow each other, there are many users who try to participate by using the hashtag, but who do not yet have either social capital or content capital in their participation in MTBoS. And, that even if one does have the associated features of these, much of the activity in MTBoS is reificative, with rather infrequent occurrences of negotiation. Since negotiation is crucial in the growth of the collective, consideration should be taken by stakeholders to work at findings ways to occasion negotiation more often. While the limitations of this study may affect these findings, the continuum between reification and negotiation remains evident among all activity in MTBoS, and it is clear that both of these aspects are necessary in the life of the collective.

**Answering the research questions**

In summary, based on the above-discussed themes, I now reveal the key learnings most pertinent to the two research questions that guided this study.

**RQ1:** *What is the nature of ‘MTBoS’ and how does it emerge from ‘the people of MTBoS’?*

MTBoS is a complex collective of autonomous individuals who engage in public communication around mathematics teaching via social media (i.e., Twitter). Although it is composed of a constantly in-flux population of contributors, with a permeable boundary that allows for a shifting membership, various forms of engagement, and diverse social locations, it can be viewed as a united collective that emerges as greater than the sum of its parts. This occurs through reification and negotiation of ideas, which
link agents with ideas. By looking at the ideational network formed by the social network, various sources of coherence are evident, which are formed by either reified, reifiable, negotiable, or negotiated ideational artefacts that are held together by various kinds of relations. These relations offer space for either implicit or explicit reification or negotiation. As such, the ideational landscape that emerges from the communicative activity of individual agents, coheres around certain topics that either induce negotiation or that become taken-as-shared. The whole that emerges from this activity is greater than the sum of its parts because even though contributions are made individually, the reoccurrence of similar ideas and patterns of activity develop into sources coherence. Amplification processes further escalate these patterns of recurrence, making some ideas more prominent in the network, and ultimately forming the collective’s identity.

In this research, MTBoS is a complex social collective interested in practical implementation strategies for inquiry-based mathematics teaching practices, tasks that emphasize multiple pathways to a solution, approaches to humanizing mathematics classrooms through valuing teacher-student relationships, and the various facets of mathematics teaching as a profession. Mathematical content is most often used as a vehicle for communicating pedagogical strategies; however, at times, it becomes of central interest when agents engage in recreational problem solving. Such instances of recreational problem solving can develop into pedagogical discussions as well. Overarchingly, agents take interest in connecting with each other socially, and social dimensions are inextricably intertwined with the nature of ideational contributions.

Given the constraints of the communicative medium (i.e., the Twitter platform) that allows for asynchronicity, publicity, and unreciprocated relationships, agents have varying opportunities for visibility by others and awareness of others. These variances define four social locations (newcomers, observers, influencers, and leaders), which each evidence different forms and modes of engagement. Therefore, the people of MTBoS form the collective through the contributions they make, which are fundamentally intertwined with their social relations and what those relations make them privy to. While leaders both push ideational boundaries and evidence social responsibility for upholding the collective, newcomers have yet to discover sources of ideational coherence and do not yet see the social network as a support system. Further, while influencers find ways to give back to the collective through offering various resources without necessarily aligning with prominent ideas in the collective, observers promote themselves as being
ideationally aligned without necessarily trying to serve others by revealing the details of their practices. As such, a diverse and bottom-up emergent social landscape provides the necessary conditions of redundancy and diversity among agents, which forms the basis on which an ideational network can thrive.

With such a social landscape and including the notion that agents may be driven in their contributions by inter-personal aims such as being considered a ‘good mathematics teacher’ or similar, various ideational possibilities arise. More specifically, ideas in MTBoS end up being either negotiated or reified. While reification provides stability, negotiation offers dynamism. However, while the potential for either of these may exist, it may not always be seized. Some ideas are negotiable or reifiable but go on unquestioned and taken-as-shared. Given that opportunities for negotiation are fundamental to the ongoing development of the collective, there is a looming need for more agents to take on leadership positions in which they are willing to be vulnerable enough to question statements made by others, and to negotiate the negotiable or reify the reifiable. Without such activity, ideational activity may fail to thrive.

MTBoS is overall a place where affirmation and validation are prevalent more so than negotiation and debate. This means it is a space in which agents may feel supported in their endeavours of incorporating reform-oriented approaches to mathematics teaching, or at least, the approaches they perceive as achieving this goal. It is a place where agents may commiserate with each other socially and find peers who agree with their views. However, it is also a space where ongoing negotiation exists, albeit not nearly as commonly as one would expect. These moments of negotiation are incredibly powerful at driving novel ideational emergence within the collective, and helping it steer away from ideational stagnation. While negotiation is rare, when it occurs, it can be powerful. The people of MTBoS have the agential power to spur negotiation. Therefore, it is a space that could use more ‘critical foxes’, who find ways to challenge and build on pre-existing ideas. The converse of this is that perhaps the great volume of reification and affirmation of agents’ contributions is more necessary than one would think. While negotiation is necessary, this research shows the incredible prominence of reification as a lifeline for the collective and a steppingstone for negotiation. Reified ideas ultimately drive the collective’s identity as they form into taken-as-shared sources of coherence.
RQ2: How and why does ‘MTBoS’ invoke a sustainable form of professional activity around mathematics teaching?

MTBoS is a sustainable form of professional activity around mathematics teaching not necessarily in that it is sustainable for each individual, but rather, that it is sustained as a collective enterprise in which agents may engage to various degrees and in a multitude of capacities. Therefore, its sustainability depends on the processes that shape its existence. The most influential processes on its sustainability include the ongoing formation of social capital and ideational capital that, through various forms of amplification, drive ideational authority in the collective, and in turn, determine key points of coherence for the collective. These forms of amplification are largely offered by the medium itself, and stem from the way ‘likes’, ‘retweets’, and ‘replies’ are used, offering opportunities for endorsesive, diffusive, and refractive amplification, respectively. Each of these forms of amplification are driven by modes of amplification, including pride and accomplishment for endorsesive amplification, utility and novelty for diffusive amplification, and vulnerability and curiosity for refractive amplification. This means that interpersonal dimensions are fundamentally involved in ideational amplification, and therefore, in determining content capital. However, social amplification also exists through the privileges associated with those who hold leadership roles in the collective. Namely, leaders contribute in ways that are more often ideationally aligned with the collective and also tend to exhibit greater social responsibility towards others in the collective. This has to do with not only their high visibility, but also their awareness of others in the collective. Therefore, ideas in MTBoS can be amplified with content capital or social capital. Content capital lies in the qualities within content that are determined as desirable by the collective through feedback mechanisms. Social capital lies in the content’s visibility to others through social relations. As such, social capital is necessary for content capital since if an idea is unseen, it cannot be amplified. Likewise, content capital is necessary for social capital since it is difficult to gain followings without producing content with desirable qualities. Therefore, they are intertwined and co-dependent since they require each other. And, ideational material that is repeatedly supported with both social and content capital has the capacity to gain ideational authority in the system. As ideas gain ideational authority, they define and steer the sources of coherence in the ideational network towards those ideas more so than towards ideas that merely recur without attracting additional forms of amplification. Through such processes, the historical basis
of the collective develops over time, engulfing the resilient ideas into identities that the
collective takes on at various stages of its existence. The key factors in its sustainability
and ongoing evolution lie in its social landscape and feedback loops the medium affords.

The converse of this narrative of resilience is that without forms of amplification or
ideational material to amplify, the system could seize to exist. Therefore, in addition to
amplification, both reification and negotiation among agents must occur. If too much of
one prevails, or not enough of one occurs, there may be nothing relevant for agents to
amplify, and as any system based on positive feedback loops can experience, its
resilience as a collective may dwindle. This could result in either reorganization into
something new, or a disintegration into non-existence. As such, all components revealed
in Figure 9.9 are necessary in the sustainability of MTBoS as a thriving complex
collective that is currently brimming with ongoing activity around mathematics teaching.

10.2 Learning from MTBoS

Taken together, these findings have various implications for stakeholders interested in
professional activity around mathematics teaching. Although most of these have been
mentioned in §10.1, I organize them more succinctly here, parsing them into implications
for professional development, for mathematics education research, and for MTBoS itself.

Implications for professional development

Since this research is oriented towards professional activity around mathematics
teaching, it is no surprise there are many implications for professional development
initiatives. While much research has informed the various settings that are most
conducive to engendering mathematics teacher learning, most of these initiatives involve
time, funding, and facilitation. However, mathematics teachers are on their own and
without mandate engaging deeply in frequent activity around mathematics teaching
within MTBoS as well as in other settings. The vast capacities for innovation, craft
knowing, and classroom expertise they hold when taken together supersedes that which
may be offered as a single workshop, or even in a workshop series. This potential
source of ideational material is often unacknowledged, unmobilized, and unseized.
Given that a growing body of literature indicates that teachers are often
‘deprofessionalized’ in various ways (Carlgren, 1999; Hargreaves & Goodson, 1996;
Hoyle, 1980), and that this is unwarranted given the discontinuities in their profession
that teachers adapt to each time educational reform occurs, more attention towards the naturally occurring professional activities teachers are engaging in are called for. As such, in exploring the activities around mathematics teaching in the informal online collective of MTBoS, several implications arise that are worthy of consideration by those aiming to design professional development initiatives for teachers. Since these implications essentially aim to contrast the processes for mobilizing and amplifying mathematics teaching ideas from experienced practitioners with traditional professional development approaches, I frame these implications as questions for those entertaining novel approaches to designing professional learning settings to consider:

- What opportunities for diffusive, endorsive, and refractive amplification exist?
- How are ideas surfaced from participants listened to and mobilized?
- How are opportunities for reification and negotiation supported?
- How much opportunity is there for participants to receive affirmation?
- What sorts of social structures are enabled by the kind of organizational structure that is used?
- Who holds authority over the ideational space?
- Is organization decentralized to allow for the emergence of various social locations and sources of ideational coherence?
- What is the density of the network, and how strong are opportunities for local interactions?
- How are leadership roles established, and do norms support leaders to be ideationally aligned and socially responsible to others in the collective?
- How is content privileged over social status, and what level of accessibility to ideas prevail for newcomers?
- Are mathematical engagements provoked with an awareness that mathematical thinking by teachers tends to lead towards discussing the pragmatics of teaching? Do participants have opportunities to do mathematics before engaging in theorizing about practice?
- Do the aims of the professional learning environment help support mathematics teachers in identifying and developing structures that foster delivery of content in ways that make mathematics enjoyable for students, that help teachers develop relationships with students, and that help teachers connect with each other consistently?
• What could the imagined audience be for participants? That is, who do they think they are communicating with?

• What opportunities do participants have for encountering similar ideas over and over again, resulting in a sort of ‘slow burn’ rather than a complete but short-lived immersion?

By considering these questions, which have been developed out of the results from this research, I aim to provoke subsequent negotiation about how such aspects can live within physical, hybrid, or even fully online settings. I also aim to highlight the professional capacity of mathematics teachers to engage meaningfully in thinking about teaching and mathematics, and that ways of mobilizing their tacit knowing is necessary.

@Desmos gave us time last night to choose our own adventure. Play with puzzles. Do some math art. Work with others to solve and trade math problems (especially with @eluberoff) Or just hang out on the couch and meet a new friend. This is how you humanize PD! #desmosfellows (@EulersNephew, July 13, 2019)

Wow. Giving Ts choice, trust & freedom - how novel. @Desmos rocks! (@JTJuten, July 13, 2019)

Implications for mathematics education research

The results of this research also signal implications for mathematics education research as a field. Throughout my own journey of immersing in mathematics education scholarship, particularly in happenstance conversations with other researchers at conferences, I have encountered a significant lack of awareness about the activities occurring in MTBoS even though they have continued for almost ten years. Given the high amount of interest in supporting the growth and learning of both in-service and pre-service mathematics teachers within our field, it is surprising that the natural activities occurring around mathematics teaching have not received more attention. Rather, studies continue with exploring the affordances of organized initiatives. While the possibility of designing an organized initiative that is repeatedly successful is evidently desirable due to its reproducibility and promising effectiveness, perhaps there are more nuanced aspects within settings such as that of MTBoS that can be gleaned from to enhance more organized efforts. And, perhaps what may be gleaned, is that pursuing organization from the perspective of setting constraints, or proscriptions, that have the power to engender complex adaptive behaviour is a feasible structure for valuing teacher ideas, attending to their professionalism, and occasioning ideational emergence.
It is largely recognized that the worlds of practicing teachers and academic education researchers can remain disparate (e.g., Carlgren, 1999; Hargreaves & Goodson, 1996; Leikin & Levav-Waynberg, 2007; Zeichner, 1995). As Zeichner (1995) notes:

Many teachers feel that educational research conducted by those in the academy is largely irrelevant to their lives in schools. On the other hand, many academics dismiss the knowledge produced through teacher research as trivial and inconsequential to their work. (p. 153)

In recent years, however, more attention has been taken in bridging this researcher-practitioner and theory-practice divide (e.g., Arbaugh et al., 2010; Coburn & Stein, 2010; De Corte & Verschaffel, 2002; Hiebert, Morris, & Glass, 2003). Researchers in mathematics education also take on opportunities to work with practicing mathematics teachers to engage them in the research process. For example, Herbel-Eisenmann and Cirillo (2009) engage teachers in studying their own practice and writing about their use of discourse in the classroom within reform-oriented approaches to practice. While such endeavours evidently require time, funding, and facilitation, they do occur and can serve as an effective way to bridge the researcher-practitioner divide.

However, the tendencies of these spheres to ignore each other often continue to prevail. This in part has to do with the nature of the teaching profession and the power of academic research on shifting expectations placed on teachers. As Carlgren (1999) writes, “The ‘gap’ between what is and what should be in teaching can be understood as a contradiction between teachers’ tacit professional knowledge and the demands constantly being made on teachers” (p. 44). She particularly points to the frequency of teaching reforms, their often utopian visions of teaching, and their incongruence with ground-level day-to-day practice, indicating that teachers are prone to being pegged as “conservative and unwilling to change” (p. 44). However, teachers hold a wealth of professional knowledge without which, the sphere of research around teaching would not exist. As such, recent efforts have worked towards valuing teacher experience and tacit knowing and on exploring naturally occurring informal teacher collegiality that seems to support their adaptation to various external mandates and pressures. The research presented in this thesis aims to explore one such example of informal activity around mathematics teaching. However, unlike many other studies on informal settings, the context explored in this research involves an incredibly scaled up version of ground-up unmandated unfunded informal collegial interactivity around mathematics teaching.
And the findings suggest that an incredible wealth of ideas lie within this space. But not only are the results indicative of the kinds of needs and interests that prevail, but also the strength of this sort of collective to adapt and reorganize around changing demands.

Therefore, it behooves the field of mathematics education research not to consider the naturally occurring complex activity that is occurring without mandate, funding, or researcher intervention. In this sense, the research sphere can learn from MTBoS activity towards gleaning insights that may be helpful in various ways, such as in designing successful professional learning initiatives, exploring naturally occurring dialogue between teachers, gleaning activity ideas, etc. However, the converse of this is that the field of mathematics education research has an opportunity to connect with mathematics teachers and leaders around the globe through participating and stimulating ideational emergence in MTBoS. This idea was brought forth during the 2017 Annual Meeting of the Canadian Mathematics Education Study Group/Groupe Canadien d'Étude en Didactique des Mathématiques within the working group, Social Media and Mathematics Education. During one of the day sessions in this working group, two prominent MTBoS participants came to share about their experiences in the collective, with room for questions from the mathematics education researchers in attendance. The meeting of these two worlds led to an intriguing question about not only what the research world can learn from MTBoS, but also how they can live together.

Pivotal, Steven Khan asked Nat and Alex about “what the MTBoS community wants from researchers”. Nat and Alex agreed that it is about contributing and interacting. It is important that researchers not just ‘study’ MTBoS members, but rather, that they become involved in the ways they would around their interest areas. Nat suggested that researchers should find people who post about things they are studying and interact with them. Alex added that sometimes he posts on his blog, and a researcher will comment with a reference to an article that supports what he is doing, and he finds this very helpful. (Larsen, Chernoff, & Freiman, 2017, p. 86)

The MTBoSers’ suggestions around hoping that mathematics education researchers find ways to interact and engage with those participating in MTBoS highlights an important result of this thesis around the need for occasioning negotiation, which involves not only enough redundancy among members, but also diversity. As I also advocated for in Larsen (2017a), there is space for more diversity of ideas and for members to be vulnerable enough to challenge ideas and pursue opportunities for further unpacking and questioning meanings. Without this sort of diversity, the ideational network may become
stagnant and simply resort to be the echo chamber that social media spaces tend to be associated with. Also, there are many unseized opportunities within the negotiable occurrences found even within only a week of data. However, for these to be accessed, social interactions and positions need to be nurtured, so that not only content capital exists, but social capital helps it become visible. As such, there is evidently space for both the research world and the practitioner world to learn from each other through phenomena such as MTBoS. And, there are already a mixture of both researchers, practitioners, and leaders in the space, but mathematics education researchers only make up about 5% of those tweeting to MTBoS. Therefore, there is room for more such engagement. Taken together with the findings from this research, which highlights ways in which content can become viral and resilient with far-reaching effects on practice as well as prominent needs and interests around mathematics teaching, this work illuminates opportunities for the field of mathematics education research to grow in ways that are attuned to current and emergent activity.

**Implications for MTBoS**

Given MTBoS has sustained existence for almost ten years, it is evidently important for MTBoS to also consider its own state as a collective, and how it may continue to sustain and develop. Over the past few years, the number of participants in MTBoS have continued to rise almost exponentially, which has interesting implications on the inner workings of the collective, particularly because it is a naturally emergent one with no one in charge. In some sense it is a ‘ship with no captain’, and simply continues to grow. However, this research indicates that the ‘captain’ is in the forms and modes of amplification, which direct activity through positive feedback loops that draw attention to certain features and ideas over others. In some sense, what gets attended to, is the direction in which MTBoS grows and coheres around. However, for the collective to continue to grow, it is imperative that contributors maintain the courage and energy to question and negotiate meanings that are either unclear, taken-as-shared, or seemingly divergent. Without negotiation, the collective will focus on what becomes reified, which in turn, steers the collective towards an echo chamber rather than a refraction chamber. As such, diversities are places of learning, and efforts towards negotiating diversities need to continue for the growth of the collective. As such, MTBoS, just as other professional spheres, need more ‘critical foxes’ (Kilpatrick, 2013) who continue to challenge and question presumed meanings and norms.
Also, access can be an issue for those who are new and joining, especially with such a high increase in volume over past years. It is perhaps no surprise that the collective has evolved into being structured more as different pockets of activity, either by grade levels (e.g., #msmathchat), topics (e.g., #geomchat), activity structures (e.g., #wodb), and classroom practices (e.g., #thinkingclassroom). Rather than trying to join by tweeting into the large vortex of #mtbos engagement, perhaps seeking out specific users and hashtags of interest, and focusing on interacting with those is a way to feel a sense of belonging. Each user holds important diversities that are necessary within the sphere, but in order to become visible, one must engage in gaining awareness of the activities in the collective, and in turn, becoming more visible. While this process involves work, the benefits can be far-reaching. However, it may not be a platform that works for everyone. This must be acknowledged. The fundamental structure in this collective is that of self-organization. Therefore, activity and engagement in the space must remain self-initiated.
Chapter 11  Reflecting back and looking ahead

In reflecting back on this study, several reverberations and meta-observations remain, particularly around the theoretical and methodological contributions this work offers as emergent from the singular nature of the setting studied. As well, limitations of this study are important to consider since many of them point to the various opportunities for further study, which can continue to investigate the implications of this work.

11.1 Theoretical contributions

The most significant theoretical contributions of this work lie in the nuanced extensions and refinements of complexity thinking, which were stimulated by the unique nature of the context studied. Although complexity thinking embraces and emphasizes bottom-up organization, serendipitous neighbour interactions, and decentralized control, it has not been applied to as socially and ideationally dense settings as MTBoS before. Therefore, novel constructs were developed to account for the differences in context between those which may prevail in physical settings and those that become possible when temporal and physical constraints are challenged. This ultimately involved drawing not only on complexity thinking, but also on some notions from activity theory and communities of practice as well as ideas from social media literature and the context itself. Therefore, the theoretical contributions of this thesis involve an enhancement and extension of complexity thinking by involving prior theoretical work\(^80\). As such, not only are new notions developed, an approach at networking theories is revealed.

As discussed more thoroughly in Chapter 9, ideational emergence occurs through reification and negotiation that can be either active or inert and is directed by sources of ideational divergence and convergence. Ideational activity can also be amplified through positive feedback loops that privilege certain ideas more than others, thus skewing the network. However, decentralized control allows for amplification to determine who and what has ideational authority through social capital and content capital, which live in a symbiotic relationship. Social capital and content capital are not appointed by a centralized authority, but rather, are determined by the amplification processes defined

\(^{80}\) This sort of merging of theoretical tools and perspectives is more recently referred to as the networking of theories (Prediger & Bikner-Ahsbahs, 2014; Rolka & Roesken-Winter, 2015).
by the system and used by the collective. The prominence of validation through endorsement and celebration rather than negotiation among ideational interactions indicates the value of this form of feedback. Overall, the teacher autonomy and the decentralized control driven by positive feedback loops is central to the workings of this collective. Through both social capital and content capital, ideational resilience within the collective becomes possible, and evidently can influence classroom practice. In this way, the collective works as a whole that seems to become greater than the sum of its parts.

In many ways, this reiterates and exemplifies the tenets of complexity thinking, which outline the conditions necessary for complex emergence and for complex systems to learn. However, it also highlights ways in which the connective tissue between individual and collective layers co-act to form a greater whole. Therefore, findings from this research emphasize and extend ideas presented by Davis and Sumara (2006) around complexity thinking. These pertain specifically to social complex collectives, in which agents are people and the collective is formed as an aggregate of their contributions to a shared ideational network that exhibits some sense of ideational coherence. Some of the highlights of the extensions afforded by this research include the following notions:

1. While emergence may occur in a social complex collective, recurrent attention in reproducing emergent ideas through negotiation or reification and then amplification with social and ideational capital creates a transcendent form of emergence that has more intensity and longevity, referred to as ideational resilience. It contributes to the collective’s coherence and sustains.

2. In a social complex collective, agents and their ideas can gain authority in the collective through social and ideational capital, which by nature requires agency. Ideas are carried by people who reside in different social locations. Therefore, although ideas interact serendipitously, people use their agency to choose how to bring ideas together and what social positions they take.

3. Positive feedback loops drive social and content capital, and therefore acts as an authority in a social complex collective. Feedback amplifies ideas that are most sustainable and allows them to be negotiated or reified, thus pulling the ideational network towards either stability, or instability. Stability creates coherence, instability creates randomness. Both are necessary for resilience.
While these notions are strongly derived from and remain close to tenets proposed by Davis and Sumara, they aim to nuance certain aspects by contextualizing them in MTBoS activity and conceiving of how they can explain not only emergent activity in MTBoS, but also its ideationally resilient and sustainable tendencies. While living organisms all die, some organisms sustain and thrive more prominently than others. MTBoS seems to be thriving longer than other organized professional learning initiatives, and therefore is worthy of consideration in the theoretical implications it reveals.

11.2 Methodological contributions

One of the most significant highlights of this work lies in the specific means used to illuminate the complex nature of individuals acting in a complex whole. Guided by the notion of an ‘ideational network riding on top of the social network’ from complexity thinking, as well as the innate structural features defined by the context of MTBoS itself, a particular methodology emerged. In recent years, more attention has been brought to the networking of theories (Prediger & Bikner-Ahsbahs, 2014; Rolka & Roesken-Winter, 2015) as well as networking of methodologies (Andrà, 2015) within studies in mathematics education. This literature points to the necessity of our field to draw on the theories and methods from other fields to enhance the tools we use to study phenomena. In this study, I drew on the tools from social network analysis, but adapted them to attend to the qualitative nature of the data. In doing so, I had both quantitative and qualitative data. However, the quantitative data was used primarily for organizing purposes to slice off segments for further qualitative investigation. And, qualitative approaches were also used in constructing quantitative findings. As such, my aims were to focus primarily on qualitative findings. However, the quantitative findings were used to support the process particularly because the nature of the data involved many quantitative features (i.e., likes, retweets, frequencies, replies, hashtags, etc.). In this process, there are a few significant methodological contributions worth highlighting here.

First, the construction of the ideational network from a collection of utterances is a methodological contribution that I have not yet encountered in literature. Most social network analysis approaches of Twitter data involve networks in which nodes are either users or hashtags, and relations are either directed replies or co-occurring terms. Although I attempted to use both of these approaches, neither seemed to give enough insight into an ideational network, the existence of which is proposed by Davis and
Sumara, who themselves do not offer specific methods for constructing such a network. Networks of users and their replies illuminate social relations, but these are limited by time frames that cannot indicate ongoing relationships. And, hashtag networks reveal general interests and sources of ideational coherence, but hashtags by their nature are not very specific or accurate with regard to ideas presented. Instead, by identifying ideational artefacts qualitatively from data and constructing such a network by indicating relations between artefacts as well as whether they are in convergence or divergence, produced a much more detailed picture of the ideational landscape. Although this process gets dense very quickly and involves a very tedious process, it could prove useful for smaller collections of data. In particular, it could assist in constructing and examining an ideational network from classroom collectives, professional learning initiatives, or scholarly publication networks. Further, the network analysis approach I designed towards examining the constructed social network is also unique. Most social network approaches involve user and reply relations and some include follower relations; however, in my case, I did not want all follower relations, only those within the network itself. As such, by taking a less automated approach, I was able to illuminate different social locations within the collective, which led to identifying and specifying social capital. Instead of considering a user’s total incoming and outgoing followings, only those that remained within the data set were considered. In this way, even if a user had many followers, follower relations outside of the collective were ignored. This choice improved the accuracy of identifying social locations within MTBoS and is a methodological contribution that can be used in other settings where dense collectives congregate within a social media space that allows for unreciprocated following relations.

Finally, by not attending only to one form of amplification, but all three forms, I was able to identify content amplified in different ways. This choice was closely related to the functionalities available in the medium. In some sense, this means that the constraints of the medium defined the possibilities for methodology within this study. By attending to the features available, new insights could be afforded. Rather than laying on expectations, such as seeking out specific kinds of posts (e.g., only questions, only tasks, only instances of negotiation), I was able to attend not only to what is desirable for a researcher, but also to the complete spectrum of activity in MTBoS. This decision was made to include and account for possibly undesirable forms of activity as being potentially necessary for the functioning of the more desirable forms of activity. For
instance, there are many cases of non-mathematical discourse in the collective, often very personal in nature, involving jest, ‘inside jokes’, and even expressions of exhaustion. These elements may not be considered desirable in a knowledge-sharing community (i.e., in physical settings this is seen as off-task behaviour). However, these were evidently important in the connective tissue between members. Perhaps by being more willing to be vulnerable because of feeling social support, novel ideas were more likely to surface. While this would need further empirical study, I note this here because of the methodological importance of capturing all activity rather than only the most desirable forms of it. In this way, a more complete map can be made so that more specific inquiries can be taken and be understood as components of a greater whole.

11.3 Limitations and considerations

The most evident limitation in this study is in the data collection process, which is restricted to a relatively short span of time in the life of MTBoS. Activity in MTBoS produces a sizeable source of data that is incredibly challenging, if not impossible, to capture physically and work in detail with. Not only do about 6000 users tweet around 25,000 tweets to the MTBoS hashtag per month, not all users who identify and engage in MTBoS activity use the hashtag in their tweets. In fact, I personally know of users who express being part of MTBoS, and have significant contributions to the space, but do not typically use the hashtag unless they want to direct their comment to a larger group because they have a strong enough follower base to know their message will be received by those whom they want it to. There are also users who may tweet about mathematics teaching, but who may not know about the hashtag and the collective behind it. With an ambiguously bound boundary that allows for users to move in and out of the space fluidly, this makes the idealistic ‘complete’ data set beyond the scope of analytic capability. As such, I made a selection that would allow for enough analytic possibility with the potential for representing the whole of MTBoS that was manageable. In this process, I attempted to capture tweets from various hashtags over varying ranges of time. In many cases, the sheer volume of tweets made spreadsheets unstable and files difficult to work with, not to mention the time required to sift through them. By playing around in these ways, I found that the MTBoS hashtag offered the broadest view on activity, and that a week-long timeframe was reasonable enough. Even a week of data was too much for processing in the ways I had wanted to, so I made a random
selection of 30% of the set after removing retweets. Evidently, from the results in this thesis, this nonetheless allowed for much insight into the inner workings, processes, and structures in the collective. It also gave some indication into what the collective attends to as restricted to this timeframe. Since my interests were focused on the mechanisms that drive collective activity, this served well. However, had it been possible to include multiple snapshots of time, it could have given more insight into how the collective changes over time. This would take more analytic work and was beyond the scope of this thesis but proves to be an interesting pathway for future inquiry.

Another limitation pertaining to the data collection is around my choice to remove retweets before making my 30% random selection. The intent of this was to attend to only the contributions and to home in on the space of possibility in the ideational network. Since 75% of the originally captured tweets in the selected time frame were retweets, I wanted to select only out of original tweets to avoid analyzing duplicates. In doing so, I privileged encompassing a diversity of ideas over inclusion of people and temporality. This achieved my aim of highlighting the ideational network but limited results around frequencies of occurrence. While I still accounted for frequency by using the retweet counts from the selected tweets, the chance of encountering an idea has somewhat been skewed. Had I included the retweets before making a 30% selection, I would have mimicked what a user might encounter when scrolling through tweets in the hashtag at random times. This would have been another valid option to consider, but it would have set the research on a slightly different path. By focusing only on original contributions as an initial condition, I included the widest range of ideational possibility for the analytical and computing power I had access to working on it with. And, I also maintained a focus on the collective perspective rather than an individual one.

Along this vein, I also could not account for user participation in the form of 'likes'. The list of people who ‘like’ a tweet becomes hidden by the platform after it reaches a certain number, and therefore, this data was impossible for me to access. Therefore, the initial condition for data in this research was that of a contributing participant who publishes a tweet. Although liking, retweeting, and replying were included as forms of feedback, which indicate the quality of the content of a tweet, they were also forms of engagement. However, these forms of engagement were difficult to attribute to any specific users because of the nature of what the data affords. As such, the locus of interest in this study is on tweet cluster contents. All other data around it supports this unit of analysis.
This is in part premeditated based on tenets of complexity thinking, which initiated my curiosity towards the ideational network, but also highly emergent from the data itself.

Further, beyond the nature of data, another limitation could be considered in my bias towards seeking out negotiation. While the methods were designed to mitigate this since I did not choose to make the initial condition for data as predetermined by the presence of negotiation, when approaching analysis of the data, I was consistently attending to and seeking out moments of negotiation. In some cases, instances of negotiation would have drawn my attention closer because of this. However, given the volume of data and the processes I took to attend to the data in different ways, it does not seem to have had a significant effect on my results. It is worth also considering my personal position in MTBoS and my views as a teacher of mathematics in analyzing data. Part of my methodology was becoming an insider outsider participant, which involved my engagement in MTBoS activity, so that I could understand better the language and norms in the space. This greatly enhanced the quality of my analysis, particularly when constructing the ideational network. However, there is a possibility that my personal biases could have affected some of the coding. These biases pertain to both my own beliefs as a teacher of mathematics as well as my own interpretations of what contributors were saying. This is a double-edged sword because on one hand, having an insider perspective makes the data analysis more robust and attentive to nuance, but on the other, it runs the risk of inaccuracy or skewing in interpretation. However, with the volume of data attended to, the moments in which I had to ‘second guess’ myself to question my biases were few and I also took a second pass through the data to check on such choices. Therefore, this decision to use an insider view to interpret and code ideational artefacts and relations was overall crucial in the outcomes of this study.

11.4 Opportunities for further study

Evidently, there are many avenues for further study resulting from the work presented in this thesis. These can loosely parse into those that dig deeper into the inner workings of MTBoS and its implications, those that aim to utilize the benefits of MTBoS, and those that move the results of this study forward into other contexts. At the most specific level of interest pertaining to the work and results of this study, there is clearly more inquiry that could be taken even within the same data set already collected. For instance, further categorization and investigation of the ideational artefacts and relations found in this
study could be taken towards building a more generalized and comprehensive framework for ideational networks of complex collectives. In particular, the loose parsing into ideational artefacts that are either content-related, practice-related, or connection-related could be used as an analytical tool to further examine tweets in this dataset to build a mapping of which foci are used, when, how, and by whom. To this end, perhaps tweet contents could be placed as being oriented more towards some of these than others in a sort of triangle diagram\(^\text{81}\) as suggested below in Figure 11.1.

![Figure 11.1](image)

**Figure 11.1  Aspects in ideational artefacts and relations**

While cursory examination of these aspects indicates they are scattered throughout all contributions and that no other categories exist, this could be further empirically explored and refined. What such an investigation could highlight is just how prominent interpersonal dimensions are within activity around mathematics teaching, what role they may be serving the participants, and how they play into the ideational network.

Another dimension that could be explored could be around the use of the medium and the ways in which the writing styles affect engagement. That is, perhaps it is not only the ideational content in the tweet, but also how it is presented, that attracts attention via feedback mechanisms. For instance, some contributors use available stylization tactics such as entering spaces between statements within a tweet to make it appear larger and therefore, more attractive. Or, they may choose to include vibrant media that is eye-catching. While this is generally related to typical uses of the Twitter medium, perhaps there is opportunity for examining how this affects the ways in which activity around mathematics teaching is presented and taken up. Does the way it is presented contribute to more conversation and novel ideational development?

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\(^{81}\) This triangle diagram is inspired by the work of Törner and Grigutsch (1994) as discussed in Liljedahl (2008), where such a triangle is used for mapping beliefs about mathematics.
Moreover, to focus further on interpersonal and discursive aspects, threads with high amounts of negotiation could be more deeply examined in terms of the diversities and redundancies between agents and how these pertain to the ideational network developed from this time frame. And, a closer inspection of the structures found among leaders could be taken to identify whether there truly is a single core, or many cores, in the central positions in the social network. While cursory evidence supports the possibility of multiple cores, further empirical study is needed. In particular, an examination of the ideational network created only by leaders could be carried out, with attention to the social relations and how they support and hover around specific sources of ideational coherence. Similarly, examination of the networks among only newcomers could be taken to explore pathways they take, possibly over time, in the network.

Beyond the current data capture approach, other approaches to collecting and analyzing data could be taken. Most simply, temporality could be accounted for by extending this work to repeat it over multiple time frames. This could show the stability and instability within the ideational network and highlight perturbations. In particular, it would be interesting to consider the perturbations to the ideational network before and after a significant political event such as change in leadership or policy that affects the populous of MTBoS. An extension of the methodology could also be made to include tweets in MTBoS without the hashtag. For instance, by a snowballing approach, building a set of users, and scraping their contributions and engagements within a period of time. While this is currently difficult, it may become easier when technology improves. Choices around including retweets could also be made, and another pathway to explore would be the ‘liking’ activity of particular users. It is possible to collect data of a user’s ‘likes’ currently. An interesting study would be to consider the life of specific users over time, possibly comparing users from different social locations to examine their patterns of activity. And, potentially, to also explore the effects on practice they experience through participation. In particular, investigation into how engagement in MTBoS mediates tensions teachers experience in practice and how it supports them in adapting to changing conditions such as reform-oriented mandates. While many MTBoSers report significant changes to classroom practice based on their engagement, how this happens has not yet been explored. Perhaps the sense of community they feel could be considered. And, intervention-based approaches to find ways to help newcomers make use of the space and to develop a sense of community through participation in the
network could be taken. Or, more ideationally, interventions around designing hashtags and 'viral' ideas could be attempted to more deeply explore what it takes for an idea to be influential in the space, particularly one that consequently affects classroom practice.

The results of this research could also be taken into other settings, such as professional development initiatives, professional scholarship circles, or even into classroom practice. Essentially, the findings here apply to collectives that have properties of self-organization and features of a complex system. They suggest that with the right conditions in place, a collective may occasion ideational emergence, and even ideational resilience, which can influence the collective over time not only in their identity as a collective but in their subsequent capacity for adaptation to novel conditions. Ideational resilience is desirable in many situations. In the case of professional development initiatives, principles of self-organization may be particularly impactful in the design of future initiatives. Enhancing the connection-related capabilities of participants may also aid continuity of professional activity over time. Connection-related skills seem to propel and motivate sustainable activity and may be more necessary than we think in fostering improvements to the ideational capacities of a collective. Such aspects could be considered in future design and investigation of professional learning spaces for mathematics teachers. In cases of professional scholarship circles, the notion of mobilizing ideas may be considered. Often, in professional scholarship, many ideas go unpublished, and instances of ongoing public conversation between publications are overall rare. As Kilpatrick (2013) suggests, we need more 'critical foxes' who traverse boundaries and make their ideas visible. Future empirical work could for instance examine the ideational networks in mathematics education professional publications to identify structures and potentialities for further negotiation. Some work around this has recently occurred as in one study, Hannula and Moreno-Esteva (2017) pursue mapping out citation relations between scholars publishing about mathematical affect within CERME proceedings. This work could be extended. Some of the findings from this thesis could also be used in the design and examination of classroom settings. While classroom settings are bounded by walls and limited by temporality, features of complex organization and attributes conducive to supporting ideational resilience could be considered. Overall, the findings of this thesis evidently inform an array of settings, and are important since ideational resilience is ultimately, learning, and is therefore, desirable.
Chapter 12  Afterword

My initial intent in this research was to examine professional discourse among mathematics teachers. This interest was rooted in my own ego-driven curiosity around how I could continue learning from and with other teachers of mathematics in order to combat the isolation I felt after transforming my practice in various ways. By aiming to connect with other teachers, I wanted, in some sense, to affirm for myself that I was doing a good job. We often seek to compare with others in efforts to reconcile our own actions. Only after this process of investigation do I realize just how alike my desires for affirmation were to those of many other mathematics teachers participating in MTBoS. However, this was not initially evident. In fact, when I first embarked on this journey and began reading through activity in MTBoS, I felt very distant. Something about the public nature of the medium made me feel like those who were tweeting their practices and experiences ‘had it’, and I needed to learn from them. That is, they had some sort of authority by being public in this space. However, as I began reading, engaging, and meeting the people behind MTBoS, I realized that just like myself, they were yearning and searching for connection with other like-minded teachers. The only difference was that they were willing to take a leap of vulnerability in order to expose their musings. As I also began to expose my thoughts and ideas, and engaged in conversations with others in the space, I became more like ‘them’. Without realizing it, others began treating me as an authority, which always struck me because I never felt like one. The interesting thing about personal experiences within a public space is that our own experiences are very often incongruent with the identities others project onto us based on our actions. While this is ever-present within physical settings, the Twitter medium homes in on ideas as the primary public face of activity. As such, it is incredibly conducive to ideational interactivity that can propel knowing-oriented activity and put less attention on the people behind the ideas. While who tweets matters, once a tweet is visible, what is tweeted is even more important. In realizing this, I found that there was a substantial opportunity for self-expression in MTBoS, something that is not often available within our teaching contexts. Even if no one responds, expressing our ideas and experiences can feel good. Perhaps it is the imagined community we feel connected to that creates this effect. Nonetheless, ambient effects of participation in MTBoS offer a power to occasion a sense of belonging, a connection to like-minded peers, however imagined they may be.
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