

Discussion-Based Learning in a Harkness-Based Mathematics Classroom

by

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Abstract

All across the world, mathematics education researchers and reformists are calling for collaboration, problem-solving, thinking, and communication to be at the centre of primary and secondary mathematics curriculum. However, such practices are still in the development stages and practitioners are only starting to implement them. I am fortunate enough to teach at a school where such practices are encouraged. The school has been investigating whole-class discussions as a central instructional approach in mathematics teaching. In this thesis, I analyse and classify the discussions that happened in my classes and study them carefully to determine some common themes and identify ways to make the learning of mathematics more engaging and meaningful to students. Results indicate that for whole-class discussions to be an effective way to conduct a mathematics class, the teacher's expertise plays a vital role in guiding and facilitating the discourse.

Keywords: Whole-class discussions; Mathematics education; Harkness teaching; Student-centered learning.

To my parents and family, whose support has been
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Chapter 1. Introduction

The Ministry of Education in British Columbia, in their draft of Mathematics 9 curriculum, are calling for “communicating concretely, pictorially, symbolically, and using spoken and written language in expressing, describing, explaining, representing, clarifying, modifying, reinforcing, applying, defending, and extending mathematical ideas” (BC Ministry of Education, 2015). I live this every day and truly believe that this direction is the correct one. It is now up to us, the classroom teachers, to do our best and turn these ideas into practice. Over the years, I have been exposed to and used the whole spectrum of classroom structures from sitting in rows and listening to the teacher to complete self-paced individualized learning. However, none of these approaches gave me the satisfaction I wanted or that my students wanted or needed. On the one hand, I have found that the traditional rigid structure of sitting in rows and taking notes to be lacking the engagement, collaboration, problem-solving, and communication components. On the other hand, too loose of a structure with a large component of working independently or groups does not exhibit the possibility for necessary ongoing formative assessment, as the teacher is not always present at every moment of time when students are working on their mathematical tasks, as he or she usually walks around the room. Therefore, the teacher is only able to assess a smaller sample of the learning process. Intuitively, however, it feels that the answer is somewhere in between, but closer to the latter as I see very little benefit from students sitting quietly and copying the notes for very long periods of time. Teachers possess an incredible collection of knowledge and experience that is absolutely invaluable to those who are learning something for the first time. Thus, one has to start looking for alternative methods, combining some elements of teacher-centered and student-centered environments. Ideally, such a combination of elements such as group work, independent work, exploratory activities, on-going formative assessment, whole-class discussions, student presentations and teacher examples and explanations. In addition, the use of technology can tap into vast collections of useful apps, widgets,

videos, worksheets, and online quizzes to support mathematical learning. Students now have access to an incredible variety of tools that did not exist a few years ago. As a result, students have the ability to take a very active role in their learning. They have the potential to go to a different level in their learning and have greater ownership of it, thus gaining motivation and a sense of purpose from achieving their goals. It is up to the classroom teachers to try to create the classroom environment that facilitates and employs these items and use them well.

For example, imagine a mathematics classroom where students and their teacher are talking about the idea of an asymptote of a reciprocal function in a whole-class discussion. Students are coming up to the board, writing up questions and their solutions. As this is being done, other students are giving their feedback and opinions about their classmates' work. The conversation flows naturally from student to student, with most students in the class participating and giving their opinions and ideas. And suddenly it happens. The essence of the problem is brought forward. There is the group of those who say that the graph will eventually cross the invisible line, and those who say it will not. Tension grows, arguments are thrown around, and the noise level rises. Some students are now looking at the teacher. This lesson has reached a perfect moment! What does this teacher do next: interfere, give another problem, pose a question, use technology to demonstrate something or just let it develop further? This is the phenomenon that I am interested in. It transforms the classroom from a passive learning environment to a dynamic and much more organic one.

For me these types of experiences began to occur when I started my graduate studies at SFU in 2011. The coursework was very inspiring and gave me much to think about, things to try, and people to bounce ideas from. At the same time, my school (Westridge School)¹ was exploring a similar area in preparation of our new Senior School Academic Model. From these investigations, school visits, articles and journals read, and discussions at various professional development opportunities questions began to emerge. Our administrative team found funds and released faculty to go visit schools

¹ Pseudonym

where a similar approach was being used. Teams of teachers and administrators traveled and gathered as much information as they could to be communicated back to the larger community. We were also fortunate to have teachers from experts schools visit our classrooms, give us feedback and conduct workshops with us to give us a better understanding how discussion-based teaching works, give us the opportunity to ask questions, and improve our practice.

One of these methods of instruction is called the Harkness teaching model. In the Harkness teaching model, students are seated around an oval table with the teacher sitting amongst them. Students discuss course material while the teacher acts as a facilitator of this discussion, giving feedback and interjecting when needed. This is a very crude summary of what it looks like in expert schools. It has since become a task for us to see how we can adapt this to our situation with our curriculum, student body, faculty expertise, and resources. Questions arose to start the process, “How many students per class will make the discussion productive?”, “How do we arrange furniture and equipment?” and “Do we have enough money to sustain this initiative?” Once the set-up stage was completed and we were underway with trying whole-class discussion mechanisms for running our courses, we started to wonder about the quality of the discussions: “What is a productive discussion?”, “What makes it possible?”, “What is the role of a teacher in all of this?”, “How does it affect the attitude of students towards learning and subject matter?” All of the above made the last four years very interesting and fruitful. In the next chapter, I elaborate on the Harkness model. I will discuss the history of the method, details of implementation at our school, challenges and successes along the way, and what we have found useful for our practice.

Chapter 2. Harkness

In this chapter, I present the history of Harkness and its practice at Exeter Academy in the USA, as well as our school's adaptations and experience researching, planning and implementing this unique teaching style.

2.1. Harkness at Exeter

In the 1930s, oil magnate and philanthropist, Edward Harkness donated a large sum of money to Phillips Exeter Academy in New Hampshire to help promote and implement his vision of teaching where discussion was the main vehicle of instruction and learning. Mr. Harkness left the following quote with the school.

What I have in mind is a classroom where students could sit around a table with a teacher who would talk with them and instruct them by a sort of tutorial or conference method, where each student would feel encouraged to speak up. This would be a real revolution in methods. (Phillips Exeter Academy, 2015)

The basic structure of such a classroom is a large oval table with students and a teacher seated around it in no particular order. The roles of the students and the teacher are redefined to make the teaching almost completely student-centered, with minimal input from the teacher. The teacher is merely a gauge of the discussion and is there to ensure that at the end of the class, students leave without any major misconceptions about the new material. However, the so-called delivery component of the material via teacher-centered approach is virtually eliminated during class time. Students are encouraged to talk to each other, rather than solely to the teacher, contribute as much as they can to the discussion, ask questions, prove points, take risks, and share attempts even though they may not be fully formulated. In order for this to work effectively, students must listen to each other carefully and stay engaged with the discussion at all times so as to not miss

anything vital and important. In addition, they are encouraged to criticize and correct their peers' work and ideas. One might see how this applies to English or Social Studies classes; however, it can be harder to imagine mathematics or a science class being structured this way. The success of the academic program of Philips Exeter Academy is that almost every single class offered at the school is student-centered. This includes mathematics and science. The questions become "How do teachers at Exeter achieve that?", "Can other teachers implement this way of teaching and have some success with it?" and "Will it satisfy the earlier outlined criteria for being in the middle between a teacher-centered and student-centered work?" Philips Exeter Academy has been practicing Harkness teaching for decades and is a leader in implementation of whole-class discussions. They achieve this by the means of three main criteria: small class sizes, both dedicated and qualified faculty, and the absence of a mandated curriculum and standardized tests. Figure 1 is a visual comparison of the so called "traditional" classroom and a Harkness classroom. The one-way communication of the traditional model is contrasted with the myriad of utterances of the discussion around the Harkness table. Also, a teacher is at the front of the room in the traditional classroom, whereas in the Harkness model there is no "front of the room" and everyone is positioned as an equal contributor (see Fig. 1).

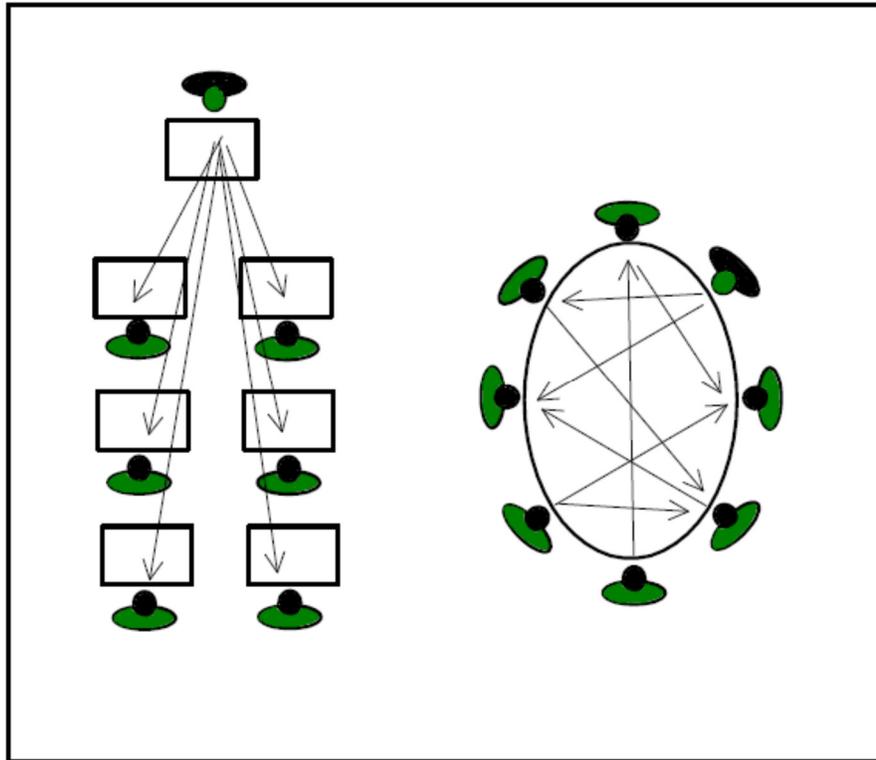


Figure 1: Traditional vs. Harkness Teaching Models

I would like to discuss the details of Harkness implementation at Exeter. First and foremost is the class size. Exeter has dedicated money to set their classes at no more than 12 students. This specific number has emerged through years of testing and observations. They have noticed that if the class has fewer than 12 students the discussion tends to be less rich and diverse as fewer ideas and opinions are circulated around. At the same time, if there are more than 12 students embarking on learning the Harkness way, then the discussion starts to become hectic with too many voices competing and creating classroom management issues. Thus, they set their classes at 12 students and strive to maintain it this way.

Second, most of the faculty at Exeter hold graduate degrees in their discipline. This enables them to critically look at the curriculum they are teaching and make good decisions on what, how and when to teach. In addition, they continue to take part in professional development throughout their careers as there are incentives in place for the

faculty to continue their professional growth. These include tuition compensation, salary increases, and the ability to move up the “corporate ladder” within the school. Exeter faculty are also true believers in the Harkness method and as a result their school has one of the highest employee retention rates.

Lastly, the absence of standardized tests is a large component accounting for the success of the implementation of Harkness method. Because of this freedom, teachers at Exeter design their own curriculum and textbooks. For example, in the mathematics department they have created problem sets for grades 9 to 12 from which students are assigned the problems to work on every day at home and discuss in class. These problems are carefully selected and presented in a particular order so that students are exposed to mathematical concepts and ideas via problem-solving. These problems serve as a lead-in to a concept, practice with a method or algorithm, or extension work. There are no chapters, sections, or units. Therefore, the order is not imposed and topics can be moved around freely where they fit best. In addition, technology can be integrated as a substitute for an algorithm that appears redundant or obsolete. At the end of the day, students are simply and truly doing and learning mathematics. Teachers decide on the appropriate time for the assessment, usually a written test, and together with the class brainstorm the ideas and topics to be included. The only measurement tool for across the board testing are the SATs and AP exams at which Exeter students excel every year, with regular admissions to the best colleges and universities of the world. The Mathematics department at Exeter has a long tradition of using Harkness teaching and their philosophy reflects this very well. The following is a mission statement of Exeter Mathematics Department:

The goal of the Mathematics Department is that all of our students understand and appreciate the mathematics they are studying; that they can read it, write it, explore it, and communicate it with confidence; and that they will be able to use mathematics as they need to in their lives. (Exeter Mathematics Department, 2015)

All of the faculty and students embrace this vision and vehicle of learning. The following are just a few of the testaments from faculty and staff at Exeter.

Just because I say it, does not mean they know it, they have to say it.

This [Harkness teaching] has been something I have never experienced, as the result of a school having a pedagogy that spanned across multiple grades has created a powerful interest of what was going on at the school and if it was not what it seemed to be then, I don't think I would have stayed here for 19 years.

The thing I love most about Harkness is that I am able to absorb all the material in class because we discuss it so thoroughly. When I do homework, I simply do homework—I don't have to learn the material... Now, when it comes time for a test I feel like I've been studying since the beginning of the term. (Phillips Exeter Academy, 2015)

Furthermore, in their final statement of Exeter Academic model they explain that

... as in most academic classes, mathematics is studied seminar-style, with students and instructor seated around a large table. This pedagogy demands that students be active contributors in class each day; they are expected to ask questions, to share their results with their classmates, and to be prime movers of each day's investigations. The benefit of such participation in the students' study of mathematics is an enhanced ability to ask effective questions, to answer fellow students' inquiries, and to critically assess and present their own work. The goal is that the students, not the teacher or a textbook, be the source of mathematical knowledge. (Exeter Mathematics Department, 2015)

It is clear that teachers who practice Harkness approach are advocates of the method. It has been around for a reasonably long time with thousands of alumni becoming very successful members of our society. Whole-class discussions are a great tool to promote collaboration, communication, problem-solving and accountability for one's own learning. Harkness teaching puts students in the centre and it is up to them where they go

from there. This means that students must take the responsibility to propel their learning by taking an active role in class discussions and adequately prepare for class by trying the problems and showing their solutions to the class. The teacher's role is to facilitate these discussions, keep records (who is participating and how much) and conduct ongoing assessments of how the class is progressing as well as how individual students are managing in their learning and understanding. Teachers often pose questions but then sit back and observe where the discussion will go given the questions they brought up. Occasionally, they will interject in cases where a mistake has occurred or in their opinion the change or a new direction to the discussion is needed. All this leads to a better and deeper understanding of the subject matter, increased abilities in communication, collaboration, and problem-solving, as well as more engagement in the classes and more authentic learning environment and experience for everyone involved.

2.2. Harkness at Westridge School

As mentioned earlier, there was a drive from my school's administration to explore discussion-based teaching in 2012-2013. We are fortunate to be able to allocate funding to implement this exciting new approach from our annual fundraising initiatives and good fiscal planning by our finance department and the Head of School. Without such support, it would be very difficult to pursue this venture. The biggest issue at stake with regard to discussion-based learning is class size, which means hiring more faculty to be able to create more sections of the same class in every grade. Before the move towards a discussion-based approach, our classes were over 20 students. In our explorations and research, we found that such large classes were very difficult to manage during whole-class discussions for the full length of class time. Our board of governors have decided to allocate money in the budget to give us classes with no more than 18 students each. The only classes that remain over 18 students are those specialized courses for which there is simply no way to create more sections due to physical space limitations. An example of such classes is Physical Education, and some electives such as Media Arts, Drama,

Music, and Fine Arts. Westridge School has decided to adapt some of Harkness practices into our academic model and day-to-day instruction. To make this happen, we needed to have the above-mentioned four criteria as close to Exeter as possible. We have grouped the last two items in one. Here are the adaptations and changes we have made:

(1) Money was allocated from the budget in 2011 to purchase five Harkness tables. In addition, our timetables were changed and more teachers were hired to as much as possible allow no more than 18 students per class.

(2) Exeter faculty was brought in to run a Harkness Institute with us before the 2012-2013 academic year began. Most of our faculty were excited to try the new approach. Throughout the year we continued to share our experiences and spent a considerable amount of time during professional development time discussing successes and struggles with Harkness teaching in our classes. This was done not only within the disciplines, but also across all departments.

(3) The third adaptation was challenging and I can only speak from Mathematics Department's point of view. Since there is a provincially imposed curriculum that we have to cover by the end of each course, and constant inspections of various kinds to ensure that we do, we do not have the luxury of designing our own problem sets or using the ones from Exeter. This was a time to get creative and through a collaborative discussion we established a method of material delivery inspired by the flipped classroom movement (Larsen, 2014).

Two teachers in our department have volunteered to make videos for the content of Math 10, Math 11, Math 12, Calculus 12, AB, and BC. The purpose of these videos is to replace the lesson component of a class that a teacher would traditionally deliver in front of the class as students listened and took notes. These videos typically have the explanation of mathematical concepts, worked out examples, and opportunities for students to try more examples once they pause the video. The solutions are demonstrated to them to check their work. The videos conclude with certain selections of textbook questions (usually of a lower level) to be done at home. These are not meant to be

completed correctly and perfectly right away, but only attempted in order to try the new ideas out and bring their attempts and ideas to class.

Students are expected to bring their work to class the next day and be ready to take part in the discussion. When students come in the next day, they are asked to put those problems and their attempts on the whiteboards or use their tablets to project them to the screen. As soon as the class starts, students get up to explain what they have done in their attempts, pose questions, listen and process the feedback they are getting from their peers. The discussion picks up from questions and issues that come up when students are going through their solutions and attempts, but then as soon as the first question or comment is raised by their peers, the discourse starts to flow between the students back and forth. This is when the potential to explore this extremely fluid and always changing classroom environment arises. Students are expected to contribute something every class, and they have two ways of doing that: coming prepared with attempts and so to speak “scripted” contribution with the aid of their notes or contributing right there and then when the discussion takes place during class.

2.3. Structural Changes to my Classroom

At Exeter, classrooms are dominated by a large wooden oval table with chairs in the middle of the room, and the walls are either covered with blackboards or whiteboards (see Fig. 2). This provides the optimal solution to the classroom arrangement so that students have access to whiteboard space to present their work. We have purchased a handful of these tables, one for each department: English, social studies, languages, mathematics, and science (see Fig. 2).



Figure 2: Harkness Table

However, these tables are very expensive and our budget only allow having five of them for now. We have an additional one in the Harkness room, mainly used as an administrative meeting room, with some classes running in there too. But if we are dedicated to the approach, we must use what we have and try to create a similar environment in all of our classrooms. What we have is an abundance of individual desks with chairs (see Fig. 3).

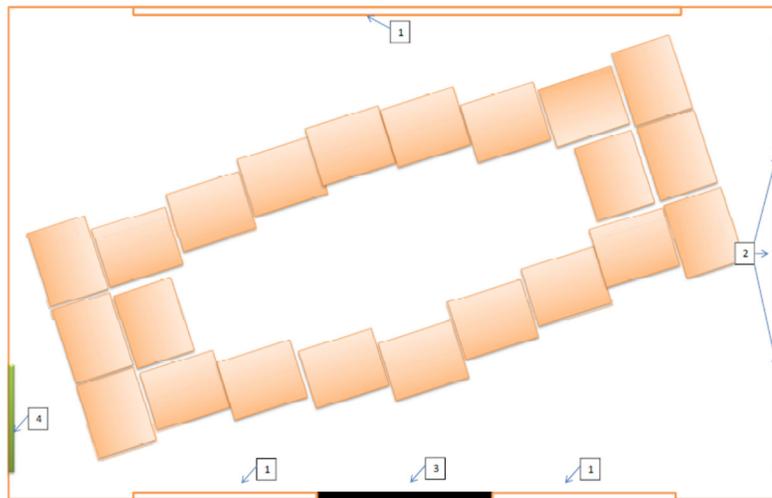


Figure 3: Desks in the Classroom

I tried finding the optimal configuration that resembled the original Harkness table and created a similar atmosphere in the classroom. I had to balance the following factors: physical dimensions of the classroom, number of students, and other classes taught in the room for the ease of transition and delegations with my colleagues on the state of the classroom space. Two configurations were tried (see Fig.4) and I provide my observations in Table 1 about the benefits and downsides. In Fig. 4 the numbers indicate the following:

Layout #1

- 1 - Whiteboards 2 - Windows 3 - Flat screen TV 4 - Door



Layout #2

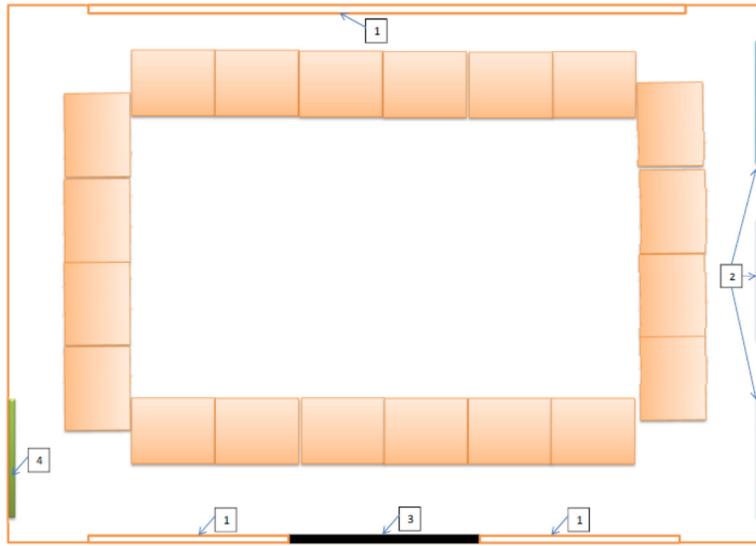


Figure 4: Classroom Layouts

Table 1: Classroom Layout Comparison

Comparison Item	<i>Layout #1</i>	<i>Layout #2</i>	<i>Actual Harkness Table</i>
Dimension of Classroom	Due to the size of the room, the tables had to be positioned diagonally.	This layout physically fit very well in the classroom, with tables positioned parallel to the walls.	Single orientation due to the rigid structure.
Participants' View of Vertical Displays	Because of the diagonally positioned table, those students seated with their backs to the TV or a board, found it difficult to see. They	The view of the vertical displays was better than that of Layout #1, as only one side would be positioned with their backs to the board or a TV. Oftentimes	Very good view of vertical display, as participants can avoid sitting at the ends.

	had to twist their chairs to be comfortable. This made taking notes harder, as they would not have a hard surface to write on.	students simply rotated their chairs to the inside of the layout and then they had a very good view of the vertical display.	
Number of participants seated comfortably	This layout seated 20 participants comfortably.	This layout seated 20 participants comfortably.	Seats 18 participants comfortably.
Access to Vertical Displays	As noticeable from the diagram above, some access to the whiteboards was very limited because of the corners of the desks.	Access to the whiteboards is good; nothing obstructs the movement of students to and around the boards.	Access to the whiteboards is very good; nothing is blocked by the oval nature of the table with no corners.
Participants' view of each other	From its shape and positioning of the desks, participants had a much better view of each other, this layout created a more intimate feeling of a single table that a proper Harkness table provides. On average, participants were closer to each other than that in Layout #2.	From its shape and positioning of the desks, participants had a good view of each other if they were opposite to each other or sitting in perpendicular rows. However, if they were in the same longer row, it provided more difficult to see each other, as there were other students in the same row blocking the view.	Participants enjoy an excellent view of each other, as the oval shape allows everyone seated at the table to see everyone else.

Because it creates the environment that is the closest to the actual Harkness table classroom organization, I believe that the Layout #1 is a better option overall. The quality of conversation and discussion seems to be more influenced by the ability of participants to better see each other as well to be closer to each other. The view of vertical displays and access to the boards is less important, as most time spent in a Harkness class is around the table. As a side note, given the configuration of the tables, the test-taking procedure had to be modified slightly. Because it is difficult to manage the movement of desks back and forth from the test setting (regular rows) and the layouts described above, therefore, we decided to explore the idea of individual dividers and found these produced locally (see Fig. 5).



Figure 5: Exam Dividers

Students were instructed to set these up before the test and take them down after. This was a very efficient practice and quickly became part of the routine. After one or two assessments students learned the procedure and automatically set up the space without reminders. In my observations, the dividers work better than a traditional separation, there are fewer distractions, because students do not see each other allowing them to better focus on the assessment. From a supervisory standpoint, since students are harder to see

with the dividers separating them, the teacher must be walking around to supervise the assessment adequately.

2.4. Role of Technology

The use and integration of technology has been a really positive reinforcement in the process of implementing Harkness teaching. The Smart Board in the classroom has been replaced with a flat screen TV equipped with AppleTV and AirParrot client enabling us to wirelessly project content from the laptops and tablets on to the wall. Not only do teachers have this opportunity, but also every single student does as well. Furthermore, we piloted a project where a class set of portable tablets (see Fig. 6) were purchased and software was installed on students' machines.



Figure 6: Wacom Tablet

Wacom tablets converted students' ordinary laptops to a full-functioning tablet PC. In this way, students were able to present their solutions on the screen and access various programs and apps in turn enriching the discussion and making it much easier to manage and facilitate the discourse. The integration of technology has provided an alternative to

constantly using the whiteboards, and the record of the work could now be kept for future use as well as being available for students who miss class. Also, when using Google Docs together with the tablets, the discussion often becomes more productive, since students are facing each other when working. Thus, everyone feels valued around the table with nobody standing in the spotlight at the board in front of the class. Consequently, this technology innovation has completely removed the need for notes as now all of the students' work can be published on the course webpage and be accessible by anyone enrolled in the course at any time. However, since there was only a single class set of these tablets available, it had to be shared between the teachers and their classes. Therefore, the primary practice has become using the teacher's tablet as the so-called "talking stick", in that it was passed around the room and students volunteered to contribute on the screen where exercises and problems appeared. These were typically from students' workbooks that they had a hard copy of with them at all times as well. It naturally evolved into an informal rule that, when someone has the tablet, the others are listening and watching what is happening on the screen. When a question comes up, students would speak up and the student with the tablet would have to defend his/her ideas. This strategy of passing the tablet around the room has given students, who are of a more reserved nature, a chance to contribute their thoughts and ideas to the class in a more structured way. The student with the tablet at their desk acted as the leader of the discussion with his/her peers listening in and contributing whenever appropriate, thus making the discussion more authentic.

2.5. Role of a Teacher

Such changes to the classroom and departmental settings does not come quickly. It has been a rather long process starting with an investigation, small scale tryouts, scope and sequence plans, and implementation stages with benchmarks set out for years to come. Our school has developed Westridge School Academic Model in which the student-centered collaborative teaching approach (i.e. Harkness teaching) fits very well (see Fig

7). Such sections as “Critical Thinking”, “Communication and Collaboration” and “Creativity and Innovation” are related particularly well and well-suited to class discussions. This also echoes British Columbia Association of Mathematics Teachers vision (BCAMT, 2015) and National Council of Teachers of Mathematics Process Standard of Problem-Solving, Communication, and Connections (NCTM, 2000).

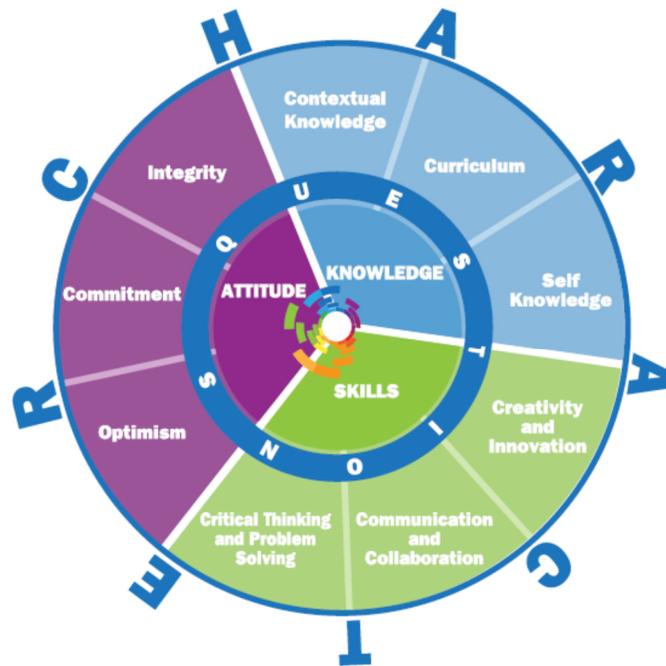


Figure 7: Westridge School Academic Model

Furthermore, it aligns well with the Core Competencies of the current Provincial Curriculum by the BC Ministry of Education (BC Ministry of Education, 2010). These competencies reassure us that we are indeed going in the right direction and when we are confronted with tough moments trying to justify our practices, we always go back to these and double check that we are aligned with curricular institutional goals. At the same time, the difference between Exeter is that we are mandated to follow our current Provincial Curriculum and teach the Prescribed Learning Outcomes (PLOs). Preparation for a discussion-based class is not an easy task. Personally, the idea of “Backwards Design” looked the most usable in this case (Grant & Wiggins, 2005). By considering the final

prescribed learning outcomes that students must learn and master, and then looking at the available materials, one can narrow down a list of questions, problems, and explorations that students could use to get to that desired result. However, since the bulk of conversation and ideas come from the students, there are various forms and directions which the conversation can take. The teacher needs to be very confident in the subject matter to be able to respond to ideas being brought up. For example, when discussing slope in the first lesson in Foundations and Pre-Calculus 10, students may arrive at a point they start talking about an equation of a line. However, in the linear plan of a standard textbook this section does not appear until lesson four. The teacher needs to decide if they want to change the plan and go right into that learning outcome. This is just one example, but there are other potential issues where the question posed may be left uncovered for a year, or if the teacher is comfortable and competent in the course material, the class can dive into that exploration when it arises. At the end of the day, as stated by the visiting Exeter faculty member Thomas D. Seidenberg, “students need to be confident that when they walk out of the classroom, they know that they have learned what they were supposed to, that is the teacher’s job” (T.D. Seidenberg, personal communication, October 2014).

2.6. Going Further

Now that the environment and setting are established, I would like to talk about discussions themselves. Sometimes class discussions are very slow as students come unprepared to class or they are tired and it is very difficult to keep the conversation going. Sometimes all I have to do is ask one question, and the discussion takes off for the next fifty minutes. Other times, the slow conversation takes off as it is fueled by something all of a sudden. Students become interested in the topic and discussion reaches a point when the class is completely engaged and many students are contributing their thoughts and opinions. I became interested in understanding this further: what makes these discussions turn around, and can the discussion be steered towards this crucial climax state?

In what follows, I will explore some of the literature on whole-class discussion in Chapter 3, my methodology in Chapter 4, data analysis in Chapter 5, my discussion on the data in Chapter 6 and finally my conclusions in Chapter 7.

Chapter 3. Literature Review on Discussion

3.1. Overview

Classroom discourse, and in particular whole-class discussions, have become a very popular topic of mathematics education research around the world. As more research advocates for alternative methods of instruction, educators seek new information about the different and possible approaches for differentiating instruction to balance out traditional teaching techniques prevalent in so many classrooms. To take an economical point of view about this gap, there is a very significant demand on information, development and ways of implementing these methods. This is why it is so important to continue to researching whole-class discussion approaches as a teaching method in a mathematics classroom. Better understanding and improvement of the method will lead to more support from various stakeholders, including education leaders, practitioners, parents and learners.

In this literature review, I will discuss a collection of mostly recent articles and books, on discussions in a mathematics classroom spanning from 2002 to 2011, and their implications on teaching and learning, as well as the possible direction for future research. The texts classify into 3 categories:

1. “Why Discuss?” - overview of some evidence on the benefits of class discussions.
2. “How to Discuss?”- some necessary conditions and attributes for good discussions to take place in the classroom.
3. “Tracking Tools and Theoretical Literature” - some new tracking tools for monitoring and analysing class discussions as well as theoretical framework used in the analysis of the data.

3.2. Why Discuss?

Nick Fiori and Jo Boaler (2003) have conducted a research study where they examined videotapes of 40 groups of three or four students working on an open-ended mathematical task for 90 minutes. The researchers developed and implemented a tool which allowed them to keep track of, and analyze, the relationships developed by students and mathematics. They looked at the following: student knowledge and proficiency with major mathematical ideas, as well as how students apply this knowledge to solve problems; if students saw themselves as mathematicians, as well as their relationship with mathematics: what students perceived “doing” mathematics means; students’ mathematical conversations and how/if they reflect the kind that professional mathematicians would engage in. By looking at these investigative areas and analysing the results (reviewing and coding of recordings) the researchers found two major issues. The first one showed that there is a large gap between the written work of a group and a discussion of the same task students worked on. Even though the written work may have appeared coherent and logical, it was missing all the “process” elements of figuring the task out. There were a number of misconceptions that came up in the discussions, but these misconceptions were completely lost in the written portion of the work, as they were dismissed in light of the so called “good copy” of the problem that students submitted when they solved the problem. The second finding confirmed that mathematical work starts on an intuitive level and then transforms into concrete and “fluff-free” summary. In addition, the authors claim that class discussions are a good element to have in a mathematics classroom as it promotes engagement and authenticity of learning mathematics, as mathematicians do. Mathematics starts with a problem and all kinds of trying and a whole lot of figuring things out before it becomes a coherent solution on paper.

A further insight into the whole-class discussion in a mathematics classroom is given by Weber, Maher, Powell & Lee (2008), where the authors share their findings and experiences as well as students’ participation and dialogue in a longitudinal study over three years. Researchers observed a cohort of 24 students during school and after school hours. They have given the students interesting and engaging mathematical tasks and problems. Students were asked to communicate their solutions and approaches as they

were completing these tasks. The researchers had two main goals: show more examples where discussions and collaborative nature of doing mathematics can benefit the learning of students and illustrate the social and environmental conditions that make discussions productive and engaging. I will talk about the second goal a little later with the following set of literature pertaining to the implementation of the class discussion method of teaching. As for the contributions of discussions to mathematical learning, the authors' theoretical basis was Toulmin's model (Toulmin, 1969), in which argumentation consists of three essential parts, called the core of the argument: the claim, the data, and the warrant. The researchers included the script of the discussion about a statistical problem of fair dice. From this sample of discussion and other evidence collected, they concluded that such productive discussions bring forward the kind of mathematics and statistics educators want their students to learn. They occur naturally through the discussion and debate about students' solutions and approaches to the given tasks. Both of the above articles and research findings echo one another: class discussions can develop some key elements of an engaged discussion and thus an engaged classroom, often unique to the approach. It is, however, imperative to talk about the specifics of whole-class discussion approach. In the next section, it will be shown how the elements of actual classroom discourse allow for successful implementation of the method.

3.3. How to Discuss?

The second finding of Weber, Maher, Powel, and Lee (2008) was that giving students sufficient time and freedom of speech created an environment where risks could be taken and productive discussions could emerge. The role of a researcher is summarized here:

The researcher's role was modest, primarily reinforcing the norms of the classroom that had been previously established. For instance, she asked students to share their conclusions and provide justifications for those conclusions and she reminded students to attend to one another's arguments. (p. 14)

Here we see the role of a teacher that was most successful for the discussion approach to teaching.

Furthermore, in her PhD thesis, Rebecca McGraw (2002) argues that for a successful whole-class discussion more voices need to be heard, and thus there needs to be some sort of control over who is speaking and how much. This role is assumed by the teacher, but not in the form of lecturing about the course content. The teacher becomes a facilitator, thus moving away from a central role in the classroom. Taking this idea further, Cory A. Bennett (2009) dives into the different content knowledge that teachers must possess in order to facilitate a productive discussion. The researcher classified a series of episodes and corresponding teacher instructional actions. He also has considered the different kinds of knowledge: MCK (Mathematical Content Knowledge, the knowledge of mathematical ideas and notions, something a mathematician possesses), CCK (Common Content Knowledge, the knowledge of mathematical ideas by general individual), KCS (Knowledge of Content and Students, knowledge of how students learn mathematical content), KCT (Knowledge of Content and Teaching, used to do the mathematical work of teaching). These four types of knowledge make up MKT (Mathematical Knowledge for Teaching). Bennett (2009) argues that all of these sets of knowledge must be at a teacher's fingertips to successfully facilitate class discussions. The researcher kept track of the type of instructional actions given by the teacher and the number of times it occurred. The results from this study demonstrate that MKT is important in the way teachers pursue student thinking. All four domains of MKT supported teachers in using instructional actions that are critical in creating opportunities for extending student thinking. The teachers' MKT supported them in deciding which mathematical ideas to pursue and how to pursue them during whole-group discussions. This type of knowledge gives teachers an ability to think on the spot as well as reflect upon the learning and classroom progress.

Finally, Margaret Smith and Mary Kay Stein (2011) create a comprehensive summary of many works on classroom discussions in their book, published by the NCTM called "Five Practices for Orchestrating Productive Mathematics Discussions". They propose the following five practices in the discussion lesson implementation sequence:

anticipating likely student responses to challenging mathematical tasks; monitoring student's actual responses to the tasks (while students are working on the tasks, thus performing formative assessment); selecting the students responses, sequencing the responses to be displayed in a specific order; connecting different students' responses to key mathematical ideas. In their findings, the authors stress the careful planning of these types of lessons when discussion is the key element. Anticipating student responses is vital to a productive discussion as this gives the teacher a playing field to steer the discussion in the desired direction, like learning outcomes. This careful planning makes the other four practices useful and actually feasible. However, teachers need to be mindful of their own input versus students' input into the discussion, since his/her voice has a tendency to take precedence and student discussion may lose its authenticity if it turns into yet another giver and receiver mode: the passive mode of learning.

3.4. Tracking Tools and Theoretical Literature

When looking at such a complex phenomenon as students' interactions, there is an absolute need for an effective and useful tracking tool. One such example is the Interactive Flow Chart, pioneered by Sfard and Kieran (2001) and adapted by Ryve (2006) and Liljedahl and Andrà (2013). Sfard and Kieran (2001) use it as a way to capture "two types of speaker's meta-discursive intentions: the wish to react to a previous contribution of a partner or the wish to evoke a response in another interlocutor." (p.17) Thus, a conversation can be thought of as being comprised of a series of invisible arrows directed to specific people and/or specific utterances. Sfard and Kieran (2001) developed a coding scheme to make these arrows visible (see Fig 8). The scheme follows two basic structures:

1. A vertically or diagonally upward arrow is called a *reactive arrow* and points towards a previous utterance.
2. A vertically or diagonally downward arrow is called a *proactive arrow* and it points towards the person or people from whom a reaction is expected.

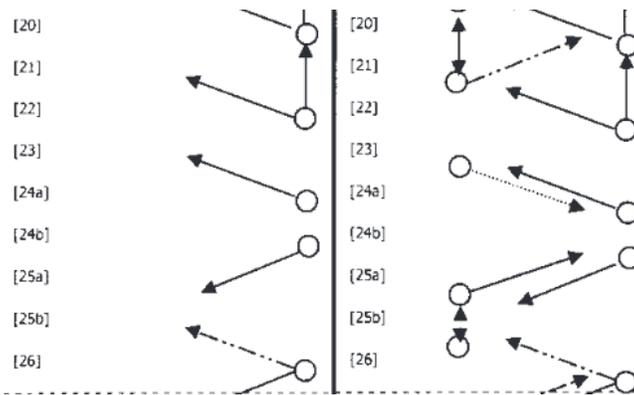


Figure 8: Sfard's and Kieran's Flow Chart Example (Sfard & Kieran, 2001)

Here from line [24a] we see a reactive arrow. The utterance represented by this arrow is a reaction to something that came from line [23]. Also, a proactive arrow from [24b] means that this utterance is meant to invoke a reaction from [25a].

Ryve (2006) focused on an important aspect of interactive flowcharts. The aim of his study was to suggest two ways of looking at mathematical discourse. Not only was he looking at the contextualized setting and simply tracking who was talking to whom, but he was also interested in the nature of utterances. He divided the types of utterances into mathematical and non-mathematical. Based on his data from a study of how four groups of Swedish engineering students worked on problems in linear algebra, he showed that “the two complementary analyses make the construction of the interactive flowcharts more coherent and transparent, and hence, more reliable” (p.1). In addition, the two analyses made it clear when a discourse was mathematically productive or not. Those that had many non-mathematical utterances were considered not productive. This provides further insight into the study of discussions in the classroom. An example of the Flow Chart from Ryve is as follows (see Fig. 9).

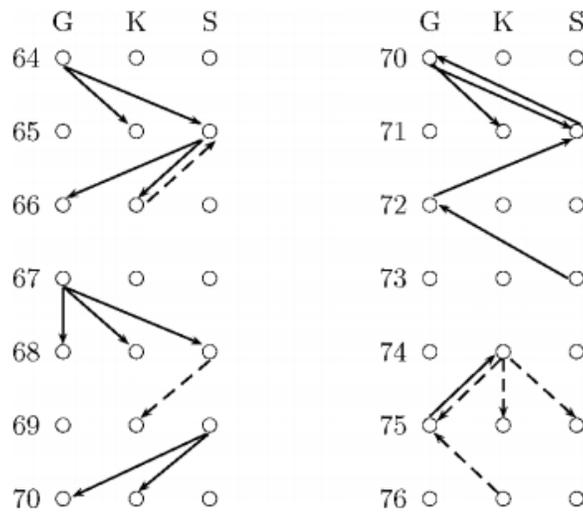


Figure 9: Ryve's Flow Chart Example (Ryve, 2006)

G, K, and S are the three students and figures [64]–[76] are the numbers of the utterances performed in the discourse. Therefore, in [64] G is addressing the other two students, K and S. The solid arrow indicates that the utterance is of a mathematical nature. In [65], S is addressing G and K leading to a reaction by K in [66]. The arrow representing utterance [66] is dashed, indicating that the utterance is of a non-mathematical nature. The utterance of [74] was interpreted as a non-directive question, meaning that K addresses himself as well as the other students. Liljedahl and Andrà (2013) have looked at learning as follows:

Learning occurring in and through relations with others driven by collectively motivated activity. Activity is a process with inner contradictions, differentiations, transformations, as well as emotions—necessary for the activity and responsible of its development. (p.1)

They have added the idea of gaze and tried to track that as well. That is, they represent, using a new set of arrows, where someone is gazing during each utterance. They use red arrows to represent the speaker and blue arrows to represent non-speakers. They also introduce a new “participant” to the interaction – the paper (P) with the problem on it. This paper holds the gaze of the participants at different times of the conversation – so much so that they do not code blue arrows when the students are looking at the paper. Unlike the arrows representing utterances, all of the gaze arrows are diagonally downward to represent the passage of time (see Fig. 10).

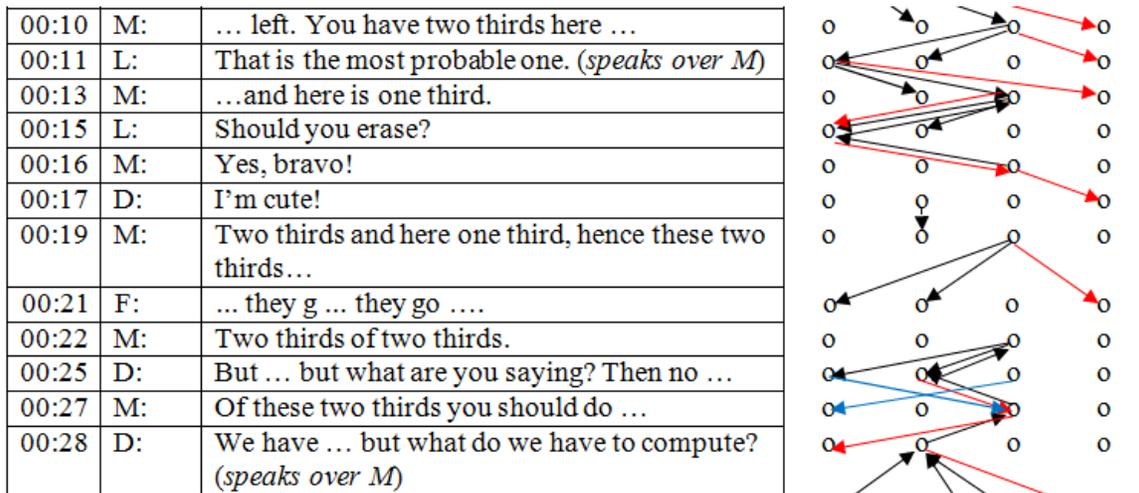


Figure 10: Liljedahl's and Andrà's Flow Chart Example (Liljedahl & Andrà, 2013)

In the Liljedahl and Andrà's analysis and conclusion they found that:

The interactions between four subject students have a turbulent undercurrent of emotions and intentions. The use of interactive flowcharts documenting the verbal interactions and the gazes gives a window into these emotions and intentions. (p. 10)

3.5. Research Questions

I believe it is important to keep in mind when looking at the material presented here not to only focus on the empirical data presented but also to reflect on the experiences attained through the process of the investigation and implementation of the Harkness teaching in a mathematics classroom. It has been a challenging and rewarding process to go through for the past three years, and it continues to dazzle me with its twists and turns: the more I explore it, the more I see to explore further. I should point out that this approach is a very unique one, and I have not been able to find anyone else in the world combining Harkness teaching and elements of a flipped classroom yet. There has been no literature dealing directly with this combination that I could find. It creates a feeling that there are not that many practitioners using whole-class discussions for now. However, my colleagues were able to develop some partnerships and connections with those schools that are doing similar things, and also those individuals who are interested to learn from our experiences. I was fortunate to speak about my findings and model the Harkness class at a couple of conferences in the past year. The first one was the annual BCAMT conference in the Fall of 2013. Those sessions were very well attended and the participation level was very good. In these sessions, I tried to model the Harkness discussion among the conference attendees to give them an idea what a whole-class discussion may look and feel like in a mathematics class. These sessions turned out to be very well-received and generated excellent discussions afterwards. Many fellow mathematics teachers were excited to learn about Harkness teaching as they asked very good questions, relating to what this might look like in their schools and classrooms. My second talk was at Hawaii International Conference on Education 2014, where I shared my early results from the data collection and some general facts about Harkness teaching in mathematics. This talk was pretty well attended and good discussion resulted in the end. As this conference was in the United States, common core competencies were brought up and all the connections between them and what my colleagues and I are trying to achieve with the approach to instruction and teaching in our classrooms. This kind of feedback tells me that this research is of value to people in our field. It seems interesting to both practitioners and academia members.

As I kept engaging with whole-class discussions, I noticed that there are different levels of engagement from my students. Sometimes the discussions would take off and the entire class would be completely involved with the noise level escalating in the room. And sometimes the discussion would run flat with only a handful of students participating while the rest of the class sat passively with little or no signs of interest in what was going on. I started to wonder if there are ways to analyse this data I was observing every day. Can we classify the types of behavior, conversations, moments of engagement or disengagement, more intimate discussions between smaller groups of students, types of questions and answers, as well as types of responses given? From my own experience, I have noticed what I call the “climax” of a discussion. These are the moments when the room is overflowing with ideas and everybody is engaged in the conversation. My research questions are as follows: What leads to these climaxes? How can we make sure that the curriculum big idea is a central focus? Subsequently, can we classify the types of discussions that happen? Is there a way to analyse discussions further and learn from their analysis? What mechanism can be used to keep track of utterances and identify climaxes and levels of engagement in the room, and how and why do they happen?

Chapter 4. Methodology

4.1. General Setting: School Profile

Our school is a K-12 university preparatory institution, a member of Canadian Association of Independent Schools, with a very good reputation in the local and wider communities. We are also fortunate to have a team of dedicated and very professional faculty. All of our faculty have at least a minor degree in the subject matter they teach, with the overwhelming majority holding majors and graduate degrees in their disciplines. In addition, the school invites relevant professional development speakers and facilitators every year to help us develop and refine our approaches to teaching on an ongoing basis. Personal professional development is encouraged with funding provided for leave coverage, registrations and travel expenses. There are just under 800 students combined in both the Junior and Senior schools, additionally, the Fraser Institute reported our rank as of at least top five schools in the most recent five years. This rating includes all the secondary schools in British Columbia. Estimated tuition costs per student are \$19000, depending on the extra-curricular activities they choose to pursue. Overall, every family that send their children to this school are expecting a rigorous and accountable educational program to be delivered.

4.2. My Classroom: Layout and participants

4.2.1. Physical Space

As noted earlier, for the research presented here the physical layout of my class can be seen in Fig.11, in which the students and the teacher were seated around the table.

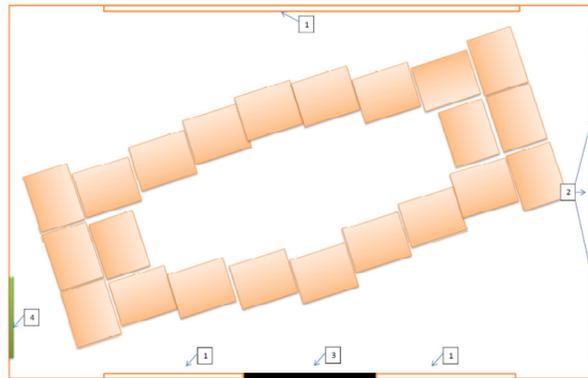


Figure 11: Preferred Classroom Layout

4.2.2. Participants

In the year of my data collection (2012-2013), I had one Grade 8 mathematics class (18 students), two Grade 9 mathematics classes (18 and 22 students), one Grade 10 mathematics class (11 students) and one Foundations 11 class (9 students). (Note that one class is over 18 students, as timetable did not allow this class to be 18 or less) These students come from middle to upper class families. Parents of Westridge School are very concerned and attentive to the education and practices of the school. How their child is educated is very important to them and they often contact teachers inquiring into the progress and results of their child. Thus, it is expected that students will be more motivated to learn and take part in class activities. They respect and trust their teachers and value their contributions and professionalism.

4.2.3. Typical Lesson

The year of 2012 was Westridge's School first year of implementing Harkness teaching in its classes and thus it needed a bit of time to get comfortable before the discussions became spontaneous. Students were slowly encouraged to talk about their ideas, thoughts, opinions, solutions and approaches. More of a group work approach was used at the beginning of the year where students were put in random groups to work together on problems and exercises, and then vertical spaces (white boards and windows) were used to present to the class. Students have become accustomed to writing their solutions

and attempts on problems on the whiteboards. Overtime these presentations looked and felt more than just a giver and receiver of knowledge: the dialogue was emerging between the members of the presenting group and the rest of the class. When compared to student presentations before class discussions, they tended to be only the showcases of students' work. Now students got more comfortable asking each other questions and did not look over to the teacher for feedback. To provide some structure for this (consistent with McGraw's research, 2002), in particular for the junior classes, a rubric was created by our collaborative efforts in the department (see Fig. 12). This rubric is used to assign an effort grade at the end of each term. Students are asked to use this rubric to self-assess their discussion skills at the end of term, then submit it (usually electronically) for their teachers to look over. Teachers make comments and assign a grade for the effort in class with both students and teachers agreeing on it after a discussion.

Harkness Skills Rubric

Name: _____

Self-Score: _____

Teacher's Score: _____

Compromise Score: _____

	1	2	3	4
Criteria	Almost Never	Sometimes	Often	Almost Always
Come in ready to work, listen, think and contribute				
Talk directly to your classmates, not to the teacher				
Ask questions				
Answer questions from your classmates				
Contribute ideas and different solutions on all the homework problems				
Offer to show something that you tried that maybe didn't work. Put challenging problems on the board, even if not correct				
Point out connections to other math ideas and problems you have learned				
Speak up when you don't get it and would like to see/hear it again or from another point of view				
Help find errors at the board				
Stay involved in the discussion				
When finished working on a problem, look to help others				

Figure 12: Harkness Rubric

For the most part, students were much harder on themselves, so oftentimes their final grade was higher than their self-reported one. After winter break, the classes felt like they were vastly improved. Typically, a class would start with a discussion about homework. If there were questions, students knew to put those on the board as soon as they came in the room before class started. This meant that students, who had those questions, would put their attempts at solutions up and go through them, explaining what they did, and where they had an issue. Most of the time we would have one or two problems to go over. This proved to be a very fruitful exercise for everyone to both contribute and also correct their mistakes. After that, there would be some sort of an exploratory activity or a problem to work on either individually or in groups to get some exposure to the new idea. Typically, there would be a reading or a video that students were supposed to engage with at home prior to coming to class. Not all students came to class prepared. However, it did not take long for students to realize that discussion was not going well for those who were not prepared. Thus, the homework completion rate went up naturally, as students understood that there was lecturing going on in class. Those who came prepared felt that they were able to engage with the problems and exercises, often becoming leaders during whole-class discussions. When students spent some time on the problem or exploratory activity, I would pose a question to the class or ask for a volunteer to start the discussion. Normally, the question would be on my tablet, so I would pass it to the student or a group who volunteered to give their interpretation or solution to the problem. If someone else had something to say, they would pass the tablet to that group or individual. As mentioned earlier, the tablet acted as a “talking stick”. I will elaborate more on this in Chapter 5. After we had exhausted the problem or exploratory item, students would be put into groups and each group would be assigned a question from their workbooks from the section that was the focus of the day. They had to put the solution either on the board or the tablet and be ready to present to the class. Presentations followed, with each group leading a discussion, while the rest of the class listened and asked questions, and made notes, etc. At the end of class, there would be some sort of a summary discussion or a collection of questions to work on until the end of class, at which point the new homework was assigned, whether it was a video or a reading, or questions to explore. The summative assessment would be done at the end of each chapter via a

chapter test. Also, a variety of other formative assessment was used during class. For example, in-class assignments to be discussed, and online multiple choice quizzes provided feedback right away to students and gave a natural transition to discussion of common mistakes and misconceptions. These quizzes were particularly well-received during exam review at the end of the year when students needed a quick reminder of concepts learned during the whole year.

4.3. Research Procedure

4.3.1. Data Collection: Flow Charts

Some of the classes were videotaped with two cameras: one stationary at the back of the classroom and one portable web camera attached to my ear. This way my data was collected on a larger scale, capturing the entire classroom as well as on a smaller more personal scale recording the conversations that were close to me or involving me as well. It also captured the direction of my view at all times, thus giving me an opportunity to notice what I was paying attention to during class. The recordings were transcribed and then, using the Flow Chart tracking tool, discussions were analysed resulting in a handful of emerging themes. During analysis, I focussed on identifying engaged versus unengaged discussions. I asked questions, such as: “What prompted the engagement?”, “When did the engagement occur?”, “What was the student’s body language in the engagement?”, and lastly “What is a teacher’s role in discussion and when is it effective to step in and when is it not?” I have modified the Flow Chart tool to suit my needs and the classroom environment I have created. The purpose of this tool is to keep a record of the discussion happening in the classroom, the type of utterances and the direction of the replies. I attempt to identify the “climaxes” where the level of engagement is increased and most students are involved in the discourse. The analogy to this as I see it is as follows: if you pretend that each participant is a light bulb, when they are actively engaged (can be seen from their body language as well as direct participation in the discourse), the

room lights up with the activity, as the number of utterances and number of people taking part in the discourse drastically increases to a crescendo. Types of information I kept track were: type of utterance (math or non-math related) (Ryve, 2006), direction of the utterance (concrete individual or the whole class), direction of gaze (Liljedahl & Andrà, 2013), location of tablet (oftentimes the tablet was sent around the room and students used it as main presentation tool and a talking stick when leading a discussion). Below is the legend of the modified Flow Chart tool used (see Fig. 13).

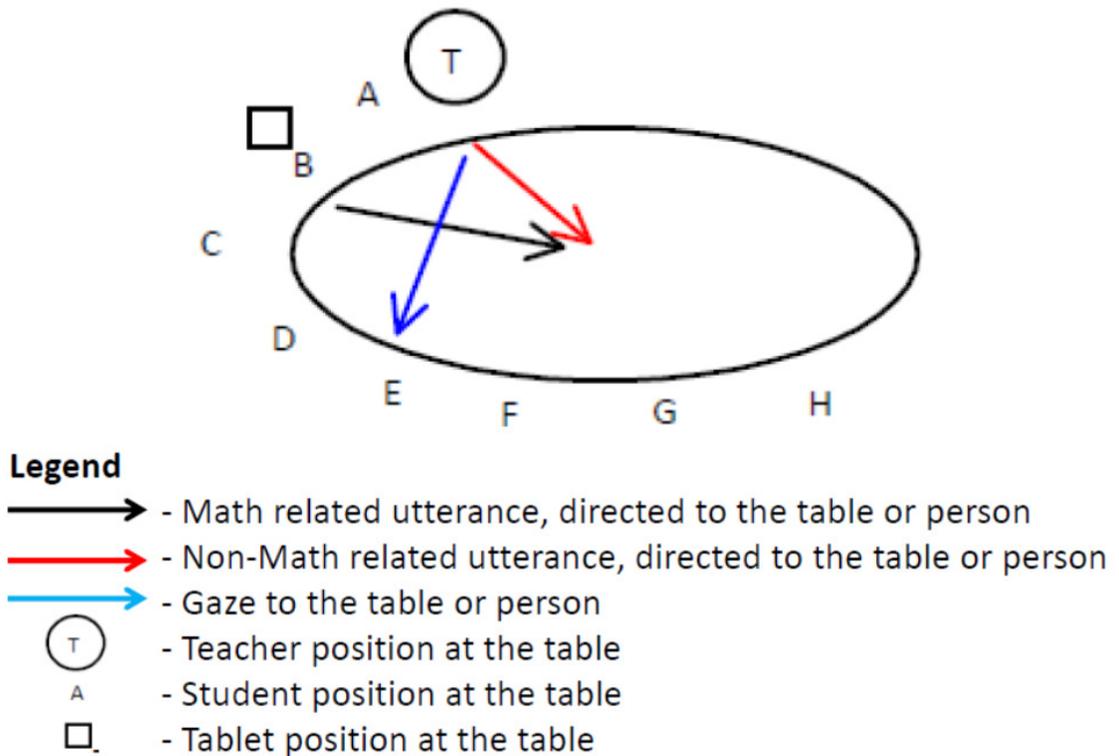


Figure 13: Modified Flow Chart

Below are examples of discussions from a Math 8 class as they are discussing ideas about statistical data and its representations. The data is presented in its transcript form and flow chart form (see Fig. 14 and 15).

Time, Name, Utterance	Diagram
[0:44] T: Do we agree with what E said? [0:45] D: Yeah! [0:45] A: No!	
[0:46] E: What?! [0:47] T: Why not? [0:48] H: Yeah...?	
[0:49] A: If you look at the H2O of the graph, it goes up by less, so it is less of an incline... [0:57] E: Yeah, but so does the other one too... [0:57] A (speaking over E): the other one is not as much so it looks like it goes up more. [1:00] T: OK, good.	
[1:02] T: There is some validity in both. [1:04] A: And was not even here that day, E! [1:08] D: It seems like bottle shell is more...	
[1:10] T: So, going all the way to question d) now. Which one? [1:16] C: H2O! [1:17] B: H2O!	
[1:19] G: Mountain! [1:20] A, B, C: H2O! [1:21] G: Mountain! [1:22] T: Wait a minute, this is kind of interesting, I didn't expect that... Why?	
[1:26] D: If you didn't look at the numbers, the H2O looks like it rises more rapidly... [1:31] G: That's bad! Which is bad! [1:34] F: If it's higher, then they get more money.	
[1: 36] - [1:47] All talking, unable to pick out individual utterances	
[1:47] D: Oh, I thought that like it shows how good the company is doing. [1:52] G: No! [1:53] T: If you take this as a perspective of a stock market, then... [1:59] B: So, its Mountain?	

Figure 14: Sample Discussion 1

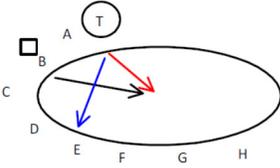
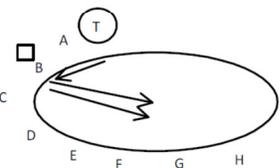
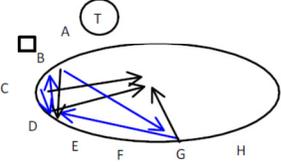
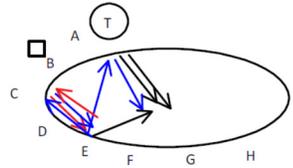
Time, Name, Utterance	Diagram
<p>[0:02] B: So for 1a it was...</p> <p>[0:05] T: Shush!!!</p> <p>[0:07] B: it was list two things you know from the bar graph, and so we noticed that trumpet is the most popular instrument and clarinet is the least popular instrument.</p>	
<p>[0:18] B: And then for 1b, list two things you notice from the circle graph we know that trombone and percussion are both at 18 percent, and that the flute and the saxophone are in like the middle range from the lowest and the highest. What are... 1c, what are some advantages or disadvantages of a circle graph?</p> <p>[0:46] T: Yeah...</p> <p>[0:47] B: Oh no, bar graph... It was, so an advantage was like the total, you can tell the total amount of students, and its not completely accurate, because like, it's not as exact. And then 1d was some disadvantages and advantages of a circle graph. An advantage is that you can tell clearly the sections like they are very individual, and then the disadvantage is that it does not like say the total amount of students. And then 1e is yes you can use the line graph because it's pretty much the same as a bar graph, just showing it in different way.</p>	
<p>[1:42] D: How you could use a line graph if there is no time here?</p> <p>[1:46] G: You mostly use it for time, but not always.</p> <p>[1:50] B: You could use it.</p> <p>[1:53] C: It basically becomes a bar graph</p>	
<p>[1:56] T: It's not very useful, but you could. And one way to make it happen is what you do, you take the bar graph, then you just connect the tops.</p> <p>[2:02] E: Yeah, that's what I told F!</p> <p>[2:05] C: And save the planet...</p> <p>[2:07] E: You could!</p> <p>[2:10] T: Very good, number 2.</p>	

Figure 15: Sample Discussion 2

As I was coding the data, I focused on the origin of each of the climaxes. That is, for each episode of discussion I have identified how the highlighted episode began and what in particular served as a starting point of discussion that led to a climax. In the next chapter, I discuss the analysis of these and other such coded transcripts. Since the lessons happened in my classroom, I refer to myself as “the teacher” in my data analysis.

Chapter 5. Data Analysis

5.1. Introduction

In this chapter, I will outline my research findings and data collected over the course of 2012-2014 school years. Also, I will analyse the data and present my findings showcasing new revelations about discussion-based teaching and learning.

As seen from the methodology chapter, I have adapted and modified the flow chart model to track the discussions in my classes. As data was collected, I was trying to classify and find emerging themes from the video recordings and the coded information via flow chart diagrams. As a quick reminder, below is the legend for the diagrams (see Fig. 16).

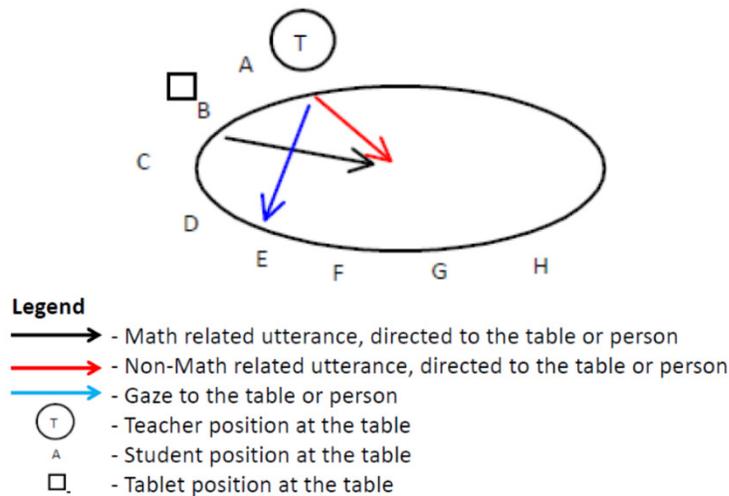


Figure 16: Flow Chart Legend

5.2. Discussions

In this section I will present a number of discussions coded with the help of the modified interactive flow chart. The following categories have emerged as I was coding the data: Teacher Poses a Question, Student poses a Question, Breakthrough, and Student-led Explanation This classification came to me, as all of the coded discussions would fall into one of these categories. Below I will try to identify as many details as possible in order to gain a better understanding of these complex entities.

5.2.1. Sample Discussion 1

The first discussion is from a Math 8 class, while they are working on identifying statistical discrepancies and misrepresentations in various examples from a worksheet they were given in class. The worksheet is projected on the screen and the teacher has the tablet the whole time. Students have the paper copy of the worksheet. Groups chose their spokesperson to present their findings and solutions to the class. The question students were working on is below (see Fig. 17).

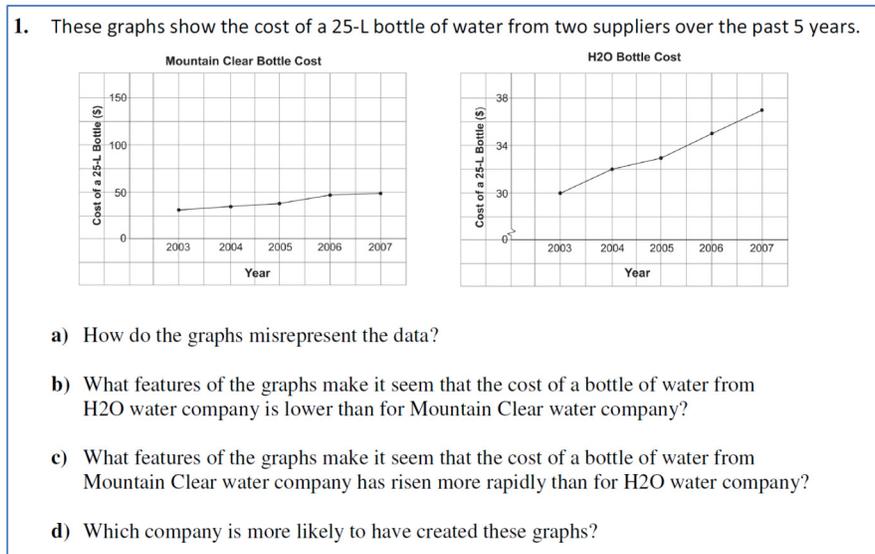


Figure 17: Students' Questions from Discussion 1

The modified interactive flow chart coding is presented below (see Fig. 18):

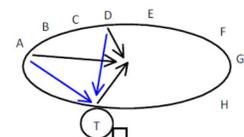
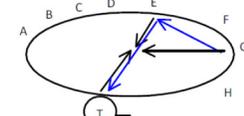
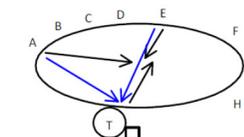
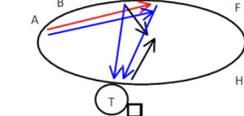
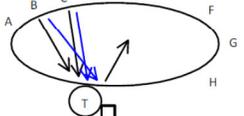
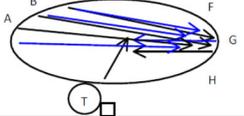
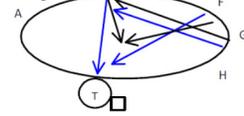
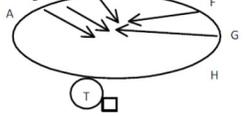
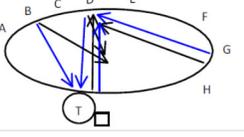
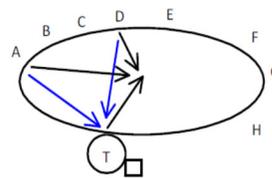
Time, Name, Utterance	Diagram
<p>[0:44] T: Do we agree with what E said? [0:45] D: Yeah! [0:45] A: No!</p>	
<p>[0:46] E: What?! [0:47] T: Why not? [0:48] H: Yeah...?</p>	
<p>[0:49] A: If you look at the H2O of the graph, it goes up by less, so it is less of an incline... [0:57] E: Yeah, but so does the other one too... [0:57] A (speaking over E): the other one is not as much so it looks like it goes up more. [1:00] T: OK, good.</p>	
<p>[1:02] T: There is some validity in both. [1:04] A: And was not even here that day, E! [1:08] D: It seems like bottle shell is more...</p>	
<p>[1:10] T: So, going all the way to question d) now. Which one? [1:16] C: H2O! [1:17] B: H2O!</p>	
<p>[1:19] G: Mountain! [1:20] A, B, C: H2O! [1:21] G: Mountain! [1:22] T: Wait a minute, this is kind of interesting, I didn't expect that... Why?</p>	
<p>[1:26] D: If you didn't look at the numbers, the H2O looks like it rises more rapidly... [1:31] G: That's bad! Which is bad! [1:34] F: If it's higher, then they get more money.</p>	
<p>[1: 36] - [1:47] All talking, unable to pick out individual utterances</p>	
<p>[1:47] D: Oh, I thought that like it shows how good the company is doing. [1:52] G: No! [1:53] T: If you take this as a perspective of a stock market, then... [1:59] B: So, its Mountain?</p>	

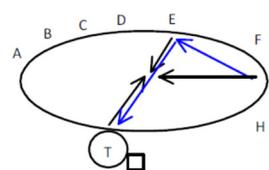
Figure 18: Flow Chart of Discussion 1

[0:44] T: Do we agree with what E said?
 [0:45] D: Yeah!
 [0:45] A: No!



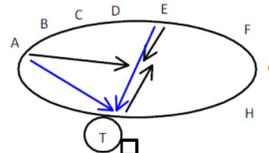
Student E is leading the discussion, when the teacher steps in with a question [0:44] “Do we agree with what E said?” (8) Both A and D are looking at the teacher when speaking, but the whole class is hearing this [0:45]. This prompts E to express the confusion and

[0:46] E: What?!
 [0:47] T: Why not?
 [0:48] H: Yeah...?



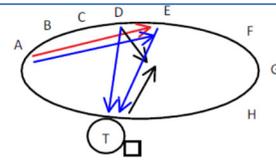
disagreement, while looking at the teacher for support. [0:46] The teacher genuinely wonders why students are divided in their opinions [0:47]. H enters the discussion at this point, demonstrating some disagreement [0:48]. A defends her point of view. Sfard and Kieran (2001) call this type of utterance reactive, where a student is reacting to the statement of his or her partner; this is important for the flow of the discussion [0:49]. She

[0:49] A: If you look at the H₂O of the graph, it goes up by less, so it is less of an incline...
 [0:57] E: Yeah, but so does the other one too...
 [0:57] A (speaking over E): the other one is not as much so it looks like it goes up more.
 [1:00] T: OK, good.

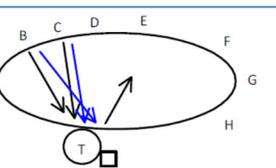


notices that the scale in the H₂O graph is mislabelled and thus creates a visual representation of higher increase. She goes up to the screen (not evident from the modified flow chart), and points it out for everyone to see. E is trying to defend his point [0:57], but A is determined to prove him wrong and continues her explanation over E [0:57]. As Weber, Maher, Powell and Lee (2008) determined, a classroom needs to be an environment where risks and mistakes happen, and it is not uncomfortable for the learners to make mistakes. This will encourage discussions and promote the natural flow of discourse. Teacher gives an approval statement [1:00]. As noted by Bennett (2009), teacher knowledge must dominate and be engaged at all times for discussions to be

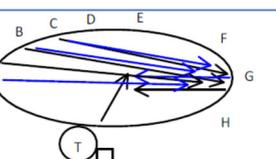
productive so that the teacher can assess and facilitate the discourse in the correct direction.

<p>[1:02] T: There is some validity in both. [1:04] A: And was not even here that day, E! [1:08] D: It seems like bottle shell is more...</p>	
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The teacher continues to maintain a neutral state [1:02], giving more opportunities for further discussion. A goes on a personal attack, while looking at E, saying that E was not there on the day when we talked about the idea of changing scale and how it affects the graphs. Here I use Sfard's and Kieran's (2001) idea to track the non-mathematical utterances, as it is a very important aspect of a good discussion. As seen here even though the utterance is non-mathematical, it is adding the human aspect to the discussion, making it more authentic. Keeping track of these allows the researcher to see a clearer picture of the discussion, as one can look at the level of engagement in the discussion. E continues to look at the teacher waiting for the ruling. [1:04] D chimes in, but it is unclear what he means by his statement. The teacher now is trying to identify the split of opinions in the class and calls for an answer from the group to decide which company made these graphs. Once again, we see the invoking of teacher knowledge (MCK) as per Bennett (2009) [1:10] C and B both call out H2O, both looking at the teacher [1:16], [1:17].

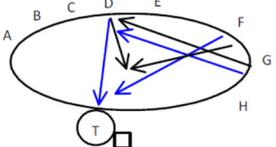
<p>[1:10] T: So, going all the way to question d) now. Which one? [1:16] C: H2O! [1:17] B: H2O!</p>	
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G now disagrees with the majority [1:19] and A, B, C are all almost yelling at G. Then the gazes are turned away from the teacher and are directed to G [1:20] who seems to be with the minority, but continues to stand his ground [1:21]. We see a demonstration of yet another example of how by creating the classroom environment where original thought is encouraged and supported by all (Weber, Maher, Powell, & Lee, 2008) . The

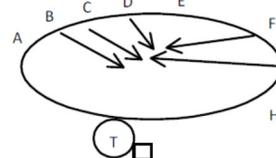
<p>[1:19] G: Mountain! [1:20] A, B, C: H2O! [1:21] G: Mountain! [1:22] T: Wait a minute, this is kind of interesting, I didn't expect that... Why?</p>	
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teacher is caught by surprise as it seemed like the reasoning why H₂O was a better option was shared with the class, however, it did not convince everyone: he asks the “why?” question at this point [1:22].

D brings up the explanation again, looking at the teacher and addressing the whole class [1:26]. G is now getting restless that the whole class is misled by the question and she expresses her emotion very direct to D [1:31]. Fiori and Boaler (2003) indicate that there are significant gaps between the written work and discussion work. This is a good example when misconceptions are put forward and flushed out to deal with during discussion. Such an approach indicates where misconceptions are and allows to address them right there on the spot. F pitches an idea, looking at the teacher [1:34].

<p>[1:26] D: If you didn't look at the numbers, the H₂O looks like it rises more rapidly...</p> <p>[1:31] G: That's bad! Which is bad!</p> <p>[1:34] F: If it's higher, then they get more money.</p>	
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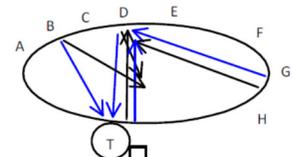
At this point the class breaks down into mayhem, and it is impossible to pick out the individual utterances [1:36-1:47]. Liljedahl and Andrà (2013) talk about the importance of emotions in discussion as they need to be taken into consideration when analyzing the

<p>[1: 36] - [1:47] All talking, unable to pick out individual utterances</p>	
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discourse (p.10). The students' chaotic outbursts go on for about 10 seconds, but there is evidently shift in the thinking as they realize the error they made in the interpretation of the question.

Student D now realizes that he was wrong with the interpretation of the question and gives in to the idea posed by G [1:47]. G, now feeling the advantage, confirms [1:52].

The teacher tries to give D some reassurance that given a different interpretation, it is possible to argue the idea G had originally. Now B gives reassurance that the class is mostly convinced that G was right all along [1:59].

<p>[1:47] D: Oh, I thought that like it shows how good the company is doing. [1:52] G: No! [1:53] T: If you take this as a perspective of a stock market, then... [1:59] B: So, its Mountain?</p>	
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This example demonstrates just how much depth of discussion, complexity of conversation and negotiation of social order can occur in such a short period of class time. It is amazing to see how engaged the class is in this stretch as they resolve a complex question with multiple points of entry and interpretation. The student exhibited a complex use of vocabulary, articulated their arguments very well and came to the right conclusion with minimal teacher input. In this first discussion when Student E is leading the class, the question posed by E engages the rest of the class, creating immediate tension. This divides the class into two groups: those agreeing with Student E and those that do not. The question became very personal to many students and they did not want to let it go that easily. In such a case a teacher's role became that of an observer, steering the discussion a bit here and there so that they did not get off-track. It is informative to note that in this discussion the gazes of students were to each other, they do not seek the approval of the teacher as they were too engaged with the argument; they forgot that they were being evaluated and were genuinely trying to resolve the question. This question continued to be an engaging piece of conversation until the class resolved it with the actual calculation of the slope (this piece is not presented in the Flow Chart above). This was an excellent example of an engaged discussion with tension and, later, a resolution that came from doing mathematics. I have no doubt that this is a much better way to learn: most of the ideas came from the students and they defended their point of view. The teacher's involvement was minimal and it did not directly supply the "expert" opinion. There was no need for that, as the students themselves were able to come to the correct conclusion. This is a good example of what I call the "climax of discussion": when the majority of the class is taking part in the discussion either by talking or actively listening and exhibiting

body language reinforcing that. I call this type of discussion *Teacher Poses a Question* because the starting point was induced by a question coming from the teacher.

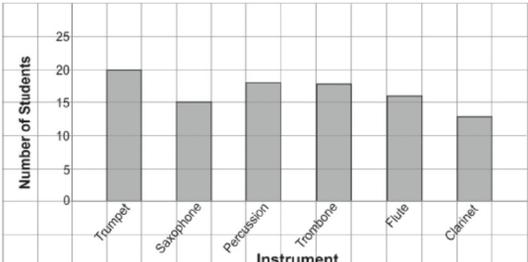
5.2.2. Sample Discussion 2

The second sample discussion is from the same class of Math 8, but on a different day of the same unit on statistics and probability. The class set-up is the same as in the previous section with the exception that the tablet is with student B. The task students are working on is presented below (see Fig. 19).

Choosing an Appropriate Graph

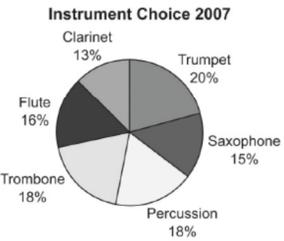
1. Each graph below shows the instrument choice for the 2007 Grade 8 Kitsilano Band Program.

Instrument Choice 2007



Instrument	Number of Students
Trumpet	20
Saxophone	15
Percussion	18
Trombone	18
Flute	16
Clarinet	13

Instrument Choice 2007



Instrument	Percentage
Clarinet	13%
Trumpet	20%
Saxophone	15%
Percussion	18%
Trombone	18%
Flute	16%

- List 2 things you know from the bar graph.
- List 2 things you know from the circle graph.
- What are some advantages and disadvantages of the bar graph?
- What are some advantages and disadvantages of the circle graph?
- Could you use a line graph to display this data? Explain.

Figure 19: Students' Questions for Discussion 2

Also below is the modified interactive flow chart coding (see Fig. 20):

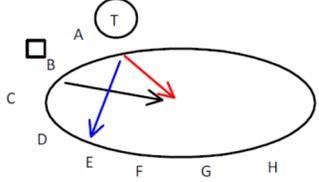
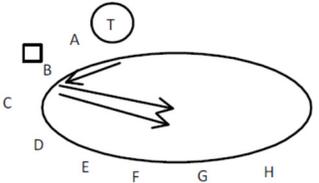
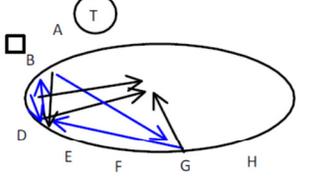
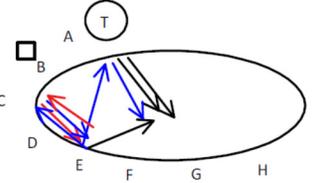
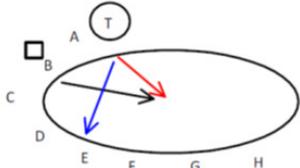
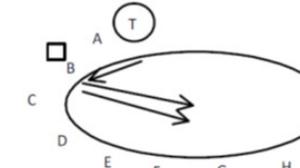
Time, Name, Utterance	Diagram
<p>[0:02] B: So for 1a it was...</p> <p>[0:05] T: Shush!!!</p> <p>[0:07] B: it was list two things you know from the bar graph, and so we noticed that trumpet is the most popular instrument and clarinet is the least popular instrument.</p>	
<p>[0:18] B: And then for 1b, list two things you notice from the circle graph we know that trombone and percussion are both at 18 percent, and that the flute and the saxophone are in like the middle range from the lowest and the highest. What are... 1c, what are some advantages or disadvantages of a circle graph?</p> <p>[0:46] T: Yeah...</p> <p>[0:47] B: Oh no, bar graph... It was, so an advantage was like the total, you can tell the total amount of students, and its not completely accurate, because like, it's not as exact. And then 1d was some disadvantages and advantages of a circle graph. An advantage is that you can tell clearly the sections like they are very individual, and then the disadvantage is that it does not like say the total amount of students. And then 1e is yes you can use the line graph because it's pretty much the same as a bar graph, just showing it in different way.</p>	
<p>[1:42] D: How you could use a line graph if there is no time here?</p> <p>[1:46] G: You mostly use it for time, but not always.</p> <p>[1:50] B: You could use it.</p> <p>[1:53] C: It basically becomes a bar graph</p>	
<p>[1:56] T: It's not very useful, but you could. And one way to make it happen is what you do, you take the bar graph, then you just connect the tops.</p> <p>[2:02] E: Yeah, that's what I told F!</p> <p>[2:05] C: And save the planet...</p> <p>[2:07] E: You could!</p> <p>[2:10] T: Very good, number 2.</p>	

Figure 20: Modified Interactive Flow Chart Discussion 2

Student B is leading the discussion and she also has the tablet. She is pointing at a spot on the graphs with a cursor as she is trying to explain her reasoning [0:02]. However, the class is not quite ready to listen, so the teacher steps in with a “shush”, looking at E, as he seems the culprit of the noise [0:05]. McGraw (2002) brings up a very important point to have control and regulations for who is doing the talking and how much. Too much chaos and one person taking the spotlight for a long time are the two extremes to avoid. B continues to explain her thoughts [0:07].

<p>[0:02] B: So for 1a it was...</p> <p>[0:05] T: Shush!!!</p> <p>[0:07] B: it was list two things you know from the bar graph, and so we noticed that trumpet is the most popular instrument and clarinet is the least popular instrument.</p>	 <p>The diagram shows a circle graph with eight segments labeled A through H. A cursor is pointing to segment E. A red arrow points from the center to segment E, and a blue arrow points from the center to segment D. A small square labeled B is outside the graph, and a circle labeled T is above it.</p>
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She keeps going with her explanation, which is very good both mathematically and logically [0:18]. We see a very good example of Toulmin’s model of claim, data, and warrant (Toulmin, 1969), as this student goes through a very good logical argument. She

<p>[0:18] B: And then for 1b, list two things you notice from the circle graph we know that trombone and percussion are both at 18 percent, and that the flute and the saxophone are in like the middle range from the lowest and the highest. What are... 1c, what are some advantages or disadvantages of a circle graph?</p> <p>[0:46] T: Yeah...</p> <p>[0:47] B: Oh no, bar graph... It was, so an advantage was like the total, you can tell the total amount of students, and its not completely accurate, because like, it's not as exact. And then 1d was some disadvantages and advantages of a circle graph. An advantage is that you can tell clearly the sections like they are very individual , and then the disadvantage is that it does not like say the total amount of students . And then 1e is yes you can use the line graph because it's pretty much the same as a bar graph, just showing it in different way.</p>	 <p>The diagram shows a circle graph with eight segments labeled A through H. A cursor is pointing to segment E. A red arrow points from the center to segment E, and a blue arrow points from the center to segment D. A small square labeled B is outside the graph, and a circle labeled T is above it.</p>
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continues to explain the questions, with the only feedback from the teacher [0:46]. The rest of the class at this point looks rather disengaged with the body language to confirm that: playing with hair, looking elsewhere, dozing off. Some are paying attention. Student B continues on to question 1c, 1d, 1e.

All of a sudden, Student D speaks up questioning Student B's last utterance, looking at Student B [1:42]. G chimes in to defend B [1:46]. B rebuts to G, looking directly at G [1:50]. Student C posits a useful idea to the table/group [1:53].

<p>[1:42] D: How you could use a line graph if there is no time here? [1:46] G: You mostly use it for time, but not always. [1:50] B: You could use it. [1:53] C: It basically becomes a bar graph</p>	
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The teacher gets involved and gives an idea about the connection in between the graphs [1:56]. E is glad to hear the idea he previously had [2:02] This is another example of how teacher knowledge (MCK) is needed (Bennett, 2009), as it is crucial to step in to keep the discussion going. Students C and E have non-mathematical exchange, adding an element of humor [2:05], [2:07]. Once again the emotions described by Liljedahl and Andrà (2013) make the discourse more authentic and engaging. The teacher, after scanning the room, decides to move on to the next question as there are no signs of further elaboration by anyone [2:10].

<p>[1:56] T: It's not very useful, but you could. And one way to make it happen is what you do, you take the bar graph, then you just connect the tops. [2:02] E: Yeah, that's what I told F! [2:05] C: And save the planet... [2:07] E: You could! [2:10] T: Very good, number 2.</p>	
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From the video of the second discussion, it is clear that the class was paying attention, but the atmosphere was not that of an engaged group. Most students were leaning back in their chairs and had wandering eyes. This reminded me of student behavior when the sage is on the stage (King, 1993). Even though the explanation was coming from a peer, the group was behaving very similarly as if the talk was coming from a teacher. However, all of a sudden, sparked by a question from the teacher, there occurred tension again, through disagreements and over differing opinions. This excited the room once more, with multiple utterances coming from around the room. However, this episode was not a long one and resolved very quickly. But yet again, I would argue that the learning happening there was very authentic, real and meaningful to the students. Once again, the gazes

showed that students were having conversations with one another and did not look at the teacher much. Mathematics became their own again as ideas were generated by students in real time, since students wanted to defend their point of view and come to a common understanding. In addition, it is interesting to note the location of the tablet in this discussion: it was used by Student B to present the answer to the question above. The tablet served a dual purpose as a common referent for the question to be visible to the entire class as well as a “talking stick” for the student who was presenting. The role of the teacher in this episode was once again to facilitate and keep an eye and ear on the discourse to make sure that it was productive and eventually correct. Yet again, the climax was evident and it occurred due to the question from a student this time. This type of discussion I call *Student Poses a Question*.

5.2.3. Sample Discussion 3

The third sample discussion is from a Math 9 class of 18 students. Students are taking turns explaining the questions from the textbook. The topic is Circular Geometry and, specifically, angles formed by tangents to circles. Students are divided into working groups prior to attempting these questions. Each group is assigned one or two questions. This set-up is in line with the considerations of McGraw (2002) and Smith and Stein (2011) to orchestrate productive mathematical discussions. This sample focuses on question 8 and the group of students assigned to work on it. (see Fig. 21) The question and modified flow chart coding are presented below. (see Fig. 22)

8. Point S is a point of tangency and O is the centre of each circle. Determine each value of a to the nearest tenth.

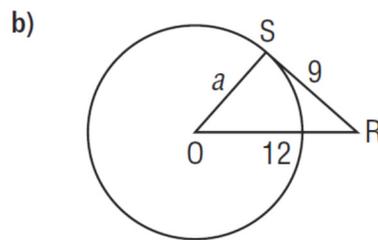


Figure 21: Students’ Questions for Discussion 3

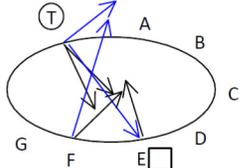
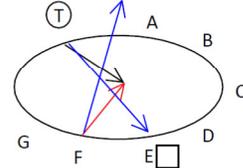
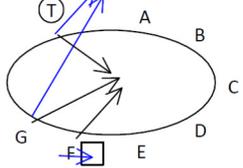
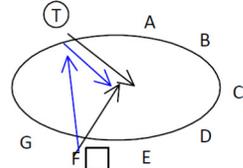
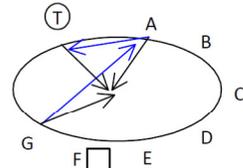
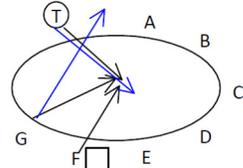
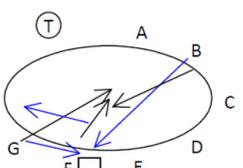
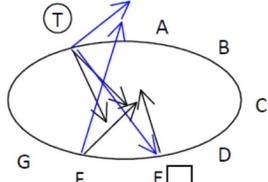
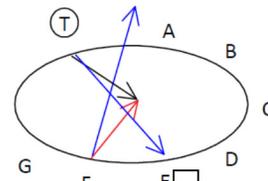
Time, Name, Utterance	Diagram
<p>[10:03] T: OK, let's look at b. [10:07] T: Label first of all where the 90 degrees is. [10:08] F: O! [10:09] E: S!</p>	
<p>[10:10] F: Atta boy! [10:10] - [10:24] <i>Students are talking off topic trying to center the picture on the screen</i> [10:25] T: Which one is the hypotenuse here? [10:30] T: Set up the Pythagorus for this, what would that be?</p>	
<p>[10:33] G: Oh there is none! Oh no no, there is! 12 is, 12 is! [10:35] T: So what will we have? [10:37] F: ninety plus... what?</p>	
<p>[10:39] F: It does not look like 90 degrees. Are you sure? [10:41] F: It looks like when I hold it this way, it is not 90 degrees! [10:43-11:15] <i>Students are chatting about the problem, can't pick out the specific utterances.</i> [11:16] T: So does anybody have an idea about this?</p>	
<p>[11:18] A: I just said my idea! 90 squared plus b squared equals... [11:21] F: 90 squared? [11:22] G: 90 is an angle, you cant combine angles and numbers! [11:24] T: Somebody has to know this!</p>	
<p>[11:26] G: 12 squared minus 9 squared equals a squared! [11:30] F: 12 squared minus 9 squared equals 63... [11:40] T: Whatever the square root of 63 is...</p>	
<p>[11:44] G: It is not 8, it is not 8, it is 7.9! [11:46] F: What is it? [11:47] B, G: 7.9.</p>	

Figure 22: Modified Interactive Flow Chart Discussion 3

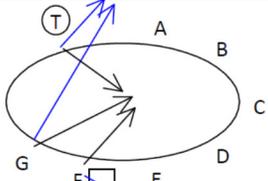
The interaction begins with part b). Student E has the tablet and is presenting the solution to start. The group has not solved this question, as it turns out, and some prompting is needed by the teacher. This combines Bennett's (2009) and Smith and Stein's (2011) claims supporting productive math discussions with teacher's involvement. The teacher starts off by asking a question to Student E [10:07]. Student F is calling out the incorrect answer first [10:08], but is quickly helped by Student E [10:09], providing a useful reactive response (Sfard & Kieran, 2001). The students are gazing at the

<p>[10:03] T: OK, let's look at b. [10:07] T: Label first of all where the 90 degrees is. [10:08] F: O! [10:09] E: S!</p>	
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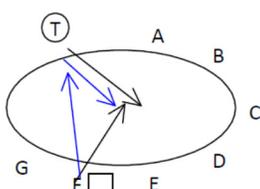
screen on the wall in front of them. Student F appreciates the correction and lets E know that [10:10], showcasing a non-mathematical utterance (Ryve, 2006) and showing positive emotion, which is valuable in this case to support each other (Liljedahl & Andrà, 2013). Then students take 14 seconds to re-align the screen and figure out what is the best position of the tablet and a stylus. The tablet has travelled to Student F at this point. The teacher steps in once again with a couple of prompts [10:25] [10:30]. After which the discussion becomes a bit fractured. The focus is lost for 20 seconds. It takes the teacher

<p>[10:10] F: Atta boy! [10:10] - [10:24] Students are talking off topic trying to center the picture on the screen [10:25] T: Which one is the hypotenuse here? [10:30] T: Set up the Pythagorus for this, what would that be?</p>	
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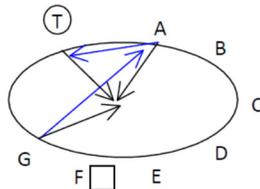
a couple of tries to get it back (Bennett, 2009). Suddenly Student G realizes that it is indeed a right triangle and lets everyone know that [10:33]. This "aha" experience is a key element in the discussion to keep it going. Something triggered this response from Student G, emphasizing ideas from Fiori and Boaler (2009) that there are misconceptions and

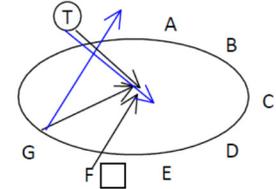
<p>[10:33] G: Oh there is none! Oh no no, there is! 12 is, 12 is! [10:35] T: So what will we have? [10:37] F: ninety plus... what?</p>	
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gaps between the written work, presented on the screen and discussion. The class is stuck once again for a bit, so the teacher asks another question [10:35]. F attempts to gather his thoughts, but really is just repeating what was whispered around the table, then realizes that something is not right [10:37]. It is very powerful here to observe the errors in the thinking and the lack of understanding of mathematics in front of them. However, F is still confused about triangle being a right triangle [10:39]. He is trying to figure out why it is so, not connecting the newly learned concept that tangents make 90 degree angles with radii [10:41]. This question creates another burst of utterances that are indistinguishable [10:43 – 11:15]. The teacher tries to bring the class together again [11:16], not giving up on the group, but also not giving out the answer.

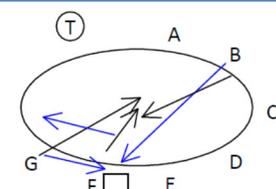
<p>[10:39] F: It does not look like 90 degrees. Are you sure? [10:41] F: It looks like when I hold it this way, it is not 90 degrees! [10:43-11:15] Students are chatting about the problem, can't pick out the specific utterances. [11:16] T: So does anybody have an idea about this?</p>	
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Student A attempts to explain her idea, but quickly realizes that she is not on the right track. She is committed to that idea someone else brought up before, confusing the degree measure with the lengths [11:18]. F, now realizing that that idea is not correct, catches onto what A is saying [11:21]. Then G has an epiphany that others are confusing angle measures and lengths [11:22]. This is in line with Liljedahl and Andrà (2013) in developing the idea of the transformation of thinking where the learner goes through a metamorphosis and all over sudden changes his or her schema of the concept. This happens almost instantaneously. All of the sudden the entire class understands the idea and are completely in sync with the leader of the discussion.

<p>[11:18] A: I just said my idea! 90 squared plus b squared equals... [11:21] F: 90 squared? [11:22] G: 90 is an angle, you cant combine angles and numbers! [11:24] T: Somebody has to know this!</p>	
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<p>[11:26] G: 12 squared minus 9 squared equals a squared! [11:30] F: 12 squared minus 9 squared equals 63... [11:40] T: Whatever the square root of 63 is...</p>	
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G sets up the Pythagorean Theorem to answer the question [11:26]. F repeats the same statement as he is writing this on the tablet [11:30]. Teacher reassures the class that they need to find the square root of 63 [11:40]. G carries out the mental calculation and square root approximation to arrive at 7.9 [11:44]. F records this on the screen [11:46].

<p>[11:44] G: It is not 8, it is not 8, it is 7.9! [11:46] F: What is it? [11:47] B, G: 7.9.</p>	
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It was eye-opening to see this discussion unfold from the moments of complete disorientation to achieving the correct solution. This supports Fiori and Boaler's (2009) argument that students need to be given a chance to think like mathematicians and struggle with the problems. Solutions should not come easily, or if they are easy, they are not simple and in perfect form right away. Discussion provides the opportunity for students to do mathematics as they are talking, often coming to the correct conclusion. However, the path to get there is far from smooth and straight-forward.

In this third discussion, the climax is less evident from the flow chart, but it is clear when watching the video. It happens when Student G realizes and sets up the Pythagorean Theorem. The class becomes alert again and most students are paying attention and are completely engaged in the process. Moreover, it is great to see students go further than anticipated and establish the square root to one decimal place without any prompts. However, in this discussion there were significantly more teacher questions and getting the group back on track. I refer to this type of discussion as *Breakthrough*.

5.2.4. Sample Discussion 4

The fourth sample discussion took place in a Math 10 class of 9 students. The homework for this class was to watch a video on the number of solutions in a linear system. In class, students were presented with a collection of questions from the textbook. They had to collectively solve them as a class by passing the tablet around to demonstrate their solutions. The question and the modified flow chart coding are presented below (see Fig. 22 and 23):

10. Marc wrote the two equations in a linear system in slope-intercept form. He noticed that the signs of the two slopes were different. How many solutions will this linear system have? Explain.

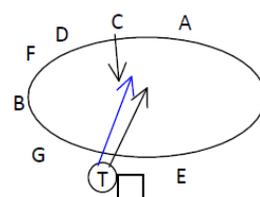
Figure 22: Students' Question for Discussion 4

Time, Name, Utterance	Diagram
<p>[0:10] T: Number 10 please, tell us about number 10. [0:13] C: One solution. [0:15] T: Explain.</p>	
<p>[0:19] D: One is positive, one is negative, so they will be going in opposite directions, so it's going to cross-over at a point. [0:26] T: OK, do we care about the y-intercept? [0:30] D: Not really...</p>	
<p>[0:32] C: Well, if they are the same, then it will be infinite. [0:33] G: If it shows you where it turns... [0:35] D: But even if it has the same y intercept...</p>	
<p>[0:38] T: True, but we know that the slopes are different, do we care at that point? [0:42] D: Even if it has the same y intercept, they will cross on the y axis together at the same point. [0:45] A: Yeah, you are too good!</p>	
<p>[0:47] T: Very good, I like it! [0:49] C: It's good! [0:51] T: I did not expect you people to arrive here that quickly.</p>	

Figure 23: Flow Chart for Discussion 4

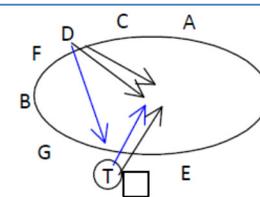
For this question the tablet ended up with the teacher. The teacher prompts the class to start looking at question 10 [0:10]. A different set up now, given Bennet's (2009) and Smith and Stein's (2011) teacher knowledge (MCK) and teacher's role in having productive discussions. C very quickly jumps in with his answer [0:13], showcasing the reactive utterance described by Sfard and Kieran (2001). The teacher asks to elaborate on this, as he is looking at the table [0:15]. Student D offers her explanation very

[0:10] T: Number 10 please, tell us about number 10.
 [0:13] C: One solution.
 [0:15] T: Explain.



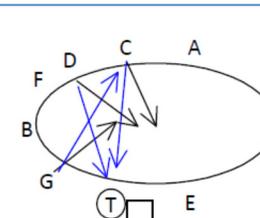
articulately [0:19]. Teacher quickly poses another question to pursue this and brings up the y-intercept, once again looking around the table [0:26]. The importance of teacher knowledge, as indicated by Bennet (2009), is imperative here. The teacher must use their expertise and knowledge to continue to guide the discussion and maintain the level of

[0:19] D: One is positive, one is negative, so they will be going in opposite directions, so it's going to cross-over at a point.
 [0:26] T: OK, do we care about the y-intercept?
 [0:30] D: Not really...



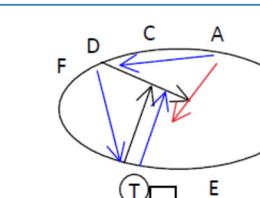
tension in the room. Student D is quick with her reply [0:30]. Student C is trying to remember what he knows about y-intercepts and infinite solutions, but here it is out of context, as slopes are identified to be different [0:32]. G offers her opinion, but it is not

[0:32] C: Well, if they are the same, then it will be infinite.
 [0:33] G: If it shows you where it turns...
 [0:35] D: But even if it has the same y intercept...



clear what she means by her statement [0:33]. D does not give up and pursues her point of view [0:35]. The teacher is trying to gather more opinions and dismisses D's comment for a moment [0:38]. However, D is persistent and finishes her thought [0:42]. Toulmin's

[0:38] T: True, but we know that the slopes are different, do we care at that point?
 [0:42] D: Even if it has the same y intercept, they will cross on the y axis together at the same point.
 [0:45] A: Yeah, you are too good!

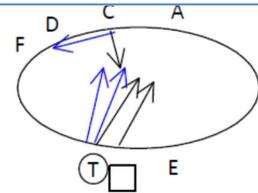


model is apparent once again, by way of the evidence of data, claim and warrant. At this point in the discussion, the class realizes that Student D is correct. There is a little pause which results in everyone having a reflective moment. Then A gives D the satisfaction of being correct [0:45]. The teacher supports the acknowledgment with his approval [0:47]. Student C also is impressed with D's thinking [0:49]. This is another example of a non-mathematical utterance. These non-mathematical utterances are important in this context, as they clearly identify the state of the class and the appreciation of the argument presented from a peer. There is absolutely no doubt about their authenticity and level of understanding of this idea with the rest of the class. Lastly, the teacher offers his compliments to the class [0:51].

[0:47] T: Very good, I like it!

[0:49] C: It's good!

[0:51] T: I did not expect you people to arrive here that quickly.



In the fourth episode, the students were more mature and thus exhibited a collegial approach to discussion. They took turns talking and there were a number of pauses to think and reflect. Once again, the climax is less evident from the flow chart, but it most certainly occurs when Student D reasons about on the extension question from the teacher regarding equal y-intercepts and different slopes. The class reacts differently by being quiet and thinking for a moment, then realizing the brilliance of D's words. Then they give her recognition of this. I call this discussion type *Student-led Explanation*.

In the next chapter I will discuss the analysis of these discussions and outline a number of common themes that emerged from the data.

Chapter 6. Discussion

6.1. Discussion-based Class

A successful discussion-based class is not just a class with discussion as the sole vehicle of in-class instruction. It starts much earlier with the classroom set-up and careful planning of lessons. The teacher selects the tasks and problems ahead of time. Each problem and task represents a specific learning outcome. It is imperative that the teacher identifies the learning outcomes that students must achieve during each class. The teacher should brainstorm the possible students' contributions and identify the course of action given each of those contributions. As discussion evolves, the teacher must facilitate the direction of the discourse so that the learning outcomes are addressed and discussed in detail. At the end of the lesson, students must walk away with a good understanding of the learning outcomes. Also, they need to feel confident that they understood the concepts and are able to keep going further into the course building up the new knowledge.

6.2. Before Discussion: Environment and Teacher Knowledge

As discussed, all of these elements were present in my classroom during the 4 episodes presented. In order for the whole-class discussion teaching method to be successful, the classroom environment must be established to facilitate such an approach. From the physical space to class size, technology and available equipment, resources available at school and individual teachers, all of these factors need to be considered to facilitate discussion. Particular attention must be paid to the furniture layout with a close resemblance of the oval table proposed by the Harkness model. The teacher is seated with the students and is taking the role of both the discussion participant and facilitator. Students need to be able to see each other at all times and have adequate working space. In addition, technology is very helpful for facilitating the talking occurring around the table

as discussed in Chapter 2, for example when a tablet played a role of a “talking stick”. Also, the whiteboards are very useful for students to present their solutions and need to be accessible at all times. Each and every one of these aspects is vital to establishing the student-centered classroom.

The emotional and behavioral aspect of the classroom also cannot be overlooked. Students need to feel comfortable enough in the classroom to take risks and speak up. They need to offer their opinions freely without reserve. From Weber, Maher, Powel and Lee (2008), we note that creating an environment where students are given sufficient time and freedom of speech is needed for productive discussions. This classroom is a place where risks could be taken and productive discussions emerge from that. The role of a teacher must be established as modest, primarily reinforcing the previously agreed upon norms of the classroom. For instance, when a teacher asks students to share their conclusions and provide justifications for those conclusions, he or she must remind students to attend to other’s arguments.

In all of the episodes shown in Chapter 5 we observe students express their opinions and ideas freely. Many of them are erroneous, but that does not stop the learners to continue to suggest them. But it is exactly these attempts and ideas, even though they are not correct, that lead the discussion to its climax. The teacher is observing all this unfolding and is able to deal with these as they come, sometimes deciding to address them, sometimes to let them go.

Extending the role of the teacher further, there is an absolute need for the professional to be highly competent not only in the discipline of mathematics, but also in the pedagogy. As Bennett (2009) suggests, MCK (Mathematical Content Knowledge), CCK (Common Content Knowledge), KCS (Knowledge of Content and Students), KCT (Knowledge of Content and Teaching) combine to form Mathematical Teacher Knowledge (MTK) and all of these must be at a teacher’s fingertips to successfully create the extending episodes that occur during class discussions. As seen from the examples of the discussions presented earlier, there were instances when the teacher had to think on the spot about something she/he had not anticipated in her/his preparation. One particular

instance of this is in Discussion 1 when the class digresses into a chaotic state and is separated into two groups with conflicting opinions. The teacher explicitly tells the class that he did not anticipate such a response. A failure to act accordingly can be very detrimental and “put out the fire” in the discussion where the class lost focus and the discussion lost its energy.

6.3. During Discussion

One of the main purposes of whole-class discussion is that it verbalizes the thinking process of a learner. It is an extremely powerful process, particularly in the process of students thinking as mathematicians and showing evidence of misconceptions (Fiori & Boaler, 2003). Students get the most authentic experience when they are taking part in the whole-class discussion since they are talking through their thinking. This is the way mathematics is done by mathematicians. Mathematics starts with a simple question or consideration, then on the board or paper with very rough notes and scribbles as they go through different cases of the problem they are working on. Only after some careful considerations and most likely a conversation with a dozen colleagues does this new mathematics get to start its life on paper. By verbalizing their thinking, students show many possible misconceptions they may have about a mathematical concept. This is evident from Discussion 3, in which the class has a common misconception about the right triangle. Only after a student questions the idea, the class figures out that they were incorrect in their thinking. Discussion becomes a very important tool in the hands of a teacher to understand where the struggles are and address them on the spot. Fiori and Boaler (2003) also note the difference present in the verbal and written mathematical argument. The discussions in my classroom confirm just that: the misconceptions are very evident and are much easier to deal with when they show up during the discussion versus in any written work the students produce. Once again, from Discussion 2 students’ misunderstanding of the question surfaces and is evident right away. It is exceptionally useful to the teacher to address this issue right away. Not only does it clear up the

misconception this student had, but it is very likely that someone else in the class had a similar issue, and therefore the clarification takes care of that too.

Another important element of the whole-class discussion is the process and practice of constructing of a good mathematical argument. Based on Toulmin's model (Toulmin, 1969), students get to present very often in front of their peers and thus go through the necessary steps of creating a well-thought out and supported arguments with the elements of data, analysis and conclusion. Discussion 2 is a very good example of a student presenting her well-put together argument. She proceeds step-by-step through her reasoning logically as the class listens. Such an approach is key to helping students become better learners and collaborators when working with their peers. This kind of discussion work often transitions into the question of who speaks and when. McGraw (2002) outlined that there need to be clear expectations and rules to govern the discussion so that it is productive and all the voices are heard. The use of a rubric of expectations and the tablet as a "talking stick" prove very useful in satisfying this criteria for a successful whole-class discussion. Students are often reminded of the proper conduct explicitly or through the actions of the teacher.

Once the rules of the discourse conduct are established, we transition to the types of utterances students exhibit. As noted by Sfard and Kieran (2001), utterances can be partitioned into the reactive and proactive categories. From my observations and evidence presented in the sample discussion, the vast majority of students' utterances are the reactive kind. This helps the teacher to plan for the lesson by anticipating what kind of ideas and questions will be generated as students are responding to the task presented to them. Students are very good at reacting to what is unfolding in front of them, but they do struggle with thinking ahead and seeing the big idea. This is where teacher can play a bigger role in the discussion to provide the proactive utterances based on his or her expertise and knowledge. The only instance where students succeed to offer proactive utterances is during the group presentations when they had some time to work on the problem and prepare. A very good example of this is Discussion 2, where a student is going through her thinking as she is answering a question on which their group had spent a considerate amount of time before the presentation.

Lastly, Ryve (2006) and Liljedahl and Andrà (2013) situate the necessary human aspect of classroom engagements. They focused on non-mathematical utterances, emotions and gazes. In my data, these played a very important role, as emotions, gazes and non-mathematical utterances allow the teacher the ability to gauge the room very quickly to assess what is happening with the discussion. It is very evident when the class is engaged in the conversation or when they are disengaged and not paying attention. Even though the non-mathematical utterances do not contain mathematical content, they can play a crucial part in the discussion. As seen from Discussion 4, they are a perfect tool to see the level of understanding of the rest of the class after the classmates had praised their peer for delivering a great argument and explanation. The teacher had no reason to ask any follow-up questions, since it was self-evident that the student giving the explanation had all of her classmates agree and understand what she meant.

6.4. After Discussion: Common Themes

So what are the common pieces of this puzzle? What are the characteristics and attributes that are the same for the four types, leading to the climax of the discussion? Can we replicate these and make them happen so that the climax of the discussion corresponds to the big ideas of the lesson?

6.4.1. Common Denominator 1: Tension

The one thing that is common to all of these climaxes is tension in the classroom. Whether it comes from conflicting pre-conceptions or misinterpretation of the question, not doing sufficient preparation at home, not paying attention in previous class, or missing class altogether, one thing is certain: tension creates and leads to climaxes in discussion. In all four cases tension is present, varying in its intensity, but always there. Below I present a table of examples of tension in each category of the discussions presented in Chapter 5.

Table 2: Types of Tension

Discussion Type	Tension Example(s)
Teacher asks a question	Teacher asks: "Do we agree with what E said?" Right away the class replies with some yes's and no's. Tension is very explicit.
Student asks a question	Student D asks: "How can you use a line graph if there is no time here?" The tension is at least in this student's understanding, but it is very public now, so there is a good chance someone else has this same concern. Tension is moderately explicit.
Utterance exchange to breakthrough	Tension in this discussion is less evident, but it is all around: the presenting group is stuck and the rest of the class cannot find a way to keep going for a long time. This creates a very awkward moment, and the class breaks into a chaotic exchange of utterances.
Student-led explanation	In this case the tension is between the student who understands the question fully and the teacher who is looking for other contributions from a different person to diversify the discussion.

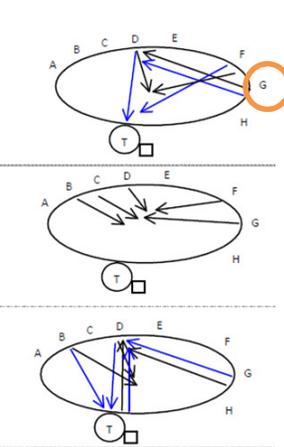
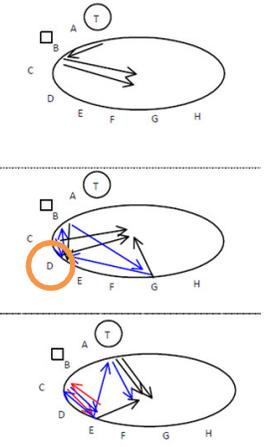
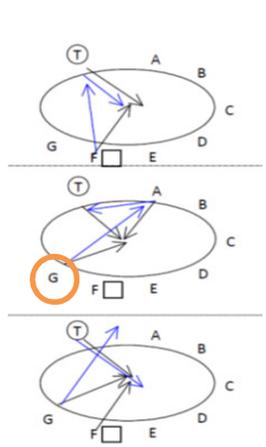
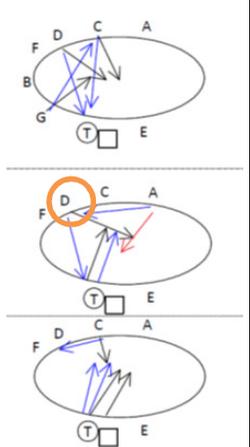
These instances of tension create buildup towards the climax in every one of these types of discussions. Often these tensions emerge directly as consequences of misconceptions and misunderstandings the students have. If discussion was not in place in the classroom, these misconceptions may have gone unnoticed and unchecked.

Interestingly enough, tension fuels the discussion and keeps it going. It makes discussion more engaging for the students and the teacher. Moreover, it is in our human nature to be curious and try to “figure things out”. A question or misconception, leading to tension and disagreement provides an opportunity to resolve it in a cooperative setting. In all of the four examples of the discussions presented, we see just that: the class comes to a mutual understanding and agreement, evident from the observations such as types of utterances, body language, tone and voice of students, as well as volume and side chatter. Having tension is absolutely essential for a productive and engaged whole-class discussion.

6.4.2. Common Denominator 2: The Discourse

“That’s bad! Which is bad!” “How you could use a line graph if there is no time there?!” “90 is an angle, you can’t combine angles and numbers!” “Even if it has the same y-intercept, they will cross on the y-axis together at some point.” These pinnacle questions and statements are examples of the voice that changed everything in the discussion. I have extracted these moments from the four discussions to demonstrate the power of these when looking at the flow chart. A shift becomes evident on the diagrams. I have circled the student producing the key utterance in each set of diagrams.

Table 3: Four Discussions Summary

Discussion 1	Discussion 2	Discussion 3	Discussion 4
			

In all these cases the key utterance creates reactions either by settling the tension or by escalating it even further. But regardless, these key utterances are providing an essential turn in the discussion that leads to the climax. Not only do they carry a very powerful mathematical meaning, but they are also the centre of the climax of each of the four discussions. Moreover, each and every one of these is a voice of a student and not of the teacher. It makes an important difference when it comes from a peer and is much more meaningful to the students. From my observations of these sample discussions and from reading various literature regarding peer learning, retention and understanding are increased and thus result in authentic and enriching learning. (Boaler & Staples, 2008) We see evidence of this in every discussion presented in Chapter 5. There are “aha” instances in every episode presented, when we see either a mathematical utterance confirming of understanding, or a non-mathematical one. Also body language and tone of voices are good indicators of understanding. Whole-class discussions generate a whole myriad of these voices and it requires teacher knowledge (MKT) (Bennett, 2009) to sort through these on the spot and pick out the key one. Sometimes the “picking” happens

naturally and the class realizes on their own that the idea from that voice is very important is powerful and useful. On some occasions, however, a teacher must intervene and point out the importance of student's idea. In the end, teachers do possess the needed expertise and knowledge to teach their students and propel the discussion and learning forward.

6.4.3. Teacher's Approach and Patience

If there is no tension, discussion simply becomes one person telling the story and the rest listening (or pretending to do so) passively, either agreeing with everything being said or not paying any attention. By interpreting the activity in the room through careful assessments of what is being done and by evaluating the quality of discussions and students' work, a teacher needs to identify which of the levers needs a move: increase/decrease the challenge or give students more skills to come back to tension. As seen from the four episodes above (as well a good collection of other instances not presented here) a careful teacher-posed question sparked a very rich and engaging discourse. Thus I propose that the following are necessary and sufficient conditions for maintaining the discussions and leading to the climax:

1. Use student work at all times and find the key utterance. It creates ownership and accountability.
2. Prepare questions/problems that create tension, where multiple approaches are possible.
3. Assess the room at all times by looking at the body language, student output, type of utterances (math-related vs. not math-related) to determine if they are bored (task is too easy) or confused (task is too hard and they are lacking skills to grapple with the problems).
4. Intervene by either making a task harder (posing an extending question).

The most important part of the "extracting" of the key utterance is the teacher's patience. The discussion may end if a teacher does not have enough patience and intervenes too

early. Such an outcome is the single most detrimental act a teacher can perform. It shuts down the discussion very quickly and prevents it from reaching the learning potential it could have. The teacher needs to exercise patience and wait for the key utterance. The teacher needs to trust their students that they will persevere and deliver that much needed breakthrough. One great example of this instance is the Discussion 3 when the teacher says “Somebody has to know this”. In this instance, the teacher kept probing because there were multiple occasions when the class lost focus. The teacher was trying to maintain composure in the class. Many practitioners would be tempted to get involved and provide the answer to the question. However, the discussion would stop right then and it would bear no fruit for the students. Also, such approach gives students the message that the answers will be provided no matter what. Not only does it destroy the students’ drive to keep working on the problems when stuck, but also it reduces the tension, which is much needed for a successful discussion as outlined above. But the teacher presses on and eventually the key utterance comes out! In this case students arrived at the correct conclusion themselves and therefore they could take ownership of this achievement.

A teacher needs to plan very carefully which questions/activities to include and in what order. In addition, given the nature of the approach, there is a lot of thinking on the spot. Successful implementation requires an excellent subject knowledge as well as teaching knowledge of multiple approaches, students’ misconceptions, potential questions and responses. This is a very hard task to take on, but an incredibly rewarding one when these moments of engaged classroom and unforgettable moments of seeing learning happen before one’s eyes in one of the most authentic ways possible occur more and more frequently. They are a pure joy in the hearts of all educators and something we must strive to achieve at all times.

6.5. Summary

I have created the following flow chart that summarizes the work and the findings. I call it the *Tension-Patience-Utterance Chart* (see Fig. 25). This chart summarizes the content of Chapter 6. Everything starts with setting up the environment and planning the lesson. The teacher prepares problems and tasks for students to engage with. The teacher identifies the learning outcomes that need to be discussed and understood by the students. Then, during the discussion, the teacher facilitates the process by constantly reflecting on the types of utterances, body language, and who is speaking and how much.

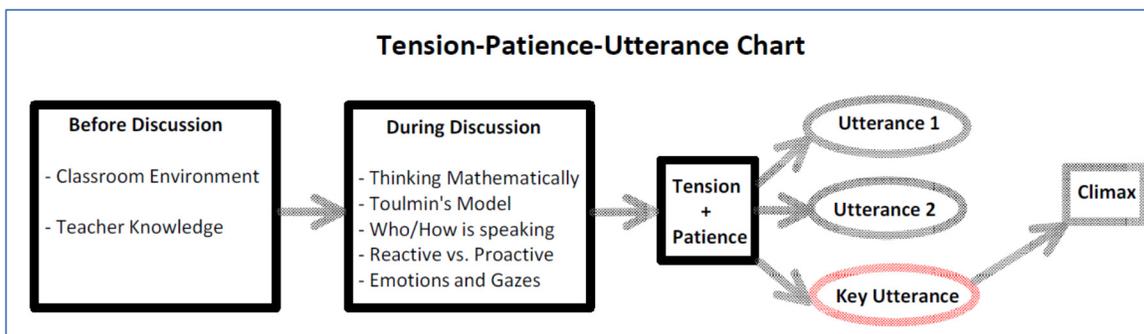


Figure 24: Tension-Patience-Utterance Chart

If the concept provides engaging for students, there will be some sort of tension present. It might involve a misconception or misunderstanding, or a new concept that students have not seen before. Oftentimes tension sparks the discussion and multiple utterances come forward with their explanations of how they understand this tension. If the teacher exhibits patience and keeps encouraging the students to defend their points of view, the key utterance resolving the tension eventually will surface. This is classified as a climax of the discussion and represents the big idea that the task was chosen to bring forward.

Chapter 7. Conclusion

7.1. Research Questions

In this thesis, I have outlined and described my work of three years from learning about the process to researching and implementing class discussions as a primary vehicle of classroom learning in mathematics. I have never been so excited and happy to see this process at work, giving me real data and supporting all that background research. The questions that I set in front of me while undergoing this work were: What leads to these climaxes? How can we make sure that the curriculum big idea is a central focus? Subsequently, can we classify the types of discussions that happen? Is there a way to analyse discussions further and learn from their analysis? What mechanism can be used to keep track of utterances and identify climaxes and levels of engagement in the room, and how and why do they happen?

I have identified four types of discussions and how they initiate and unfold (*Teacher Poses a Question; Student Poses a Question; Breakthrough; Student-Led Explanation*). Each of these types of discussions exhibits a climax: a clear episode of when the learning and understanding are happening. This involves some kind of realisation of previously misunderstood or different interpretation of a concept or idea. I have identified some commonalities within these four types of discussions and came to the conclusion that climaxes are brought forward when there is tension in the group of students. At that point it is up to the teacher and his/her expertise to elicit all the possible interpretations and ideas from the students and supported by teacher and subject knowledge as well as patience, find the student voice to drive the learning forward. Such a switch to discussion-based learning creates another example of a student-centered classroom. I am convinced that learning in this way creates much more meaningful connections and understanding of the subject matter.

7.2. Other Contributions: The Coding Tool

There were a couple of ideas that came out of this research as a consequence of looking at my data and analysing it. First is that tension is a very important element of discussion. When most students in the class have an opinion or idea about what's being discussed, chances are there will be some tension in the group. The teacher must use this to his/her advantage to try to assess where the tension is exactly and try to bring it forward in a discrete way to steer the discussion. This leads to the second idea that I identified as very important in having productive discussions: the key utterance. This key utterance is the explanation or idea that holds the answer to the tension existing in the room. With careful planning and preparation, the teacher should position the learning outcome to be covered in the lesson to be the context for the tension. When voice is understood by the teacher, and sometimes the students themselves, miracles happen in the minds of the learners.

7.3. Growth as a Researcher

This project was the first major research initiative that I have undertaken. From the first day of obtaining the research ethics approval the project has been exciting. I enjoyed learning from the start to the finish of a research project at a university level. I had to collect the forms from every single parent of my students to allow them to be filmed for my data collection as well as move furniture on my classroom and try different configurations of the layout. Also, I have run into a problem of finding very little literature and research done about Harkness teaching. There are no works that I know of that deal with mathematics education and Harkness teaching. This proved difficult since many areas had to be looked at from a new perspective. I have learned about the coding and the transcription of data, as well as using the existing models and adjusting them to suit my needs. It was a tedious process, but it was worthwhile to see the data transform and sort itself out into the four categories. These categories emerged almost on their own and then while studying each one, it was very rewarding to see the commonalities and differences between them. This

kind of a research is very appealing to me. It is very exciting to explore the data not necessarily knowing what the results will be and what would result from it. It would be very interesting to continue looking at the whole-class discussions given the results of my data and see if the climaxes can be predicted. Moreover, if the climaxes could be “artificially” put in the timeframe of a lesson so that the most important aspect of the concept being looked at is centralized within the climax. This would include a careful analysis of the tasks and possible outcomes of students’ ideas with the anticipation of tension around the issue we want them to think about and learn. I think this further direction of research could be very fruitful, particularly considering the development of the new curriculum and the big ideas that associated are with it.

7.4. Growth as a Teacher

This project has been an astonishing journey for the teacher as well. I have established a classroom where students are comfortable sharing their ideas and are not afraid to be wrong or make errors, and one in which students critique each other’s work in a very thoughtful and constructive manner. They trust that I will make sure that their understanding is acknowledged, and in turn, I trust them that they are doing their very best to try to achieve their full potential. I have integrated technology in a way that stimulates good discussions. With the use of a tablet as a talking stick, students take turns explaining their reasoning and look for their classmates’ opinions, feedback and contributions. I am confident that my students are engaged and enjoy being in my class. They have developed a better approach to major assessment portions of academic courses and their test anxiety has declined. From the analysis of the discourse using the Interactive Flow Chart tool it is evident that good questioning and smart facilitation is key to keeping the discussion engaged. I would love to spend more time observing and recording these classes, to dive deeper into reasons why and how these climaxes are reached, and to be able to anticipate them better to plan instruction. One answer is clear: the discussion becomes engaging when there is tension and many students buy into the idea being

discussed. It becomes personal for them; they go out of their way to defend their point of view and genuinely want to find out the correct answer not for the sake of extrinsic motivations, but for the sake of personal satisfaction and learning. The teacher must exhibit patience and wait for the key utterance to surface. This key utterance will produce the climax and authentic and deep learning will happen.

I would like to end with an anecdote from the beginning of 2014 teaching year. It was the second class of Pre-Calculus 11. Students were given a video to watch at home and some questions to try. When I walked in the room and sat down, all I had to ask them was "So how was the homework?" They went on to talk about arithmetic sequences for 75 minutes, non-stop. When I walked out of the room, I thought "We have learned something today..." Not all lessons, unfortunately, end like this, but I am eagerly working toward a time when they will.

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