STUDENT USE OF LANGUAGE IN FRENCH IMMERSION MATHEMATICS

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ABSTRACT

There is a challenge in juxtaposing the principles of numeracy, which is grounded in language-based problem-solving, with those of a French Immersion program which promotes bilingualism and communicative approaches. This case study examines the written output from mathematical problem-solving situations of 27 grade 6/7 students from a French Immersion middle school class. A qualitative analysis was performed and revealed that students use language in a wide range of ways to communicate mathematical ideas. Two themes emerged. First, students tend to vary along a narrative-symbolic continuum. Second, individuals differ in the ways in which French affects communication, ranging from a lack of competence in their second language, to obscuring their mathematical understanding, to their various motivations for codeswitching. In conclusion, language use depends upon personal styles, abilities and the perception of the roles of the mathematical, French and English languages. Educators need to accept the ramifications of these forms of plurilingualism.

Keywords: Numeracy, Second Language Education, Bilingualism

Subject Terms: Mathematics Education, French as a Second Language

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CHAPTER 1: INTRODUCTION

Traditionally, the teaching of mathematics in French Immersion has mostly been synonymous with teaching mathematics in French, with little discrepancy in the approach with teaching mathematics in English. The reason why there was such little discrepancy might have been related to the perception that mathematics is independent of the language used to teach and learn it. Therefore, the choice of language should not hugely impact upon the methodology. Mathematics, after all, is considered to be predominantly about numbers, not words. With the emergence of numeracy, however, communication and language begin to carry more weight. The ability to describe one's thinking process and to use language to problem-solve become a new emphasis. In contrast with the traditional approach of having the teacher walk students through a list of procedures followed by a practice session, more recent ideas about teaching and learning have favoured student-centredness, constructivism, discovery learning, and problem-solving (McAskill, Holmes, Francis-Pelton, & Watt, 2004; National Council of Teachers of Mathematics [NCTM], 2000; Pépin & Dionne, 1997; Van de Walle & Folk, 2005). Many of these qualities are finding traction in the emerging notion of numeracy around the world. Research on the implications of numeracy in mathematics education is both abundant and compelling. Similarly, French Immersion is a well-developed area of education research (see for example, Gajo, 2001; Lapkin, Swain, & Shapson, 1990; Tardif & Weber, 1987).

However, little has been done to apply these pedagogical trends in mathematics education to a program such as French Immersion. Rarely does numeracy research mention its application in a second language context and conversely, literature on mathematics education in a second language is scant at best. The few references that exist simply claim that Immersion does not have a detrimental effect on the learning in content subjects such as mathematics (Bournot-Trites & Reeder, 2001). The rare exception is research that questions the juxtaposition of the second language factor upon conceptual understanding in mathematics (Pépin & Dionne, 1997) but their only conclusion is the need for more research. At least, in the case of the latter, a consideration is made regarding the various ways of measuring and defining mathematical understanding.

Whereas French Immersion is mostly known as a method of learning a second language in Canada, little is said about how it aims to integrate content with language. Recent research is beginning to examine this perspective more closely (Lyster, 2007; Gajo & Serra, 2000). The purpose of this thesis is to take a closer look at how content and language co-exist and how the aims of the French Immersion program can be coordinated with those of numeracy in mathematics Education and vice-versa. To do this, chapters two and three will attempt to provide a thorough but concise background for the reader who may be unfamiliar with either area. In chapter four, I will pose some research questions that will bring these two areas of teaching together. This section will also explain in detail the methodology used to examine these questions. In chapter five, I will discuss and analyze the results task by task. In chapter six, I elaborate on two themes that emerge across all the tasks. And finally, chapter seven will conclude the thesis and offer some recommendations.

CHAPTER 2: NUMERACY

For the longest time, mathematics has been perceived as a pillar of objective truth, which stands independently in an infallible world of abstract ideas. In fact, some philosophers of mathematics describe it as following an absolutist paradigm (Ernest, 1991). However, this view of mathematics is increasingly under attack. Along with questions about the nature and the doing of mathematics (Davis, Maher, & Noddings, 1990) comes a change in the way it is taught. More and more, mathematics is seen less as a canon of knowledge, conventions and rules and more as a way of thinking. This chapter will examine some of the changes occurring in mathematics education with a focus on the emergence of the idea of numeracy and its definitions. The section will then elaborate on the importance of problem-solving as a mental activity. Finally, this chapter will also discuss how communication and problem-solving work in tandem.

2.1 Definitions and origins

Throughout this thesis, I alternate between the terms mathematics and numeracy. However, the two should not be confused with each other. Numeracy is a goal of mathematics. In referring to numeracy, I am making allusion to this particular aspect of mathematics education. Having said this, the term numeracy has a wide array of meanings. According to Hoogland (as cited in McAskill et al., 2004), the term numeracy is more commonly used in Europe and the United

Kingdom whereas mathematical literacy is more popular in North America, although not as much in British Columbia. It has also been noted that in countries such as Australia and South America, the two terms are used synonymously (McAskill et al., 2004). Despite the preferences of various parts of the world for one or the other term, there does not seem to be any consistent differences in the uses of either mathematical literacy or numeracy. For reasons of simplicity, I will assume that they are sufficiently synonymous. There are many definitions of numeracy or mathematical literacy. Two important features seem to reoccur: problem-solving or the application of skills, and the ability to communicate mathematical ideas.

I have chosen to combine the two ideas of problem-solving with application of skills. Although they are not synonymous, problem-solving is an essential component of numeracy and the two terms are sometimes even assumed to be interchangeable. The ability to apply a particular mathematical skill in a context is very much related to problem-solving. This latter term appears in ten out of the twenty-three various definitions offered by McAskill et al. (2004). In several other definitions, the applicability of skills is the emphasis. For example, the Ontario Literacy Coalition explains that "numeracy not only incorporates the individual's abilities to use and apply mathematical skills efficiently and critically, but also requires the person to be able to interpret and communicate about mathematical information and reasoning processes" (as cited in McAskill et al., 2004, p. 58). Similarly, Ciancone explains that "like literacy, numeracy is not a case of one's either being proficient or not, rather individuals' skills are situated along a continuum of different purposes and levels of accomplishment with numbers" (as cited in McAskill et al., 2004, p. 57). An important element in this definition is the emphasis on the purpose of the skills, rather than the mastery of them. In Steen's list of five necessary dimensions for setting numeracy goals (as cited in McAskill et al., 2004), the first one is practicality. Thinking that requires synthesizing, analyzing and evaluating is also mentioned in the list of essential elements of mathematical literacy as identified by the Mathematics Council of the Alberta Teachers' Association (as cited in McAskill et al., 2004). As was mentioned previously, the term problem-solving embodies many of the skills inherent in numeracy: applying individual skills effectively and critically, applying judgment to new situations, being able to relate problems with previous experiences, engaging critically with mathematical information, etc.

In numeracy, it is not enough to solve problems and to apply skills in authentic contexts. The second important aspect is the ability to communicate about one's own thinking process. This relates to the idea that a numerate individual should be able to share ideas with other people and relate to others of a community. Furthermore, the importance of communication validates the process of doing mathematics and not only the end products. Several definitions note the role of communication skills in numeracy as an extension of the ability to interpret and engage with mathematical material. In fact, communication skills or the ability to express oneself mathematically appear in thirteen out of the twenty-three definitions listed by McAskill et al. (2004). For example, the American

Heritage Dictionary of the English Language describes numeracy as "the ability to think and express oneself effectively in quantitative terms" (as cited in McAskill et al., 2004, p. 57). Others add the importance of [communicating] appropriately by using descriptions in words, graphs, symbols, tables and diagrams (McAskill et al., 2004). And the Mathematics Council of Alberta Teachers' Association (as cited in McAskill et al., 2004) also emphasizes the importance of "communicating using the richness of the language of mathematics". In other words, thought is not sufficient. Inherent in numeracy is the idea that the expression of one's thinking is just as important as the answer.

Although I have listed two main features of numeracy, there are many other features that should at least be mentioned in passing. Numeracy is also connected to the social dimension and has to do with how individuals use their abilities to interact and function in a numerical, technological world. "Numeracy includes a range of skills that are necessary for initial survival in a new country and for functioning as a fully literate person" (Ciancone, as cited in McAskill, 2004). Other such terms used in various definitions as "everyday activities", "societal context", "successful within a technological world", "typical member of the culture or subculture", "constructive, concerned and reflective citizen", etc. (McAskill et al., 2004) suggest that being numerate extends beyond the classroom. Numeracy encompasses abilities and dispositions that transfer and endure throughout adulthood and shape our citizenship. In fact, some definitions of numeracy include a sense of appreciation for what mathematics can do. The Mathematics Council of Alberta Teachers' Association mentions the importance of "appreciating the utility and the elegance of mathematics" (as found in McAskill et al., 2004). Moreover, Steen's list of the goals of numeracy makes it obvious that there is more to mathematics than just numbers and computation (as cited in McAskill et al., 2004). The five dimensions of numeracy he includes are: the practical, the civic, the professional, the recreational and the cultural.

2.2 Nature of problem-solving

In the following section, I will elaborate on the problem-solving aspect of numeracy: how this process occurs in the mind, the environment in which this happens best and the kind of thinking that emerges from it.

2.2.1 Mathematical thinking

Schoenfeld (1985) defines a problem as "a task that is difficult for the individual who is trying to solve it. It should be an intellectual impasse rather than a computational one" (p. 74). Many times, when adults or educators observe less experienced learners solve a problem, there is a gap between the two perspectives. For the adult, it may not be an intellectual impasse. This sometimes prevents her from being a good guide for the child. While the traditional teacher attempts to close the gap by telling the student what he has done wrong, the constructivist leader tries to meet him half way, allowing him to discover and use his own logic. Davis and Maher (1990) describe the process of mathematical thinking in five steps. They are summarized as follows. 1) Build a mental (data) representation or visualization of the situation. 2) Search for a previous representation or construct a new knowledge representation that can be used to make sense of the problem. 3) Map the route that connects the data representation and the knowledge representation. 4) Review this mapping. 5) Apply technical devices, such as calculations and transformations, to solve the problem. The key idea for Davis and Maher is the process of making the connections between data representation and the knowledge representation. In order to solve a problem successfully, the pupils must first have identified a correct picture of the situation. Next, they must make the proper connection between this situation and knowledge they possess about mathematics. These knowledge representations must be constructed by the learner himself and cannot be transposed from one person to the other. The example used in Davis and Maher's paper is the following simple word problem. “Mary had 3 dolls. For Christmas, she received 3 more”. First, the child must construct an image of the action of possessing and receiving dolls. Next, she must associate this with the notion of addition. While this connection is obvious to the adult, it may not be for the learner still making sense of what it means to add. In this article, the authors warn us that taking these constructions for granted make it difficult for students to become proficient problem-solvers (Davis & Maher, 1990). In addition, the careful construction of these representations through experiential, discovery learning is vital in creating a bank of useful knowledge for future problem-solving and learning (Van de Walle & Folk, 2005).

2.2.2 Learning environment

Many features of problem-based learning environments follow a socioconstructivist model. Jo Boaler, in her book Experiencing School

Mathematics (2002), describes two contrasted environments for learning mathematics at Phoenix Park and Amber Hill schools. Phoenix Park was based on constructivist ideas. First, it was a student-centred environment. Students are the active participants of their own learning (Boaler, 2002). The teachers' role at Phoenix Park was rarely to instruct or to be the purveyors of knowledge. Second, learners are encouraged to help each other and to discuss their points of view. Third, the process was valued over the final product and the explanation of their thinking was more important than the answer. In this constructivism-inspired setting, knowledge is not an absolute (Doolittle, 2000). It is the "result of a learner's activity rather than of the passive reception of information or instruction" (von Glasersfeld, 1991, p. xiv). The fact that knowledge stems from the learner rather than being delivered by the teacher also relates to student-centred learning. The process of gaining understanding is an end in itself. In numeracy, the knowledge being constructed relates to discovering patterns in mathematics and making connections with other ideas, rather than mastering calculation-based skills and procedures.

In contrast, the traditional school Amber Hill from Boaler’s book preferred a teacher-centred approach with limited social interactions and focused on covering each of the individual skills as prescribed by the curriculum. In the end, both schools faced the government issued standardized test. While Phoenix Park students had not had the chance to prepare for all the topics from the test, they did significantly better than the Amber Hill students in the problem-solving sections. Moreover, even when given procedural questions that they had not prepared for, the Phoenix Park students were more likely to be able to make educated guesses than their Amber Hill counterparts. In other words, a problem-based learning approach gave the learners more tools that they were able to use and adapt according to the situations. Conversely, the more traditional delivery method relied heavily on having covered every topic and did not have as much of an impact on the subjects' mathematical judgment. Moreover, students' reactions seem to suggest that the skills learned in the Phoenix Park classes were more likely to be familiar and applicable in the real world (Boaler, 2002).

Quite likely, there were many factors involved in the positive outcome of the experimental group at Phoenix Park. One of the central influences is the role assumed by the teacher. Although the student-centred classroom does not necessarily look like a lecture, it does not mean that the teacher is not instrumental in orchestrating effective learning situations and encouraging helpful dispositions. According to Yackel and Cobb (1996), the role of the teacher in an inquiry-based learning environment is quite crucial. She is the one who helps develop and negotiate the sociomathematical norms of the class. By this, they suggest that the teacher has a pivotal role in shaping the values of a problem-based learning environment. These sociomathematical norms include ways 1) to indicate what counts as acceptable mathematical explanation, 2) to increase sophistication in reasoning, and 3) to encourage the growth of intellectual autonomy (Yackel & Cobb, 1996). Unlike the teacher-centred classroom, an inquiry-based environment would have students contribute their opinions and points of view on problems being posed. In these discussions, comments and points of view are gently coaxed into following increasingly sophisticated chains of reasoning. Ultimately, the learners have freedom to discover new forms of knowledge, with the teacher acting as a facilitator, slowly transforming them into independent thinkers. So, the instructor must act as a facilitator for deeper-level and independent thinking, while at the same time, must be able to step back and allow students to take charge of their learning.

Interestingly, Maher and Davis (1990) found that when the teacher interfered in the problem-solving process in an attempt to re-direct the learners towards a correct answer, there was no transfer of the teacher's intervention when her solution had no meaning for the students. Instead, when the children were re-tested a year later using the same problem, the results suggested that it was their own engagement in the situation that transferred and remained durable. Given the results of this study, the teacher's role is not an easy one to summarize. It is neither so simple as to leave learners to their own devices, nor is it to intervene when they have not reached the desired conclusions. First, it is the teacher's role to choose problems which invite pertinent investigations. Second, it is also desirable to support the students in their mission by modelling and encouraging the kinds of questions that will further their understanding. Third, she must then back away and become an observer of the students' unique ways of processing. Understanding and acknowledging these processes are vital in creating an environment which is conducive to learning.

2.2.3 Creativity and divergent thinking

An important outcome of using problem-solving in mathematics is the type of learning that occurs. Problem-solving cannot be practiced by the passive learner. It requires a commitment and involvement of the imagination. Despite the set of heuristics prescribed by Polyà (1945/1990) for solving problems, Perkins (2000) likens the process to searching for gold in the Klondike: seemingly impossible, but potentially rewarding. In describing the experience of mathematical discovery, Hadamard (1945/1954) even distinguishes between the conscious and unconscious processes operating. In other areas of creative thought, from the arts to mathematical discovery and even gold mining, many writers have sought to describe the steps in reaching an inspiration, an epiphany or an Aha! moment (Wallas, 1976/1996; Hadamard, 1945/1954). In the same vein, Perkins (2000) lists 5 steps to breakthrough thinking: "1) the long search, 2) little apparent progress, 3) precipitating event, 4) cognitive snap and 5) transformation" (p. 13). In order to allow this form of discovery to occur, two conditions must be met: first, the problems given must offer a significant challenge and second, the problem-solver must be granted time and freedom to explore her own ideas and attempt different solutions. Ultimately, the goal of problem-solving is not to solve every problem that exists, but rather to foster a kind of insight and comfort with this mode of thinking, in the hopes that it will lead to a new perspective on mathematical concepts.

In the end, it is this transformative experience in mathematics that is sought in problem-solving. In addition to the cognitive reorganization that may occur as the result of discovering a solution to a problem, others also feel that the satisfaction from reaching an Aha! moment can have profound effects on student motivation, independently of the accuracy of the answer (Liljedahl, 2005; McLeod, 1988). McLeod explains that the "intensity of reaction to interruption is relative to the degree of organization of student's mental activity. The blocks that interrupt problem-solving may lead to intense emotions" (1988, p. 23). The emotional intensity of an activity may be correlated to the quality of learning. In fact, several researchers are also of the opinion that struggle and being wrong can lead to more profound discoveries (Mason, 1982; Perkins, 2000). Again, this presupposes an environment in which struggle and so-called failure are accepted as learning experiences.

2.3 Communication in mathematics

The second important theme in numeracy is the communication of mathematical ideas. This is valued for two major reasons: first, communication allows one to share ideas with others, and hence, expand on one's own abilities. Second, the vocalization or writing of one's thoughts can help gain access to deeper modes of thinking. In this section, I will describe these two forms of communication and how they relate to the experience of numeracy.

2.3.1 Dialogue and informal language

In allowing students to take charge of their learning, we must also discuss the language in which they negotiate with their peers. Earlier, I mentioned the importance of allowing students the freedom to choose the methods by which they attempt problem-solving. Similarly, the words they choose must also be their own. Pimm refers to two forms of language used in learning: "exploratory talk and discourse-specific talk" (as cited in Adler, 2001, p. 72). The former is an informal type of communication that allows the learner to focus on the ideas rather than the correct terminology. The latter is a more formal language used as the students gain more experience in the area. Adler points out the added challenge of the second language situation, because on top of helping students transition between informal and formal modes of communication, teachers must also consider the extra challenges involved in moving from the familiar first language to acquiring a second one (Adler, 2001). Second language issues aside, Zazkis (2000) supports the notion that allowing students to alternate between formal and informal codes, a practice referred to as codeswitching (CS), can be beneficial. In fact, we will discuss more about this term codeswitching later when dealing with second language education. Indeed, researchers support the idea that informal language can be particularly helpful when developing sophisticated mathematical ideas by validating students' expression of their thoughts in their own words (Barwell, 2005; Street, 2005). Despite the strong support for the use of informal modes of communication, some defend that there is an equally important role for formal language. Zazkis (2000) argues that using precise, specialized and correct mathematical vocabulary is tied to strong understanding. The obvious conclusion is that learning depends upon and shifts between both levels of language (Barwell, Leung, Morgan, & Street, 2005) and the teacher's role is to manage and to mediate the shift between the two focuses (Adler, 2001).

Although, it should be noted that trends in applied linguistics seem to emphasize that formal mathematical language, as it is often seen as the pillar of a concise and precise science, may not accurately represent the experience of construction of knowledge which is often ambiguous and exploratory (Barwell et al., 2005). So, while formal language can help anchor some conceptual development, mastering vocabulary should not be seen as being synonymous with learning (Barwell, Leung, Morgan, & Street, 2002). Instead, the role of language is to create a common forum in which to discuss the ambiguities and multiple perspectives on mathematics.

2.3.2 Journalling and assessment

Although dialogue is an effective way of jumpstarting the problem-solving process and can also be the locus of learning, it is difficult to use as assessment. New forms of learning are inexorably tied to new forms of evaluation. Previously, assessment of mathematical understanding was mostly based upon speed and accuracy of procedural exercises (Peterson, 1988). More recently, the use of formative assessment examines the ongoing process of forging new learning and offers beneficial information to both the learner and teacher (Black & Wiliam, 2004). In formative assessment, qualitative information is conveyed to the student with the intent of informing their future practices. Conversely, this information can also help the instructor to guide her future practice. This sometimes takes the form of rubrics and performance standards, as they are referred to in British Columbia (BC). Rather than representing success and failure quantitatively, formative assessment is more descriptive of the specific successes and weaknesses of the student. This is difficult to do with multiple choice or objective questions that demand one specific correct answer. Instead, journalling has become an interesting option for both gathering information on students' learning and communicating their areas of weaknesses in a productive way (Burns & Silbey, 2001). With journalling, students are expected to convey their understanding with words, graphs, diagrams and charts. In contrast to a short answer test, a journal can reveal more information about the learner's thinking process, and in so doing, can inform the teacher about the areas of weaknesses that need to be addressed as a class.

Journalling can do a lot more than provide a rich source of assessment. The regular practice of putting thoughts to paper can significantly change the way we think (Burns & Silbey, 2001). Similarly to dialogue, it can be a sort of communication with one's self. Pimm (1987) explained that writing "externalizes thinking by demanding a more accurate expression of ideas. By writing something down, it then becomes outside oneself and can be more easily looked at and reflected upon" (p. 116). Writing differs from dialogue in that it forces the author to be explicit, since the writer cannot use non-verbal expressions or point to objects or situations that are self-explanatory in an oral situation (Dougherty, 1996). It can also act as a record of your thoughts, allowing connections to be made with similar situations in the past or in the future (Mason, 1982). In some occasions, journalling can also be the vehicle for creating new hypotheses, or conjectures, to be tested (Mason, 1982). These theories, as they become confirmed or denied, are the stepping stones to transforming one's conceptual understanding. This metacognition, an awareness of one's own thinking, does not come automatically. Mason (1982) viewed metacognition as a reward for perseverance through the practice of reflection. Perkins (2000), while he does not refer directly to metacognition, intimates at the possibility of 'upping the luck' by sorting problems into categories, as one becomes more experienced in solving them. The gradual process of noticing patterns gives a sort of insight or awareness of the familiar chains of reasoning, thus transforming problem-solving into problem-identifying and problem-categorizing challenges. Some suggest that the key to developing this metacognitive awareness rests on the types of problems given and the provision of metacognitive terminology (Arno, Baiget, Cots, Irun & Llurda, 1997). Being familiar with cues that can point the attention to one's own thinking, then, could possibly increase one's awareness of the problem-solving situation.

When children begin learning about journalling, researchers suggest that teachers encourage them to write about what they did rather than what they were thinking (Burns & Silbey, 2001). Describing the procedures they used may be a good way to start learning to communicate. As they get more practice at being detailed and specific in their journalling, teachers can make comments about their thinking and encourage them toward more sophisticated thinking (Burns & Silbey, 2001). Within the practice of journal writing, there can be many variations ranging from informal free writing to formal and structured essays. Dougherty suggests three topics for journals: “(1) mathematical content, (2) process, and (3) affective/attitudinal" (1996, p. 557). Similarly, Liljedahl describes these three voices in journal writing: the narrator, the mathematician and the participant (Liljedahl, 2007). In distinguishing between each of these personas, the authors of these journals can shift between perspectives and as they become better writers, they also grow more aware of the range of their experience in problem-solving. In addition, the opportunity to reflect upon the affective experience can play an important role in fostering positive attitudes towards problem-solving. Unlike the objective and short answers given in the traditional tests, writing can allow personalities to shine through (Ciochine & Polivka, 1997), revealing information about the learner that can inform educators about individual needs and learning styles.

Together, problem-solving and communicating mathematical ideas form the pillars of numeracy. Slowly, these values in mathematics education are gaining credence and popularity in BC's public education system. Textbook publishers and government standards are also adapting to these new principles. In addition to these new challenges, French Immersion teachers are concerned with balancing the goals of numeracy with those of learning in a second language setting. In the next chapter, I will examine the French Immersion program and its unique features.

CHAPTER 3: FRENCH IMMERSION

French Immersion (FI) is a second language education program in which students acquire French by using it to learn other subjects such as mathematics, science and social studies. In the French Immersion class, the issues in learning any subject are compounded by the second language factor. In this third chapter, the purpose is to give the reader a theoretical framework for a better understand of the implications and designs of this second language education program.

Similarly to the previous section on numeracy, one of the central themes of this chapter is the shifting of paradigms. This shift in second language education has lead to the advent of new pedagogical approaches, which are not all commensurable with each other. The aim here is to describe this turning point in the development of second language education, to better understand how this field has evolved and perhaps anticipate where it is headed in the future.

Inexorably, second language education is connected to bilingualism. While the development of bilingualism may appear as an obvious consequence of second language acquisition, the term bilingualism itself deserves some attention. This chapter will explain how the definition of bilingualism has seen a radical change in the last twenty years. This in turn, impacts how we view the objectives of second language education and its relationship with the teaching of content subjects such as mathematics in a second language. After an overview of these approaches, I will expand upon Communicative Language Teaching

(CLT), a widely accepted approach, which incorporates many recent educational perspectives. In particular, some of the implications of CLT require a change in the way the role of the teacher and classroom environment are perceived. After theoretical perspectives on second language education have been laid out, I will describe the forms which French Immersion takes and discuss how it incorporates these approaches and views in the classroom. I will end this chapter with an important discussion about the role of the languages in the second language classroom, and mainly the role of the first language.

3.1 Definition of bilingualism: from mastery to usage

Since the beginnings of the French Immersion program in 1965 in Québec, the definitions of bilingualism have seen many changes. This evolution in what it means to be bilingual is significant because it impacts on the goals of the program as well as how its learners are perceived. We will see that one major shift, although no longer new from the perspective of research, has wide-reaching implications in the manifestations of the Immersion class. These implications are only slowly being seen in practice. Simply put, bilingualism is the ability to speak two languages. In the twentieth century, Bloomfield and McNamara were two of many researchers who proposed definitions for bilingualism. They were at opposite ends of a large spectrum. The former defined a bilingual person as one with native-like fluency in two languages (as cited in Grosjean, 1982); the latter suggested that one only needed minimal skills in any one of the four areas of language - speech, reading, writing and listening - to qualify (as cited in Grosjean, 1982). Many other definitions were proposed which lay somewhere between these two extremes. In the case of Bloomfield's definition, the true bilingual was equivalent to two monolinguals and therefore, was very rarely seen. Because all these definitions depended upon the degree of mastery of the second language, it was always difficult to draw the line that distinguished the bilingual person from the monolingual one. In the end, determining this bilingual status was a rather subjective decision, highly dependent upon the definition chosen. Despite the plethora of definitions proposed, each claiming to have a better way of deciding the extent to which the second language needed to be mastered in order to be considered a bilingual user, the common thread was that all of the definitions were based upon the degree of mastery of that second language. In fact, previously, the ideal bilingual mind was seen as equivalent to the sum of two monolingual ones, the two minds being exclusive of one another (Grosjean, 1982). This ideal bilingualism, one which is perfectly balanced between the two languages, will later be criticized as a flawed representation. According to Grosjean (1993), there will almost always be a certain disequilibrium between the two because each language grows out of different needs and contexts. Moreover, he states that "la coexistence et l'interaction des deux langues ont créé en lui un ensemble linguistique qui est difficilement décomposable en 2 monolinguismes[1]" (Grosjean, 1993, p.15) Following this idea of balanced bilingualism, there appeared the notion of a semilingualism, in which the speaker had less than mastered the second language; or double semilingualism, in which case, neither first nor second

[1] Given the bilingual context of this thesis, I have chosen not to translate direct quotes or the titles of articles and books.

languages were well used. In the literature, there is a seeming dichotomy between positive and negative views of bilingualism (Lüdi & Py, 2003). Along with these perspectives comes terminology which researchers have either come to adopt or shun as representing the corresponding perspective on the debate, including fossilization, interlanguage, interference, etc. A description of all these terms and their connotations would be very lengthy and is not completely necessary for this thesis.

In 1982, François Grosjean offered a definition that was based on a different set of criteria. In essence, he defined the bilingual person as someone who uses two languages in his everyday life (Grosjean, 1982). This was a new way of seeing bilingualism because it was not intended to represent an achievement so much as a process. Bilingualism, then, is not so much a status to achieve once and for all, as a way of communicating and thinking that can be used in varying degrees in different contexts. Cavalli (2005) adds to Grosjean's definition by explaining that "il s'agit là d'une utilisation diversifiée et complémentaire [des] deux langues suivant les domaines d'usage; […] les niveaux de compétence sont alors directement proportionnels aux besoins de communication de la personne" (p. 12). In other words, one can be more bilingual or functional in one area than another. It is not necessary to master both languages in all areas, as suggested by Bloomfield, nor only one, as suggested by McNamara, to be bilingual. In fact, the degree of mastery may fluctuate within the individual, according to the context of communication and the particular needs of the moment. It is therefore possible, according to Grosjean, to be bilingual, yet to have varying levels of skills depending on the area, the context, and the people involved in the communication. Lüdi and Py (2003) support this idea in saying that " le bilinguisme était moins une situation stabilisée qu'un ensemble de pratiques langagières " (p. 107).

This new way of seeing bilingualism becomes a major paradigm shift in second language education. Even though Grosjean's definition was formulated over twenty five years ago, it is not clear that its implications have made their way convincingly to the classroom (Cummins, 2000a). In fact, the simple shift from mastery to usage has come to represent an important dichotomy in second language education research. Gajo (2001) explains the difference between a monolingual and bilingual view of bilingualism in saying the following.

> Un point de vue bilingue sur le bilinguisme implique donc qu'on le décrive comme relevant d'une compétence originale, qui ne correspond pas au collage de deux compétences monolingues mais qui se présente par sa propre homogénéité. Toutefois, il faut souligner que cette compétence porte sur un répertoire linguistique complexe articulant la connaissance de deux ou plusieurs langues. Ce répertoire se distingue alors par une relative hétérogénéité. (p.128)

According to Gajo, a monolingual view of bilingualism suggests the sum of two monolingual competencies. In contrast, a bilingual, or plurilingual view requires a complex linguistic repertoire in which the two languages are neither separable nor exclusive of one another. In essence, a plurilingual view of languages forms a sort of linguistic mosaic pieced together by an assortment of tiles that, together, form a larger picture. Monolingual and bilingual perspectives come in conflict with each other when behaviours found in the bilingual speaker are not recognized and validated. Too often, these behaviours are perceived as deficits or impurities in the mastery of the second language (Gajo, 2001).

For many years, researchers and linguists believed that languages were compartmentalized in the brain (Cook, 2001), each language occupying a distinct area. It was also previously believed that bilingualism meant the "splitting of cognitive potential" or, worse, "a diminution of intellectual capacities," (Dewaele, Housen, & Li, 2003, p. 28). Since then, researchers have established that there are many benefits to being bilingual, both cognitive and social (see for example Lapkin, Swain, & Shapson, 1990; Hamers & Blanc, 1983, etc.). In contrast to this idea of linguistic segregation in the brain, Cummins' Interdependence Theory (1996, 2000a) suggests that competence in the first language will enhance the development of a second, particularly when it comes to Literacy skills. In other words, the development of all the languages practiced by multilingual learners is interrelated and should not necessarily be considered segregated and independent. FI students have had more opportunity to compare the differences between French and English language structures (Cummins & Swain, 1986). Similarly, they may also have better abilities to infer meaning from a text, since this skill is independent of the language of the text. Gajo describes this metalinguistic potential as a feature of the bilingual mind. He argues that being bilingual allows a stronger awareness of phonological and linguistic difference, a greater capacity for analysis and abstraction as well as greater mental flexibility (2001). Despite the evidence that bilingual learners make these comparisons and have these Literacy skills, other researchers feel that further exploiting the commonalities between languages would be beneficial (Bailly, Castillo, & Ciesanski, 2003; Cummins, 2000a). Many others have also advocated for the importance of developing metalinguistic or interlinguistic abilities and strategies (see for example Vienneau, 2006; Py, 1992; Bailly et al., 2003; Cavalli, 2003).

There are certain implications to the notion that languages are not in fact compartmentalized in the brain. First, learning a new language may affect prior conceptions of other languages (Cummins, 2000b). Second, when a bi/plurilingual speaker makes use of her knowledge of a certain language, it may be a deliberate choice to restrict her usage of all other languages, depending on the context, audience and the circumstances of the exchange (Baker, 1996). Third, if languages are not compartmentalized, then the bi/plurilingual speaker would store knowledge of all languages together. How this information may be organized is probably dependent upon the user and her exposure to various communicative situations. As Sabatier (2008) suggests, "le répertoire verbal des individus plurilingues apparaît clairement comme un répertoire unique, mais complexe, car pluriel, hétérogène et composite de par ses composantes linguistiques et l'éventail des possibilités qu'il présente" (p. 114). She adds that the development of this linguistic bank is neither systematic nor complete and is a function of the particular communicative needs that emerge from problematic situations. To be plurilingual, then, does not imply the presence of a multitude of monolingual sets of knowledge, but rather one complex, heterogeneous amalgamation of several pieces of each language. By extension, any assumption that knowledge of language can easily be segregated one from the other does not acknowledge the complexity and sophistication, but also the uniqueness of the plurilingual mind.

3.2 Approaches to second language education

Bilingualism and plurilingualism are increasingly becoming the norm rather than the exception in all parts of the world. In fact, it is claimed that over 60% of the world's population speaks more than one language (Cavalli, 2005; Richards & Rodgers, 2001). It follows that there is a long history of methodologies in second language education. The next section will give an overview of this history, ending with a description of a currently popular approach called Communicative Language Teaching (CLT). Second, we will see how the French Immersion program began, what it entails and how it fits into these theories about second language education. Third, we will examine some views about the role of the first language in these approaches. The relationship between first and second language is key here. Fourth and lastly, we will discuss how communicative competence is replacing mastery as a goal in plurilingual education. This communicative competence in a second language will have important implications in the teaching of content subjects such as mathematics.

3.2.1 Description of several approaches

The field of second language education is highly influenced simultaneously by developments in psychology, linguistics and education (Richards & Rodgers, 2001; Varshney, 2005). Richards and Rodgers (2001) propose three views of language learning: 1) *a structural view*, 2) *a functional view* and 3) *an* *interactional view*. Under the structural view, the objective is to master the phonological and grammatical units and to understand the operations and lexical items which form the new language (Richards & Rodgers, 2001). This view is in line with McNamara and Bloomfield's idea of bilingualism as the mastery of a second language (as cited in Grosjean, 1982). The Grammar-Translation Method is one example of a structural approach. Its aim is to maximize the "benefit from the mental discipline and intellectual development [through the] detailed analysis of grammar, application by translating and manipulation of morphology and syntax" (Richards & Rodgers, 2001, p. 4). The Grammar-Translation (G-T) Method was used extensively in North America and Europe since the eighteenth century and was considered the standard approach, particularly when teaching Classical languages like Latin (Varshney, 2005). In fact, it was seen that the purpose of learning Latin grammar was an end in itself. The Grammar-Translation method focused on reading and writing, through the application of grammar rules, where accuracy of translation was the priority. As contact with foreign languages became more common, the need for the oral component led to the development of new methods, such as the Direct Method and the Audiolingual Method. Rather than practicing the translation of texts from the second language (L2) to the first language (L1) and writing texts in L2, the Direct Method used only the L2, as it was considered that being surrounded by the target language would naturally help the speaker assimilate it, just as a young child might learn her first language (Varshney, 2005). Similarly, the Audiolingual method placed more emphasis on the oral component of the language. This approach continued to emphasize grammar, but rejected the L1 as a point of reference. Also, these methods all use a teacher-centred approach (Tremblay, Duplantie, & Huot, 1990).

Slowly, as trends and reforms pushed away from the notion of mastery of the discrete parts of a language, there was less focus on grammar and more on communication. Moreover, Grosjean's functional definition of bilingualism (1982) focused more on using the language to learn, rather than learning a language to learn a language. Richards and Rodgers (2001) described this as a functional approach to language learning, in which the major objective is using language to express meaning. In other words, the goal of learning language was to use it to communicate. Language learning was no longer simply a mental exercise but a means with which to interact with a French-speaking audience. For several years near the end of the twentieth century, many creative methods appeared suddenly which turned away from grammar-based approaches. From this prolific and experimental period for teaching methodologies, emerged the Communicative Language Teaching Approach (CLT) in the late 1960s. It has come to represent a widely accepted model for language learning as well as the cornerstone of an important paradigm shift (Richards & Rodgers, 2001). However, it differs from previous methods in that it is described as an approach based on a set of values and principles, rather than techniques. Because of this, the range of its application can be quite broad, depending on the interpreter of these principles. In this learner-centred, experience-based approach, CLT focuses on the process of communicating, with all its obstacles and strategies, which can carry more importance than the message communicated and the structures employed. Furthermore, the act of interacting becomes somewhat more central in the communicative experience.

3.2.2 French Immersion description

French Immersion is a second language program that first appeared in Quebec in 1965 as an experimental approach. At the time, social and professional opportunities for French speakers and English speakers were unequal. A French speaker tended to have access to higher paying jobs than Anglophones (Halsall, 1998). For this reason, the Anglophone community was eager to gain advantage by learning French. However, second language programs of the time were mostly based around the Grammar-Translation Method, which may have helped English speakers translate written French texts, but did not offer much in terms of oral fluency or comprehension. In other words, they could not use their French to communicate in job situations. The basic feature of FI was that students learn French by using the new language to learn about subjects such as mathematics, Science and Social Studies. The language of instruction is French. Within the program, there are a few variations in the level of immersion. Early Immersion begins in Kindergarten and continues until the end of high school, while Late Immersion begins in grade 6 or 7, depending on the school districts (Lapkin, Swain, & Shapson, 1990). The basic principle is that instruction is generally in French, but the degree of immersion ranges between 50 to 100% of courses taught in French, depending on the grade level and availability of qualified French instruction.

Since the beginnings of the program, French Immersion has gained much popularity as an option within the public system in Canada (Halsall, 1998). A decade or so ago, a survey from Ontario revealed that reasons to be in French Immersion included wanting to learn a second language, getting jobs and challenge or enrichment (as cited in Halsall, 1998). Although the philosophy behind Immersion was loosely based on a reaction against the Grammar-Translation Method, the techniques and approaches used in the program did not necessarily ascribe to a particular philosophy, other than to be a form of content-based instruction, which will be described in more detail shortly. In fact, there does not seem to be a clear agreement in methodologies used in Immersion. On one hand, Nikula and Marsh (1998) describe the aim of FI to be functional bilingualism and methods to be student-centred and communicative. Richards and Rodgers (2001) also consider Immersion to be a form of Content-Based instruction, which in turn, they describe as following the Communicative Language Teaching (CLT) Approach. However, according to Varshney (2005), the current immersion model was influenced by the Direct Method, which aims to surround the learner by the target language and to foster its acquisition by being immersed in the language. And others have criticized FI as being overly teacher-centred rather than student-centred (Halsall, 1998). Many researchers have described the tendency in Immersion classes to discourage the use of English, in favour of the L2 (Varshney, 2005; Nikula & Marsh, 1998). The inclusion of L1 in a second language program, in fact, is a topic of debate that is sometimes cause for embarassment and guilt for teachers (Féral & Owodally, 2003; Varshney, 2005), which we will come back to later. Despite this view, Cummins (2000a) argues that the Immersion programs should consider becoming less rigid in their separation of languages. He explains that a focus on the language and its discourse may be more beneficial and can help the learners become more aware of linguistic operations. Indeed, the French Immersion program in Canada has had many sources of influence, but there seem to be equally many variations in its application. BC, for example, differs from Québec in that French is a minority language and therefore poses an additional challenge for educators in the program. The degree of immersion is often limited to the school environment. Incidentally, BC celebrates thirty years of French Immersion this year.

3.2.3 Role of the first language in a second language program

In the area of second language learning, the debate about the role of the first language is indeed persistent. Throughout the history of foreign language teaching, whether in Canada or elsewhere around the world, the importance of the first language has alternated between being crucial, to detrimental, to neutral, and back again (Varshney, 2005). In this section, I will trace some of the roles of the L1 throughout the various approaches to second language teaching, as it oscillates between being a negative, neutral and positive influence. Also, the next section will discuss the idea of interlanguage, a large grey zone of linguistic development, which is subject to a variety of perspectives from educators and researchers.

As teaching methodology evolved since the nineteenth century, so too did the role of the first language. At first, according to Lado, it was perceived as having a negative influence, and being the cause of errors in the new language to be acquired (as cited in Varshney, 2005). Later on, with the advent of Error Analysis, the image of the L1 turned around and became a positive one. Its role was to explain the significance of errors made in the target language. In other words, the more familiar L1 was a benchpost against which to orient one's learning a new language. In the meantime, researchers were also interested in the learner's level of readiness in acquiring a second language. The first language was no longer seen as a hindrance to learning a new one at this point. In 1972, Selinker used the term *interlanguage* to validate the process in which a learner moves from being monolingual to bilingual (as cited in Varshney, 2005). Lüdi & Py define interlanguage as "l'ensemble des connaissances intermédiaires qu'un sujet a d'une langue seconde qu'il est en train d'apprendre" (2003, p. 114). This term and the acknowledgment of this continuum of learning have had profound implications. On one hand, it was possible to maintain that learners needed to move away from their error-making habits and approach a more native-like mastery of the target language. On the other hand, the intent of the term Interlanguage was to give licence to the behaviours characterized by bilingual learners (Lüdi & Py, 2003). Rather than seeing deviations from the norms as undesirable practices to be eradicated, the interlanguage theory recognized that a learner of a second language has unique skills that stem from prior knowledge. These skills are distinct from the practices of a native speaker. In fact, one of the popular criticisms of the Immersion program is that its students often fail to measure up with native users in the areas of writing and speaking

(Halsall, 1998; Cummins, 2000a). In defense of these students, Cook (2001) argues that it is not fair to compare the abilities of a native speaker with those of a second language speaker; rather, they should be compared to those of an L2 speaker. Conversely, the second language learner possesses skills unique to her situation which would not be found in native speakers. The notion of an interlanguage validates these differences rather than expect L2 learners to become native-like speakers. In this sense, the interlanguage theory does not see L1 as a hindrance to learning a second language. Instead, it suggests that the L1 is the starting point that educators need to take into consideration in their curricular planning. Moreover, the influence of the L1 in L2 learning cannot and should not be denied.

Interestingly, after this acceptance was established, Krashen's Monitor Model appeared to shift the view once more. In one of the five hypotheses of his model, his idea of the Comprehensible Input, described previously, suggests that the L2 may have a more strategic role to play (as cited in Varshney, 2005). By placing more attention on the level of the L2 material provided, the theory shifts focus away from the use of L1 as a way of dealing with challenging material (Varshney, 2005). Instead, it encourages the educator to be more strategic in choosing level-appropriate material. One reason to support this view is the belief that students may revert to using the L1 if they are at a loss when facing difficult material in L2. Although the Monitor model shifts attention away from the role of the L1, it does not necessarily discourage its use. Rather, it proposes an alternative way of making second language acquisition more effective.

The debate around the use of L1 is still very present in today's Immersion classrooms (Cummins, 2000a). Teachers can be quite passionate in defending the need to exclude English in a French Immersion class, so much so that Cook (2001) considers it to be the mainstream in twentieth century methodology. He highlights the fact that descriptions of the Communicative Language Teaching approach, what is considered to be current and widely accepted, fails to mention the role of L1 at all. There is apparently a discontinuity between views. Cook (2001) traces some of the sources of the anti-L1 attitude. Many of these reasons find their roots in topics we touched upon previously, but they bear repeating here. First, there was the belief that L2 acquisition should mimic L1 learning and therefore the existing L1 should be ignored. However, the learning of L2 is a priori different from L1 by virtue of the fact that L2 comes after L1 (Dewaele, Housen, & Li, 2003) and the learner already possesses the experience of acquiring the first language. Next, languages were thought to be compartmentalized in the brain, and L2 learning should not be linked to their L1 knowledge. However, researchers now defend that words of different languages refer to one same conceptual system and vocabulary repertoire (see for example Gajo, 2001; Sabatier, 2008). Cook’s analogy (2001) explain this well.

> Learning an L2 is not just the adding of rooms to your house by building an extension at the back: it is the rebuilding of all internal walls. Trying to put languages in separate compartments in the mind is doomed to failure since the compartments are connected in many ways. (p. 407)

Despite the developments in linguistics research and language acquisition theories, the feeling of wanting to keep L1 at bay are still persistent in the classroom. Several reasons for avoiding the L1 are based on unfounded expectations. Researchers have alluded to the feelings of guilt and embarassment associated with allowing the L1 in an Immersion class (Cook, 2001; Féral & Owodally, 2003). These feelings of guilt may be created by an unspoken policy that the Immersion teacher should only ever be speaking in French to her students. While little research has compiled data to support this theory, observation of Immersion teachers leads me to notice two groups consisting of those whose mother tongue is French, in which case, speaking French may be easier than English; and those who are bilingual and may feel that their assignment in FI is continent upon their use of French. In other words, by using English in FI, they may feel as though they would be judged by the former group, or worse, that they are not fulfilling their professional duties. Varshney (2005) argues that other reasons why teachers may be compelled to use L1 are associated either with affective factors, such as frustration and fatigue. When students have difficulty understanding the teacher in French, it seems like an easy solution to simply switch to English. Similarly, at the end of the teaching day, when patience is low, some teachers will revert to English as they feel they have more control over the class, or perhaps their own bilingual minds may prefer to use the L1 to save energy. Cook (2001) argues that little is done to undermine this anti-L1 attitude. The absence of L1 is often the default expectation, despite the fact that it does not represent reality. Nevertheless, it is a pervasive argument against using the L1 in Immersion.

There are several arguments to support the use of the first language. Cook (2001) explains that there are four criteria that may be considered when weighing the benefits of using the L1 against the potential loss of the second language. "1) efficiency, 2) learning, 3) naturalness and 4) external relevance" (p. 413). If the use of the L1 means that a task can be completed more efficiently and the benefit of doing the same task in L2 is minimal, then it pays to use the L1. Similarly, if using the L1 can help the learner use the L2 better, it is also useful. Next, if there are situations that require that the students become more comfortable, using the L1 can also make an important difference. Finally, if the situation in question is more relevant in the L1, then persisting in the L2 may not be a good idea. Overall, his point is that the use of L1 can be strategic and deliberate, rather than random and simply a matter of convenience.

Turnbull (2001) cautions us that allowing L1 can become a slippery slope. While he agrees that some L1 use can be beneficial, he points out the difficulty in determining what an acceptable amount of L1 would be. Turnbull takes the cautious position of neither promoting anti-L1 attitudes nor encouraging its overuse. Somewhere in between, and somewhat sidestepping the question, he advocates maximizing the use of the target language. He defends that the teacher may indeed be one of the few if not only sources of the target language -- in the case of Immersion, French -- and students should not be deprived of its exposure in a variety of contexts (2001). He invites researchers to further pursue the relationship between target language and L1 exposure and proficiency in this new language.

In Cook's (2001) category of efficiency, there is a strong case for the use of L1, particularly in the practice of codeswitching (CS), where the users alternate between L1 and L2. When there is a cognitively demanding task to be completed, the added challenge of the second language may create a significant obstacle. On one hand, the teacher wants to maintain a high level of critical thinking. If, on the other, she is adamant about disallowing the L1, the result will be either that the cognitive demand will not be met and learning will remain at a more superficial level; or, that the students will revert to L1 despite all instructions not to. Several researchers have highlighted the dilemma between language use and cognitive demand (Cavalli, 2003; Mailhos, 2003; Guasch, 1997; Yeoman, 1996). In one study, Adler (2001) reports that "Calculational mathematical discourse dominated in classrooms where switching was restricted. In contrast, when the teacher herself switched into the learners' primary language in the public domain, this correlated with conceptual discourse becoming the focus of discussion" (p. 74). In other words, allowing the flexibility in choice of language may encourage deeper discussion. The question, then, becomes what is more valued in a content course: the practice of using the L2 or the higher order reflection. There are no clear-cut answers that apply for all cases, but this is where the Immersion teacher must make a professional decision. Adler (2001) insists that "it is not a matter of whether or not to code-switch, nor whether or not to model mathematical language, but rather, when, how and for what purposes" (p. 85). Again, these choices can and must be made deliberately and consciously, rather than haphazardly. Similarly, as was mentioned before, the option of codeswitching in mathematics, that is, alternating between formal and informal languages can be helpful for the learner (Zazkis, 2000).

Included under the umbrella term of Interlanguage are the many ways in which a new user of a language will be influenced by their first language. Overviews of second language acquisition theories describe in length the use of codeswitching (CS), in which aspects of the L1 find its way in a speech, sentence, phrase or even a word meant to be communicated in the L2 (Grosjean, 1982). Just as with interlanguage, codeswitching also generates both positive and negative responses. According to Grosjean, it is monolinguals who have the strongest views against codeswitching. They associate it with laziness or a lack of competence, as embodied in the term "semilingualism" (1982). Furthermore, the term has been criticized for its use as a negative label, among other reasons (Baker, 1996). Dewaele, Housen, and Li (2003) argue:

> If translinguistic markers are perceived as violations of prescriptive linguistic norms and generate social proscription in many language communities, it is because of the (culturally constructed) stereotype of monolingualism is 'natural' and plurilingualism resulted from the confusion of tongues by God and weighs on mankind like a divine curse since the construction of the tower of Babel. (p. 186)

In this passionate plea against linguistic purism, they point to a cultural bias toward monolingualism which other authors have also alluded to (Grosjean, 1982). More specifically, though, scholars argue that codeswitching can be an indication of several features other than a lack of competence. For example, Hamers and Blanc (1983) explain that codeswitching can be used for compensating as well as to maximize the efficiency of a communication.

Moreover, Moore (1996) suggests that codeswitching may be an effective learning strategy and act as a bouée transcodique. In fact, she argues that when faced with a communication impasse, the L1 can help deal with the situation in several ways, depending on the learning outcome desired. An alternation between L1 and L2 can act as a relay, helping the communication moment continue without interruption. In another case, an alternation can act as a spring-board to learn more about the target language. In other words, alternation can serve many pedagogical goals, as long as they are managed strategically by the teacher. Overall, acceptance of codeswitching is a reflection of the acceptance of the characteristic abilities used by bilingual learners.

3.2.4 Communicative competence

In the late twentieth century, language learning theories were appearing from many parts of the world. The linguist Noam Chomsky posited the theory in which the ideal language user exists in a homogeneous community and applies his perfect knowledge of the structures of language (as cited in Richards & Rodgers, 2001). In contrast, Hymes believed that a view of language as the sum of structures was incomplete and that there needed to be an element of communication and culture added to Chomsky's linguistic theory. In Britain, many scholars agreed with this and felt there was a need to teach communicative proficiency rather than be limited to the structures of language. To contrast with Chomsky's term linguistic competence, communicative competence was introduced. The ideas of communicative competence and literacy, which I will discuss in more detail later, represent the result of an important paradigm shift in second language learning. In fact, both Language and mathematics education have reached similar conclusions in their paradigm shifting. For these reasons, I chose to expand this discussion to carefully develop the explanations and features of communicative competence, its implications for the learner and also some challenges it poses to educators.

According to Richards and Rodgers (2001), communicative competence is characterized by the following statements:

> 1) Language is a system for the expression of meaning. 2) The primary function of language is to allow interaction and communication. 3) The structure of language reflects its functional and communicative uses. 4) The primary units of language are not merely its grammatical and structural features, but categories of functional and communicative meaning as exemplified in discourse. (2001, p. 161)

The first three characteristics highlight the communicative function of language, which is in agreement with Grosjean's definition of bilingualism. The fourth characteristic shows the gradual transition from the sentence as a unit of language to the discourse. Slowly, the microscopic view of language is expanding to include a whole new set of human behaviours. Communicative competence is about a certain savoir-faire, an awareness of sociological appropriateness in linguistic behaviours. Increasingly, the social nature of language plays a factor. This is supported in educational theory by such movements as socio-constructivism and cooperative learning. The terms discourse and Literacy appear as a way of describing this new set of behaviours surrounding language as it exists as a social phenomenon. According to Dagenais (2004), the uncertainty of educators' understanding of the term Literacy makes it difficult to implement change in the education system. On one hand, there are those who define Literacy in terms of a technical discourse involving teaching activities and specific behaviours; and on the other hand, others see it as a social discourse which emphasizes the awareness of how written language plays a role in society. Gee states, "What is important is not language and surely not grammar but saying-(writing-)doing-being-valueing-believing combinations. These combinations, I call Discourses with a capital D" (2001, p.526). The essential feature of language now extends beyond its structures to include decision-making and the forms of judgment being applied in social situations. Again, the parallels between the terms Literacy and numeracy are not insignificant. In both the Language Arts and in Mathematics Education, there is a growing trend towards the macroscopic skills of problem-solving which require more subjective forms of judgment, and away from technical, objective questions based around microscopic structures of the language or the accuracy of computation.

Another way of seeing communicative competence, in particular for the bilingual learner, is as the creation of original linguistic tools in the face of communicative obstacles. Lüdi & Py (2003) argue:

> La recherche sur la compétence du bilingue ne doit pas se limiter à la sauvegarde de la langue d'origine ou à l'acquisition de la langue d'accueil, mais s'étendre à la création d'un outil langagier original qui réponde le mieux possible à ses besoins propres. (p.114)

Communicative competence, then, is the sum of the skills developed as a result of overcoming communication barriers. So, more important than having a large vocabulary and mastering syntax, is the ability to solve communication-based problems. Py (1993) calls this the task-based component of a three-poled framework. Skills that allow learners to solve these problems may include "la cohésion relative, puissance réferentielle, économie des moyens, autonomie, création des formes inédites, interprétations des énoncés" (Py, 1993, p.11). These are some of the skills that are relatively unique to bilingual or plurilingual learners. For starters, cohesion is required in order for all parts of interpretations of the message to create a unified understanding, without which, the learner cannot make sense of the communication. Referential power is the ability to make approximations of meaning in a new lexical context. The more the learner is able to make educated guesses about new words' meanings, the greater her referential power, and the greater her overall communicative competence. Economy refers to the ability to process all these cognitive operations effectively all the while minimizing the expenditure of effort. If a new learner is overwhelmed by the various steps required in comprehending and processing a unit of communication, the lack of efficiency may compromise motivation and interest to continue. Next, the creation of new forms allows a learner to make educated guesses by applying the known rules upon new forms. While some of these new forms may appear erroneous to the native speaker, they are in fact an example of this competence in action and should be recognized as such. Finally, skillful interpretations are necessary when it comes to second language learning, as students are constantly placed in situations where they must make hypotheses based on their observations. With increased experience in testing these hypotheses, learners become more adept at making better judgments. All of these skills depend upon the practice of problem-solving situations in communication. Moreover, these skills assume that second language learners must be allowed some margin of error in which to develop the tools that eventually allow them to become more adept at using the target language. Py's (1993) description of the learner's framework includes: the tasks, which allow and require the development of these tools; the norms of the target language, which come in the form of social pressure to be understood by native users; and finally, the system, which allows the interlanguage to flourish. In other words, the system, the norm and the tasks are forces acting on several fronts to shape the growth of the second language learner.

Another important aspect of Communicative Language Teaching approach is the emphasis on the process of communicating. More important than the message or the form taken by the message, it is believed that the best way to learn about the linguistic system is by struggling with its use (Richards & Rodgers, 2001). This presupposes that it is a learner-centred and experiential approach. Howatt points out that "language is acquired through communication, so that it is not merely a question of activating an existing but inert knowledge of the language but of stimulating the development of the language system itself" (as cited in Richards & Rodgers, 2001, p. 155). In a way, language becomes a problem-solving tool that is honed every time it is practiced. With time, language skills become stronger as learners have had to use it in various situations. In contrast, a teacher-centred approach depends mostly on the teacher's transmission of knowledge to students.

In a learner-centred environment, Richards and Rodgers (2001) say that one of the roles of the teacher is to be a needs analyst. She must become very aware of where her students are in terms of ability, and then must adjust the level to match their immediate learning needs. The term comprehensible input, as coined by Krashen, describes this alignment of learning material with learning needs (as cited in Richards & Rodgers, 2001). He argues that educators need to be aware of the level of the material provided to the learner. If it is too difficult, students may focus too intently on understanding the material rather than responding to it. Cummins (2000a) adds that Immersion teachers sometimes interpret this comprehension literally rather critically. By this, he suggests that it is possible to understand material superficially, but not sufficiently to process it at a deeper level. Krashen also highlights the importance of providing material which is interesting and relevant to the learner, "in sufficient quantity, and experienced in low-anxiety contexts" (as cited in Richards & Rodgers, 2001, p. 22).

The other component of a learner-centred approach to second language education is the production of language. In the theory of Comprehensible Output, named as such to contrast with Krashen’s Comprehensible Input theory, Swain (1985) argues about the importance of practicing new skills by speaking, writing, using, and in particular, the importance of trying again when initial attempts fail. This allows new French speakers to test out hypotheses once they have acquired certain skills. Another perspective on this theory is that the communication situation needs to be sufficiently difficult that it will push learners to develop effective ways of communicating. If the situation does not result in some sort of obstacle to overcome, requiring more effective forms of communication, then the learner may remain satisfied with simpler, less sophisticated use of language. As Swain and Lapkin (1998) explain, "Learning does not happen outside performance; it occurs in performance" (p.321). With a learner-centred approach that creates appropriate and challenging levels of output, students are more likely to improve their language skills. Moreover, Mason says, "Successfully overcoming being stuck engenders a positive attitude and self-image and it is from the reservoir of success that good ideas and positive attitudes will come in the future" (1982, p.134). Being stuck, as he puts it, can only come if the situations demand increasingly complex communication skills. In sum, educators need to be aware of the delicate balance involved in finding a comfortable zone in which learners can still be challenged. This is reminescent of Vygotzky's notion of the Zone of Proximal Development (1986) in which he explains that children learn best when the level of difficulty is within reach but slightly above their current abilities.

More recently, some researchers have challenged this notion of learning by having to overcome obstacles. Krashen (2003) defends that language learning does not occur when students face impasses. Rather, they are more likely to learn when they understand the situation fully and are capable of contributing. In other words, he believes that facing linguistic obstacles does not lead to learning but rather the opposite. Learners that do not understand or cannot have their ideas heard by others are more likely to shut down than develop better linguistic tools. This debate has stirred quite a bit of attention in the second language acquisition field. However, these two views are simply two ends of a spectrum. Learners need to both be challenged and comfortable, and this critical balance is different for every individual.

One major concern about this approach for educators is the implication that accuracy in the use of the L2 now is less valued than communicative competence. Whereas before, the role of the educator, in a teacher-centred classroom, was to ensure a proper modelling of the second language, it has now changed to include facilitating discussions, being a resource person and providing guidance (Tremblay, Duplantie, & Huot, 1990). For Richards & Rodgers (2001), "[Communicative Language Teaching] may cause anxiety among teachers accustomed to seeing error suppression and correction as the major instructional responsibility" (p. 168). This is largely due to a discrepancy between the view of bilingualism as either based on mastery or usage. The debate about error correction is a persistent one. Advocates of CLT believe that errors are a natural part of learning a second language and are largely developmental. Others feel that leaving the errors uncorrected may lead them to become "fossilized" (Lapkin, Swain, & Shapson, 1990). This term has stirred quite some controversy. For some, it gives the impression that the learner is no longer making progress and does not take advantage of learning opportunities to improve, forever to be using faulty language (Lüdi & Py, 2003). Similarly, the term refers to a degree of mastery of the target language, and does not describe the ability to communicate for a purpose. However, making errors, while it may be an indication of non-native mastery, does not imply that the speaker is incapable of communicating in this second language. Moreover, there is no evidence that correcting students' errors is an effective way of reducing them (Swain, 1988). Nevertheless, researchers agree that a focus on both communication flow and an awareness of language structure are necessary (Swain, 1988; Lyster, 1990).

In the same vein, researchers have brought up the argument that communicative competence could be well complemented with an analytical focus on language (Lyster, 1990). In order to explain his point, he uses Krashen's Monitor Model and the difference between learning and acquisition. In this theory of second language acquisition, Krashen argues that learning is what happens when there is a conscious effort to master the rules of the language (as cited in Lyster, 1990). In contrast, acquisition occurs more naturally, just as children might learn a first language. In other words, acquisition happens subconciously by being immersed in the language and through the process of trial and error (as cited in Lyster, 1990). According to Lyster (1990), there is a place for analytical teaching if the classroom already provides opportunities to acquire language. He suggests that analytical teaching need not be synonymous to the traditional grammar teaching that was kept separate from usage. Instead, he feels that the communicative competence and analytical language skills can work together to create a more complete approach.

3.2.5 Content and language integrated learning

For some teachers, the notion of Immersion implies little more than the instruction of English native speakers using French, with no consideration for the integration of content and language. However, it is a delicate challenge to combine the two. As we have seen, the integration between content and language is not a simple matter of teaching mathematics in French. The next section describes several forms of integration between content and language as seen in second language education programs. The method of teaching second language through content, as practiced in Immersion, is often referred to as Content-Based Instruction (CBI). There are many second language programs that focus on content as a means through which to learn language and so, there are, understandably, also many ways to refer to this practice (Gajo, 2001). However, just as every program has subtle differences from one another, each designation also implies subtle differences. In an attempt to unify the many options, Nikula and Marsh (1998) argued for using the term Content and Language Integrated Learning (CLIL) because it makes clear that content and language are on equal footing. Within this broad term can be included variations such as the French Immersion program, where the extent of integration between content and language is quite significant. Other variations may have less contact between the foreign language and content areas.

Researchers debate about the relative balance between content and language. By content, most users refer to "the substance or subject matter that we learn or communicate through language rather than the language used to convey it" (Richards & Rodgers, 2001, p. 204). On one hand, some argue that content and language should work together, yet without dilution or compromise (Teemant, Bernhardt, & Rodríguez-Muñoz, 1997). On the other hand, aiming to focus on both may be problematic. Noyau (2004) argued that this bifocalisation -- having to manage both communicative content and the linguistic code simultaneously -- causes a conflict which results in the choice between one or the other goal. From the perspective of Grosjean's functional bilingualism, one might argue that the assessment of content and language are intertwined and indissociable. According to Grosjean (1982), our bilingualism is determined by how well we can use language to communicate meaning. So, from the Language Arts and second language perspectives, a less than perfect communication of one's understanding of a mathematics concept, for example, would indicate both a need to develop the language and content skills. Met (1994) supports this in saying, "Classroom-based language assessments are authentic in that they measure student proficiency in the real contexts in which language use occurs. Such assessment, in essence, has content validity" (p.176). Conversely, and more interestingly, a compromised communication of one's understanding might also be linked to a less than effective understanding of the content. However, it is also possible to have a competent understanding but to struggle in expressing it effectively. Met (1994) explains that "the more effectively one can express one's thoughts through language, the more clear and precise thinking becomes" (p. 176). So while a compromised communication may reflect a poor understanding, it may also be unrelated to understanding. Yet, effective communication may both follow effective thinking and potentially improve the quality of thinking. In fact, in the evaluation of a numeracy-based task, the communication of one's ideas, albeit in a second language, is measured alongside the accuracy of the calculations and application of mathematics. The effective communication of one's mathematical ideas is part and parcel of one's understanding of the mathematical process. Certainly, the relationship between content and language is not easy to define.

CHAPTER 4: RESEARCH QUESTION AND METHODOLOGY

Given the challenge of combining French Immersion goals with those of numeracy, my thesis examines how they come together in practice. The first section of this chapter discusses the rationale for this study. Next, I will explain my research questions and describe the participants and setting for my study. Then, I will describe the nature of my study and its methodology. In this chapter, I will also explain the criteria used to analyze the data as well as the five tasks assigned to the class which generated these data.

4.1 Rationale

There are several motivations for conducting this study. First, French Immersion teachers tend to be French specialists rather than mathematics specialists, and rarely, second language specialists. This is problematic because the FI teacher is less likely to be concerned with finding ways to fulfill numeracy objectives than he is with speaking French and maintaining a French environment in the classroom. This also suggests that FI classes are less likely to value numeracy than non-immersion classes simply because the juxtaposition of communication skills with mathematical ones may be an added complexity to the pedagogical demands of the second language teacher. This research aims to examine the reality of how Immersion students use language in numeracy in the hopes of discovering what it can do to transform or hinder their learning. Granted, the second language context surely only amplifies what is also true for learners in their first language. This research may shed light on the relationship between communication skills in mathematics for classrooms other than Immersion ones. Primarily, it is an opportunity to examine more closely how students use their second language to make sense of their learning in mathematics and how they express their understanding. Moreover, since every individual has a different balance of skills in problem-solving and in communication, it would be interesting to note how these differences play out in a context where both are required.

Second, in all the literature I have encountered which describes the FI programs, the percentage of French content decreases as students reach high school from 100% in Kindergarten to 50% at graduation. However, there is little or no discussion of why there is such a drop. The only exception is the brief explanation based on language acquisition theories at low grades (Baker, 1996) requiring a more intensive immersion. For the high grades, there are several hypotheses about this drop in French content. It may simply be that there are fewer teachers available with sufficient knowledge of French to teach more specialized disciplines such as mathematics. While this is certainly a challenge for school districts, this is not the place to address this issue. There is also a sense that a cognitively demanding subject such as mathematics cannot be done justice using a second language. Continuing to do so may jeopardize students' opportunity for scholarships and such (Halsall, 1998). Some authors have suggested, but only in vague terms that certain topics or disciplines, by their nature, do not lend well to being taught in a second language, including algebra (as cited in Cavalli, 2005). Others have claimed that Immersion students do just as well as non-Immersion students in disciplines such as mathematics (Bournot-Trites & Reeder, 2001). However, these claims are based on standardized testing and very little is said about the particular focus in mathematics. It is unlikely that communication about mathematics was included in this assessment. Despite mostly vague findings about the connection between language and mathematics, there has also been evidence that the combination of content with second language may even improve the learning in both areas (Cavalli, 2003) even at the high school level, when concepts are more complex (as cited in de Weck, Gajo, Moderato, Blanc-Perrotto, 2000). The idea is that second language creates a distance between the everyday common language and that of a specialized vernacular. This distance can help learners gain more awareness of the ideas they are struggling with. Given this perspective, I question the necessity of reducing the percentage of courses taught in French at upper levels.

This concern about the decrease of French content at upper levels in mathematics relates to my research question because if there is indeed reason to believe that the combination of second language with the learning of non-linguistic disciplines can be an effective one, then my examination of the role and need of language is evermore vital.

4.2 The research question

Although research on both numeracy and second language education is abundant, educators struggle to reconcile the two areas of teaching. There is a need to examine how language and content can juxtapose effectively. More specifically, I question not only how they can juxtapose effectively from the educator's perspective, but also how learners use one to support the learning and expression of the other. My research question is the following. **In what ways do learners use language in mathematics, in the context of a French Immersion class?** Although my question is framed within the context of Immersion, nothing suggests that the answer may not also apply to non-Immersion settings.

4.3 The participants

The subjects in this study are twenty-seven grade 6 and 7 students from an Early French Immersion program in a Middle School of a suburban region of British Columbia. The families of this cohort were mostly of middle to upper-class socio-economic status. I was the core teacher to these students and was responsible for most of their core subjects including French, English Language Arts, and mathematics. I taught three 80 minute sessions of mathematics per week. Sometimes our lessons were based on the textbook and involved the practicing of specific skills. Other times, we did problem-solving in groups or as a whole class. The guiding principles of my practice in problem-solving were outlined in chapter two. I focused on the use of open-ended tasks that encourage communication between peers and the discussion of a variety of strategies. These tasks usually do not have one correct answer and allow students to take many different directions in solving them. Furthermore, students were required to use French writing to communicate their thinking.

Before data collection began, this cohort of students had experienced few instances of problem-solving or writing in mathematics with previous teachers. Most of the early experiences in my class were exploratory in nature and did not require written responses. As such, they had not practiced much mathematical writing until the data collection began, neither from my class nor from previous teachers. In previous years, they were more familiar with textbook drills and computational exercises. Before attempting to gather information about their learning using writing, it was my intention to set up a classroom environment where problem-solving, divergent thinking, communication with peers were normal, expected and welcome behaviours. These were encouraged and modelled in my class.

4.4 Setting

In order to give the reader a better understanding of how my French Immersion class managed and functioned in a second language, I would like to paint a more detailed picture of the policies and practices regarding French language usage in my French Immersion class. When working on problem-solving and in most discussion settings, Immersion students in general tend to revert to using English. As a teacher and researcher, my views on the policies regarding English use in the class evolved as I completed research on this thesis. Originally, I was more inclined to be strict about enforcing a French-only policy in any peer-to-peer discussion or peer-to-teacher conversation. I would walk around the classroom taking note of which students followed the rule and which did not. I might also verbally remind individuals about the policy. At the end of the month, the better half of the class was rewarded in some way and the other half had extra practice in Language Arts to complete as a consequence. None of the students were surprised by this and they often accepted the consequence with few complaints. In some cases, little effort was made to get the reward. In discussions with French Immersion colleagues, opinions varied about how to deal with the challenge of getting students to speak French in the class. Some would use similar prizes or bonus marks to encourage them, while others threatened with phone calls home or detention time spent after class. Many teachers were also inconsistent in their enforcement of the language rules, depending upon their energy levels and contexts of discussion. One of my Immersion colleagues did not feel that speaking French during group discussions with peers was really necessary, as long as they interacted with the teacher in the target language and completed their assignments in French. However, he did not ever discuss this issue with his students. Some parents even voiced their disappointment at not upholding what they believed to be basic Immersion principles.

As the year went along and my research advanced, my own values shifted cautiously towards a more flexible approach. There would be conversations about the appropriateness of using English in certain contexts as opposed to others. Together, we would establish an acceptable level of expectations for the use of French. When the task was considered challenging, more English would be allowed, and when the discussions were more mundane, it was expected that French would be used. At no time was the use of English accepted when used out of laziness. There were fewer consequences doled out and at the same time, it was my impression that students improved at self-monitoring. Eventually, I devised a rubric for students to evaluate the appropriateness of their use of French and English. In the end, I felt it more important for students to make a deliberate choice than enforcing a hard and fast rule that applied at all times. This rubric "Quand l'anglais est moins défendu" [When English is less forbidden] can be found in Appendix B. Again, this shift from product to process is a pervasive theme. Another device used to promote the use of French was the metaphor of a target (see Appendix C). At the centre of the target, for 100 points, was our common goal “de viser le français” and to think and communicate all in French. It is explained repeatedly to learners that while this circle was small and hard to attain, it was still important to aim for it. For 50 points, the first inner ring represented “bonnes idées, with some English help”; for 25 points, the second ring represented “good ideas, but mostly in English” and finally for 10 points, the outer ring generously offered points for “what thinking?!”. By referring to this metaphor and visual aid, learners in my class were challenged to aim for a high level of French usage, without necessarily being punished for failing to reach the goal. With this approach, I felt that the linguistic flexibility was justified in cases where the mathematical thinking was deeper. In other words, allowing some manoeuvering room linguistically helped avoid situations of cognitive overload in the content areas.

4.5 Type of study

This study is a qualitative examination of how Middle School French Immersion students use language to solve problems in mathematics. It is a combination of action research and a single case study of a group of 27 students. According to Gall, Gall, & Borg (2005), action research is primarily concerned with addressing the "practitioner's immediate, local needs […] to promote positive social change" (p. 488). In this case, that social change involves an increased awareness of how French Immersion students use language in mathematics. This is important because it may help school districts make well-informed decisions about policies relating to French content in Immersion as well as curriculum design. On an immediate level, this research will allow me as a teacher to better understand how to provide the necessary support for balancing second language learning with numeracy. If I am to implement a program which acknowledges the unique challenges of Immersion while promoting the values of numeracy, I need to know how learners use and need language in problem-solving.

The second perspective this research takes is that of a single case study of my class. While it could have been possible to gain access to other French Immersion classes from the district, the purpose is not so much to make generalizations about the way language and dialogue influence students' learning in mathematics, as it is an opportunity to observe the various ways in which they communicate about their learning. The goal is to provide a thick description of this phenomenon. Moreover, it is a chance to experiment with theoretical claims from similar environments about second language learning. Granted, any theory about learning cannot simply be transposed from one class to the next without preparation and adjustment to the class culture. So, although some ideas were tried in my class, their success or failure are not meant to justify or contradict the theories. The point is simply to experience and manipulate notions about second language learning that were previously taken for granted. This case study also allows the opportunity to describe both emic and etic perspectives on a phenomenon that occurs on a daily basis. I found that my students were very eager to discuss their experience in French Immersion and many were equally curious about the processes involved in their education.

4.6 Numeracy task introduction

What follows is a brief description of each problem assigned to the students. The original wording in French have been included in the text and can also be found in their entirety in Appendices D through I. The selection chosen offers a variety of challenges to the students. Although each problem met certain outcomes, there was little premeditation in how each new situation complemented previous ones. The criteria used in choosing the problems were inspired by Liljedahl (2006), who lists three types of tasks: planning, fair share and estimation across large number of variables. Next, he also proposes six criteria for quality tasks: 1) accessible (low floor), 2) extendable (high ceiling), 3) degrees of freedom, 4) fixed points, 5) ambiguity, 6) problematic and necessitate communication. Although these characteristics were not matched perfectly in each task, they served as a guideline. The creation of tasks or problems in this study involved varying amounts of spontaneity and planning. Some of them had been designed, reviewed and tested by a group of teachers while others were my spontaneous invention and inspired by the lessons of that particular week. Unless specified otherwise, all work done on the following tasks, oral discussion and writing, was primarily in French. In rare and specific instances, English was allowed strategically.

4.6.1 Camions et rectangles

These two tasks were taken and translated from a reference book entitled *Elementary and Middle School Mathematics* (Van de Walle & Folk, 2005). See Figure 1 for a reproduced version in English of *Camions* (Van de Walle & Folk, 2005, p. 282).

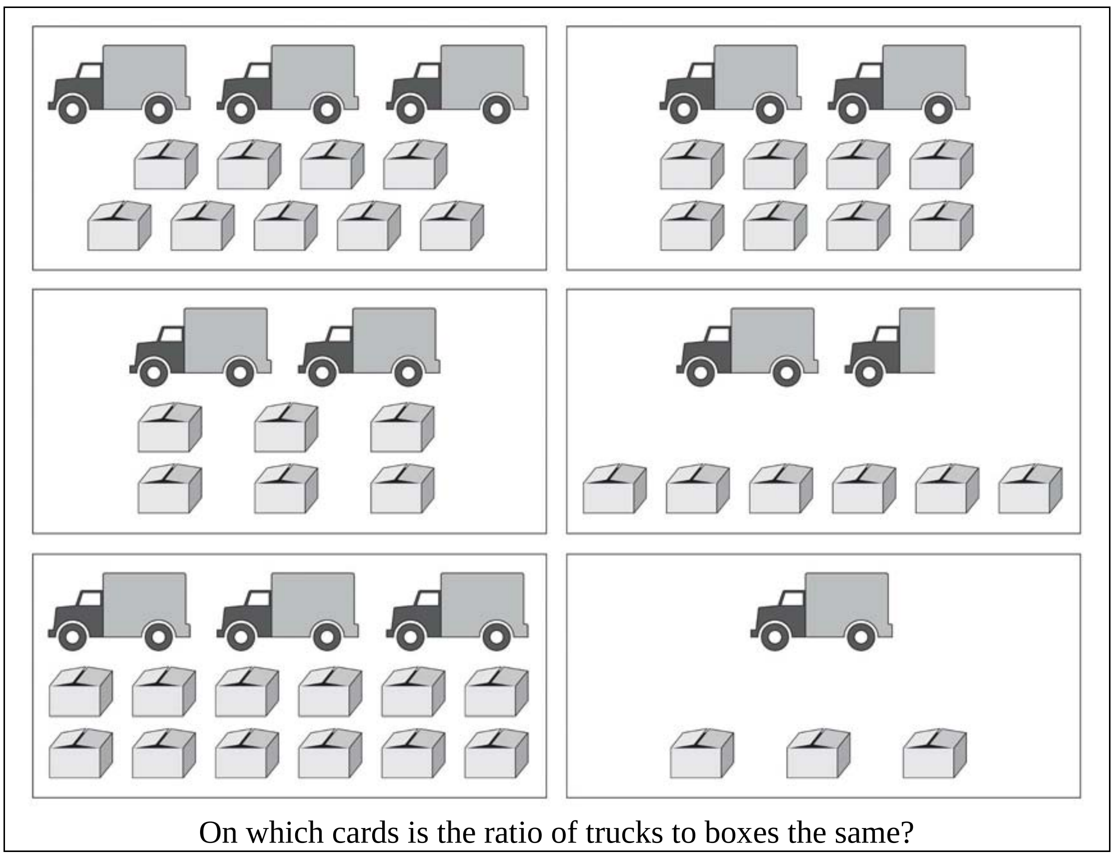


***Figure 1. Camions* task in original English version**

Camions was intended for the grade 6 students and *Rectangles* was for the grade 7s. They were grouped together because the theme was rates and ratios in both cases. However, in the grade 6 case, the two quantities to be compared were made explicit whereas in grade 7, the students had to find the two concepts being compared. The *Camions* problem had pictures of several trucks and varying numbers of boxes. The question asks to find how many boxes go in each truck if every truck should have the same number of boxes. The problem is illustrated, with a short sentence to explain the challenge. The grade 7 problem consisted of several types of rectangles (see figure 2), some long and narrow, others wide and square-like, of all different orientations. The students were asked to figure out a way of putting these rectangles into four

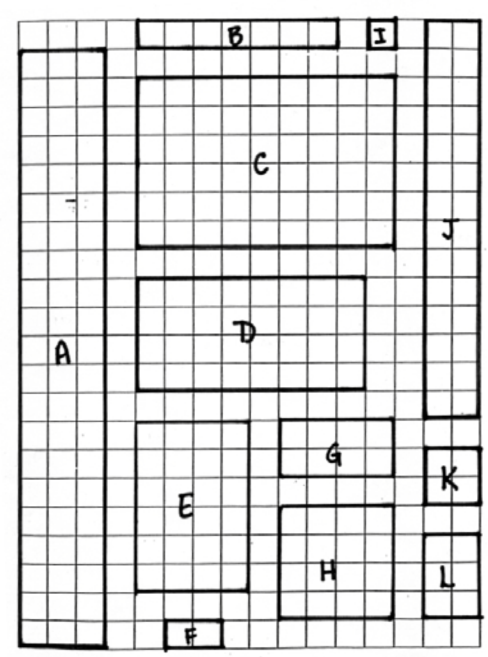


Figure 2. Rectangles

different categories, but without knowing what these categories were. They were given the hint that the square belonged in one category by itself. Previous to this problem, there had already been some discussion about ratios, but there had never been a similar problem in which length and width of a rectangle were compared. All shapes with an equivalent ratio were in the same category. After having a few minutes to discuss with their classmates, they were asked to write their explanations. In both grade 6 and 7, they were asked to share two perspectives on their problem-solving: the mathematician's view and the person's view. These were based on Liljedahl's suggestion (2007) of the three journalling personas of the mathematician, the narrator and the participant. The narrator perspective was left out for this exercise. The mathematician's view represents the procedural or algorithmic description of the problem. The participant’s view represents how the problem-solver felt as she was working through the problem.

4.6.2 Olympiques

This problem (see Figure 3) was created by a teacher colleague near the time of the 2006 Winter Olympics in Torino. It is based on the question of how to determine a winner in the Olympics. The class is given a list of a dozen countries and the medal tally as well as their respective populations. The students are asked to rank the countries and to justify their answer.

To report their solution, they must describe their plan of action and write out their explanation and calculations. Although some students knew the method by which the Olympics committee typically chooses the winner of the Games, little emphasis was placed on this knowledge.

CLASSEMENT DES PAYS AUX OLYMPIQUES

Aux Olympiques de Turin, en Italie, les athlètes canadiens ont bien réussi. Mais la question est, à quel degré ont-ils réussi? Classe les pays ci-dessous en ordre de succès. Tu peux utiliser l'information donnée ci-dessous. Justifie ta méthode de classement.

PAYS	OR	ARGENT	BRONZE	POPULATION
l'Autriche (Austria)	9	7	7	8 150 835
la Norvège (Norway)	2	8	9	4 525 116
la Corée du Sud (South Korea)	6	3	2	48 422 644
l'Allemagne (Germany)	11	12	6	82 431 390
la Suède (Sweden)	7	2	5	9 001 774
le Canada	7	10	7	32 805 041
la Suisse (Switzerland)	5	4	5	7 489 370
les États-Unis (USA)	9	9	7	295 734 134
l'Italie (Italy)	5	0	6	58 103 033
la Russie (Russia)	8	6	8	143 420 309

***Figure 3. Olympiques* task**

Instead, the problem required the students to use a method that they felt was most fair. In the journalling, they were asked to divide their work under three headings: planning, calculations and final answer. They were also encouraged to use a table if it would improve the clarity of their work. This task, by its simplicity, allows the student to demonstrate a sense of logical thinking as well as organization, rather than the application of any one mathematical skill.

4.6.3 Rénovations

Rénovations was designed by a group of teachers on a Numeracy Task Design Team (see figure 4). It was subsequently reviewed and tested by a pilot class. There are three siblings who are offered a budget for conducting some renovations in their bedrooms. Given the dimensions of their rooms and their desired renovations involving paint, laminate flooring or carpetting, students were asked to decide whether granting them their requests would be possible within the given budget. They had to calculate the area of each wall or floor and figure out how much paint, laminate flooring or carpet was needed to cover the areas. Then they had to compute the price of purchasing the correct amount of material and find the total cost. Although each individual calculation was not difficult, keeping track of all the numbers was the greatest challenge. The only skills required were arithmetic and some simple geometry. Nevertheless, this task was significantly more complex procedurally than the two previous ones. The question alone, including the instructions and the list of dimensions of each room, took up one full page. Students were given 80 minutes to complete the task and most of them brought it home to work on. They were asked to complete their calculations on a formatted 11"X17" sheet folded in half to make a booklet. This booklet was organized into several sections: planning, rough work and final answer.

LES RÉNOVATIONS

La famille Glico vient tout juste de s'acheter une nouvelle maison, et pour la première fois chaque enfant aura sa propre chambre. Puisque la maison est un peu vieille, M. et Mme. Glico veulent redécorer la maison. Les enfants voulaient tous choisir comment redécorer leurs chambres. Les parents ont accepté mais ont dit aux enfants qu'ils avaient seulement un budget de $2550. Une autre condition était que le plafond de chaque chambre devait être peint en blanc.

Magalie, la plus vieille, a demandé que sa chambre soit peinte en bleu clair. Elle veut avoir du tapis car elle a toujours froid aux pieds. Son deuxième choix de couleur serait une peinture violet claire.

Marie-Josée veut que sa chambre soit peinte violet clair. Elle veut du plancher laminé et ne veut pas de tapis. Si nécessaire, elle serait contente avec des murs jaunes.

André veut que sa chambre soit peinte en jaune. Il aimerait avoir un tapis jaune/orange, mais il serait prêt à avoir du plancher laminé.

Les prix du matériel

Peinture du plafond	$25/ boîte	Couvre 300 pieds^2	2 couches nécessaires
Peinture murs	$45/ boîte	Couvre 450 pieds^2	3 couches nécessaires
Tapis	$4/pied^2	Est vendu en largeur de 12 pieds	
Plancher laminé	$2/pied^2	Boîte couvre 30 p^2	

La taille des chambres

Chambre	Hauteur	Largeur	Longueur
Magalie	8 pieds	16 pieds	14 pieds
Marie-Josée	8 pieds	16 pieds	12 pieds
André	8 pieds	14 pieds	12 pieds

Est-ce que chaque enfant aura son choix et si non, quelles décisions prendras-tu et pourquoi?

***Figure 4. Les Rénovations* task**

On the first page, they were also given the marking criteria used. When I introduced this task, we read through these criteria and I answered questions relating to the problem before letting them proceed. We also read the long detailed question together, reducing the likelihood of not understanding the question. They were given several empty pages for rough work, although the instructions on the page specified that their calculations should still be neat and legible. The final page on the back side of the booklet had a shortened version of the question and several lines for students to explain their answers in paragraph form.3

4.6.4 Quelle roche est plus grosse?

This task (see Figure 5) was invented spontaneously. The class had been

QUELLE ROCHE EST PLUS GROSSE?
Vous avez deux roches de forme irrégulière. Vous devez décider laquelle des deux roches est plus grosse mais vous n'avez qu'une règle et les outils dans votre étui à crayons pour vous aider.
Travaillez en équipe de quatre et trouvez deux (2) moyens de résoudre votre dilemme.

***Figure 5. Quelle roche est plus grosse?* task**

studying the formation of rocks and minerals in Science class and there were several collections of different rocks lying around the room. I decided to make use of them to learn about volume. *Quelle roche* is a shortened title to the original *Quelle roche est plus grosse?* (Which rock is larger?). Two irregularly shaped rocks are given to groups of four students. They must determine which of the two is larger. Other than their rulers and the materials they already possess in their pencil boxes, they are not given any extra tools. Although the students were familiar with the notion of volume, they were not told how to decide which rock was larger. Typically, students have learned that the volume of a rectangular prism can be found by multiplying the width and length by its height. However, they have never had to figure out the volume of irregular shapes. This task also challenged the students to associate the everyday notion of size with the specialized term of volume.

In this problem, half the class was asked to journal in French and the other half in English. The idea behind this was to see if writing in French would create significant differences from writing in English. Oftentimes, Immersion teachers attribute their students' difficulties to the second language factor. By having one of the classes write in English, it may confirm or deny this assumption. Those writing in French were, however, encouraged to use English words when they felt that it would help them express their ideas better. There was much more discussion during this task than the others, since they had to work together to determine one answer for each set of rocks as a group. Unlike the other problems, where the students mostly worked alone, much of the writing of their explanations for this task was done collaboratively, even though each person was responsible for their own journal. They were allowed to discuss and share ideas, but could not copy each other's writing. It was important that each individual's understanding was assessed rather than collect one sample of the group.

4.6.5 Cultus Lake

This was another example of a teacher-designed task (see Figure 6). A school is planning a field trip to the waterslide park. In order to bring everyone there, students must decide how

SORTIE AUX GLISSADES D'EAU

Les 173 élèves en 6è/7è année de ton école intermédiaire vont à Cultus Lake pour une journée de glissades d'eau. Vous allez vous y rendre en autobus.

Le parc de glissades d'eau demande qu'il doit y avoir minimum un adulte pour superviser chaque groupe de 12 élèves à la sortie.

La compagnie d'autobus et la commission scolaire demande qu'il y ait au minimum 2 adultes par autobus.

Il y a 6 professeurs qui y vont. Les autres adultes seront des parents bénévoles.

Chaque autobus peut contenir 30 passagers.

COMMENT VA-T-ON AMENER TOUT LE MONDE AUX GLISSADES D'EAU?

Explique ta réponse en donnant des détails spécifiques sur le nombre d'autobus qu'il te faut et combien de personnes seront sur chaque autobus. Montre que ta solution peut satisfaire TOUTES les conditions.

***Figure 6. Cultus Lake* task**

many buses and adult supervisors are required, keeping in mind that there is a rule about how many adults need to be present in each bus and a rule about how many adults are required to supervise at the waterslide park. Mathematically, this task involved division and the concept of rounding. In all of these problems, the individual skills, when practiced alone, are not very difficult. What makes them challenging is the thinking required to choose the appropriate tools and the proper application in the given contexts.

The format of the answer sheet was similar to the one in *Rénovations*. The Cultus Lake problem was given near the end of the year, when our school traditionally also visited the waterslide park. Although this task could have been done at any time of the year, the connection with our own school's tradition made the problem a little more familiar and perhaps, more engaging.

4.7 Method of data analysis

Written responses were collected from the students for each of the above mentioned tasks. Several iterations of analyses were conducted on these data. In order to answer my research questions, it was important to become very familiar with the data, its range, its tendencies and its outlying cases. I found that a grounded theory approach worked well in this context. According to Strauss and Corbin (1994), with this approach, “theory evolves during actual research, and it does this through continuous interplay between analysis and data collection” (p. 273). Using the method allowed me to use findings of my analysis to reformulate increasingly better criteria for the following rounds of analysis. Rather than applying one model of analysis on the data, I performed several iterations, each based on the insight gathered from the previous ones. Having said that, however, there was much effort put into the crafting and use of the data analysis instrument used in the early iterations. This is why, in what follows, I go into greater detail about this aspect of the analysis. Methods used in the subsequent iterations of the data analysis are presented in chapter 6.

4.7.1 Initial analysis

The first model of analysis began with a rather simple hypothesis that students who were more adept at using language were also better mathematicians. In order to measure their use of language and their understanding of the mathematical concept, I started by sorting the data into some simple categories: 1) good math, good language use, 2) poor math, poor language use, 3) good math, poor language use, and 4) poor math, good language use. These categories were defined only loosely in order to get a quick sense of how students do in written work. This initial sorting revealed some interesting questions. It became clear that there were many aspects to language use. First, what qualifies as language? Are single words preferable to full sentences? Do mathematical symbols count as language? Second, is it possible to communicate mathematical ideas effectively but yet use very little word-based language? Third, what aspects of language are important in communicating mathematical ideas? And finally, how is the use of language, and in particular, a second language, related to the development and expression of mathematical understanding? While I may not be able to answer these questions, I decided to refine the categories by using the BC Performance Standards in both Writing and Numeracy (see Appendices J and K). The Performance Standards rubrics were created by BC teachers based on samples of writing and problem-solving tasks. There are four strands involved in the Writing rubric, including the meaning, the style, the form and the conventions. Similarly, the four strands of the numeracy performance standards are concepts and applications, strategies and approaches, accuracy, and representation and communication. I did not use all eight strands as they were not all relevant. As a teacher, I had never used a Writing rubric on a piece of mathematical writing in problem-solving. The exercise in itself was interesting.

4.7.2 Development of criteria

The combination of writing and numeracy rubrics became my starting point for coding the data. I devised four criteria to code my data: vocabulary, organization, reflection and mathematical concept. This is how the four criteria emerged. When examining the Writing criteria, several points were applicable to mathematics problem-solving writing. The second strand of the Writing rubric called *style* requires that the student be "clear, concrete, concise and use precise vocabulary" (Province of British Columbia, Ministry of Education, Student Assessment and Program Evaluation Branch [Province of BC Ministry of Education], 2002b). In problem-solving, using the proper specialized vocabulary is a strong indication of good understanding (Zazkis, 2000). I called this first criterion *vocabulary.* Another overlap between writing and numeracy occurs in the third strand of the writing rubric, *form,* which requires that there be a "logical sequence [and the] use of appropriate visual and text features" (Province of BC Ministry of Education, 2002b). Again, mathematical writing depends heavily on the proper organization of information as well as the use of such textual features as titles on tables and charts, the use of units when referring to measurements and the use of spacing in a piece of writing. Conversely, one strand from the numeracy rubric could also apply to writing in general. A strand called *representation and communication* which includes criteria that are similar to those of the writing rubric: "work is clear, detailed, and well-organized; creates effective charts, diagrams and graphs; explanations and demonstrations are clear, in own words, and include mathematical language; may be innovative or insightful" (Province of British Columbia, Ministry of Education, Student Assessment and Program Evaluation Branch [Province of BC Ministry of Education], 2002a). These two strands put together, resulted in my second criterion for coding, which I called *organization.* These overlaps suggest that language and numeracy are certainly not exclusive skills.

They also underline the fact that using writing-based problem-solving tasks can be an effective way to integrate learning. I also kept one criterion called mathematical concept based on the strand of *concepts and applications* in numeracy which assess the following abilities, "recognizing mathematics, grade-specific concepts, skills; patterns, relationships" (Province of BC Minstry of Education, 2002a). Finally, I added a fourth criterion to measure the use of language for the purpose of reflection. This is in contrast with words used to convey the procedures in problem-solving. Reflection includes any reference to general ideas about the nature of a task, tested and/or untested theories about the mathematical concept at play or insight into the reasoning process.

Throughout this analysis, I was constantly aware of the influence of writing in a second language. When examining French Immersion students' writing, the most apparent observation is the unconventional use of French. On one level, second language learners quite commonly make mistakes with grammar and spelling. However, on a second level, they will also make incorrect guesses about how an English word might be used or translated into French, or else, they might have difficulty putting the words in the proper order when describing procedures. The manifestations of interlanguage are hard to ignore, but not necessarily an indication of a problem. Interlanguage is the manifestation of a system students build throughout the years. Nevertheless, it is often the first impression of the French Immersion teacher. In this study, I was less interested in these examples of interlanguage, than I was in noticing the instances in which having to write in a second language might have posed an obstacle to communicating mathematical ideas. One purpose of the analysis was to uncover any interesting relationships between the use of a second language and communicating ideas. Although I did not code for these instances explicitly, they were a secondary focus.

4.7.3 Description of criteria

To summarize, the four criteria used to evaluate the students' written work, on a four-point scale are shown below in Table 1 and Appendix J. First, the vocabulary used was assessed in terms of precision, clarity and most importantly, whether it was proper mathematical terminology. Second, the organization of the written work as a whole was examined. This includes such abilities as the use of titles and physical space to separate distinct parts of the task and the use of linking words in the written sections to create paragraphs that are easy for the reader to follow. Third, the mathematical understanding was assessed by examining whether the appropriate skills were applied effectively to

Table 1
Rubric used to score students' written work

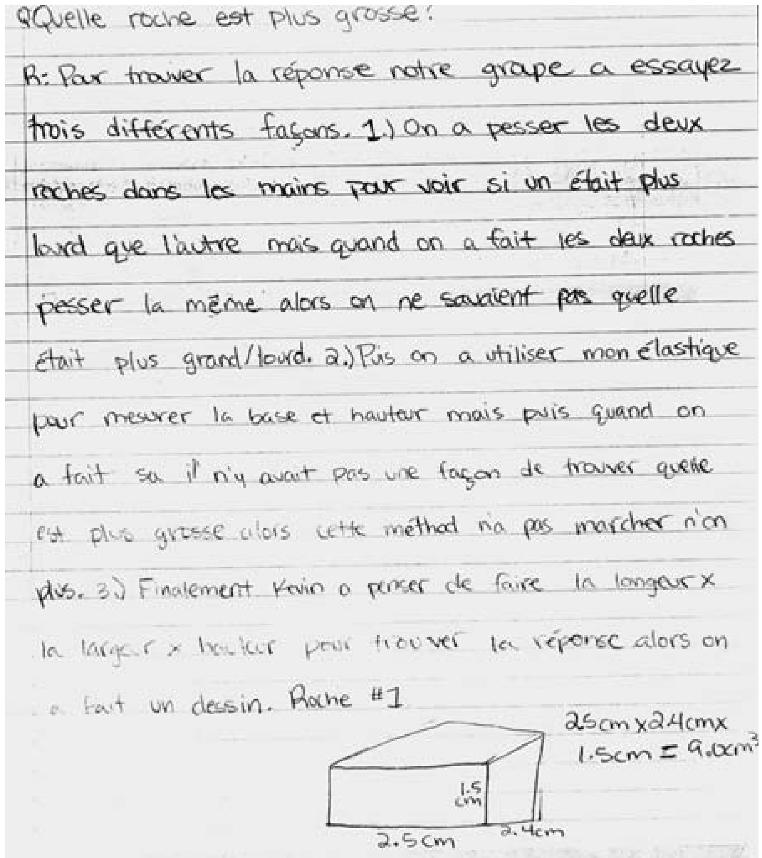
Strands	Not yet meeting expectations 1	Minimally meeting expectations 2	Fully meeting expectations 3	Exceeding expectations 4
Vocabulary	- Words chosen are often vague or incorrect - Frequent use of informal and incorrect French and/or mathematical language	- Some use of clear vocabulary, may be vague at times - Frequent use of informal or incorrect French or mathematical language	- Use of somewhat clear and concise mathematical vocabulary - May use some informal language	- Use of clear, concrete, concise, and precise mathematical vocabulary
Organization	- Few or no linking words used and various steps of problem are not clear - Work is often confusing, with key information omitted - Often omits required charts, diagrams, or graphs, or makes major errors	- Minimal use of linking words and various steps of the problem are not always clear - Most work is clear; may omit some needed information - Creates required charts, diagrams, or graphs; some features may be inaccurate or incomplete	- Use of some linking words to connect different parts of problem - Work is generally clear and easy to follow - Uses required charts, diagrams, or graphs appropriately; may have minor errors or flaws	- Effective use of linking words to connect different parts of problem - Work is clear, detailed and logically organized - Uses required charts, diagrams, or graphs effectively and accurately
Reflection	- No attempt to show insight or reasoning process	- Few attempts or inefficient attempt to use words or sentences to show insight or reasoning process	- Uses some words and sentences somewhat effectively to show insight into problem or reasoning process	- Uses words and sentences effectively to show insight into problem, un/tested theories or reasoning process
Mathematical understanding	- Unable to identify mathematical concepts or procedures needed - Does not apply relevant mathematical concepts and skills appropriately; major errors or omissions	- Identifies most mathematical concepts and procedures needed - Applies most relevant mathematical concepts and skills appropriately; some errors or omissions	- Identifies mathematical concepts and procedures needed - Applies mathematical concepts and skills appropriately; may be inefficient, make minor errors or omissions	- Identifies mathematical concepts and procedures needed; may offer alternatives - Applies mathematical concepts and skills accurately and efficiently; thorough

the mathematical situation. Finally, the fourth criterion is the use of language for thinking or reflection. This includes examples of writing that demonstrate insight into a problem, a conceptual reasoning process involved in solving the problem. This is distinct from words that are used to describe a procedure. Often, language for thinking demonstrates the reasoning behind certain decisions, or at least, begins to show some mathematical intuition.

4.7.4 Scoring samples

In order to help the reader make sense of these criteria, I will show two examples of student's work and describe how they were scored in each of the four categories. The following examples of student writing are reproduced in this thesis with all the original spelling and grammar.

The first example is from Annabelle (Figures 7 and 8) who does very well. Annabelle's writing and mathematical understanding overall are quite proficient. In the area of vocabulary, she scored 3 points out of 4. She fully meets expectations for accurately using mathematical terminology such as '*base'*, '*hauteur'*, '*grand'*, '*lourd'*. To exceed expectations, she could use the terms '*poids'*, '*grandeur'* or '*volume'*, to show that she understands the concepts being targeted. In terms of organization, she earned 4 points out of 4. Throughout her text, there are numbers to indicate the three different approaches taken. She also introduces her work with the sentence "*Pour trouver la réponse notre groupe a essayez trois différents façons*". She uses linking words such as 'puis', and



QQuelle roche est plus grosse?

R: Pour trouver la réponse notre grape a essayez trois différents façons. 1.) On a pesser les deux roches dans les mains pour voir si un était plus lourd que l'autre mais quand on a fait les deux roches pesser la même alors on ne savaient pas quelle était plus grand/lourd. 2.) Puis on a utiliser mon élastique pour mesurer la base et hauteur mais puis quand on a fait sa il n'y avait pas une façon de trouver quelle est plus grosse alors cette méthod n'a pas marcher n'on plus. 3.) Finalement Kevin a penser de faire la longeur x la largeur x hauteur pour trouver la réponse alors on a fait un dessin. Roche #1

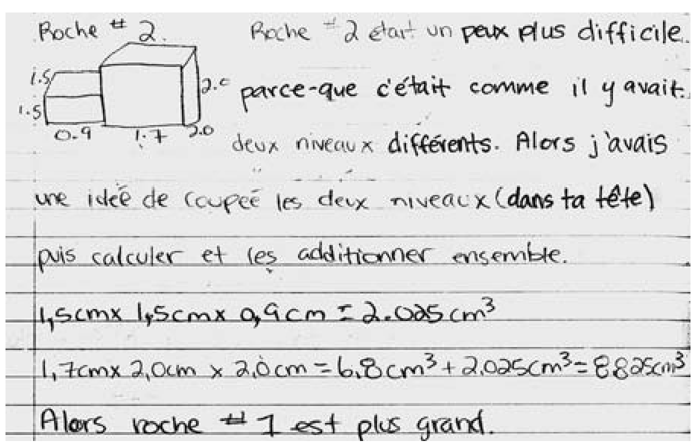
2.5cm x 2.4cm x 1.5cm = 9.0cm³

1.5 cm

2.5cm

2.4cm

Figure 7. **Annabelle's writing sample**



Roche # 2.

1.5

1.5

0.9

1.7

2.0

2.0

Roche # 2 était un peux plus difficile. parce-que c'était comme il y avait deux niveaux différents. Alors j'avais une idée de coupeé les deux niveaux (dans ta tête) puis calculer et les additionner ensemble.

1,5cm x 1,5cm x 0,9cm = 2.025 cm³

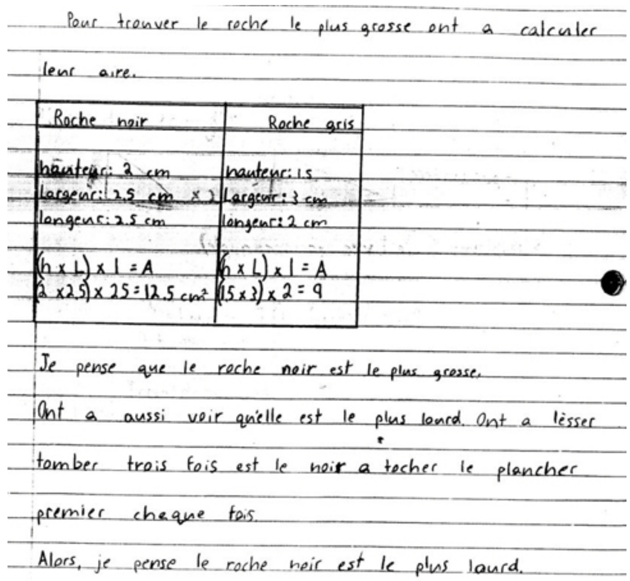
1,7cm x 2,0cm x 2,0cm = 6,8cm³ + 2,025cm³ = 8825cm³

Alors roche # 1 est plus grand.

Figure 8. **Annabelle's writing sample cont.**

‘*finalement’* to introduce her strategies. Moreover, she uses a diagram of the two rocks to clarify how she proceeded. Following her writing and calculation is easy. In the category of using language for reflection, Annabelle also exceeded expectations. Some passages of her writing are examples of using language to communicate her thinking process. Although they do not necessarily reveal extraordinary discoveries, they do demonstrate an ability to use words to convey her insight. In particular, with the second rock, she explains how she chose to calculate its volume in two parts. Finally, her understanding of the mathematical concept is very good. She fully meets expectations in this area. In the first two unsuccessful attempts, she describes strategies that show a slight confusion between the concepts of mass and volume. Nevertheless, when she does find an effective solution, it seems clear in her writing that it is well understood.

Unlike Annabelle, Craig has more difficulty (see Figure 9). In this second sample, his vocabulary score was a 2. On one hand, he misused the term ‘*aire’* when he really meant ‘*volume’*. However, he was familiar with the measurements of ‘*hauteur’*, ‘*largeur’* and ‘*longueur’*. He is also able to use the term ‘*lourd’* in describing one of the rocks, but did not quite identify the concept of ‘*masse’* or ‘*poids’*. Because of these shortcomings, he minimally meets expectations. Craig does slightly better in organization by placing the information in a table to compare the two rocks. However, other than using a table effectively, he does little more to organize a very short entry. In fact, he has written very little to explain his thinking. His description is mostly procedural and does not show any insight into the concept. He minimally meets expectations in this category.



Pour trouver le roche le plus grosse ont a calculer leur aire.

Roche noir	Roche gris
hauteur: 2 cm	hauteur: 1.5
Largeur: 2.5 cm x 2	Largeur: 3 cm
longeur: 2.5 cm	longeur: 2 cm
(h x L) x l = A	(h x L) x l = A
(2 x 2.5) x 2.5 = 12.5 cm²	(1.5 x 3) x 2 = 9

Je pense que le roche noir est le plus grosse.

Ont a aussi voir qu'elle est le plus lourd. Ont a lèsser tomber trois fois est le noir a tocher le plancher premier cheque fois.

Alors, je pense le roche noir est le plus lourd.

Figure 9. **Craig's writing sample**

Finally, mathematically speaking, he scores a 3 out of 4. He has understood the basic idea of multiplying the length, width and height to get a volume, but has not revealed any ideas about how to adapt a simple formula for the irregularity of the shape of the rock.

Based on the analysis using these criteria, a further level of analysis was performed. In examining how vocabulary, organization and language for reflection were used, I looked for emergent themes. This final level of analysis started by examining individuals whose use of these three criteria seemed extreme in one way or another. Then, I also looked at particular cases in between those two extremes, in an attempt to show a set of student work that could represent a normal range of results in the ways that language is used in numeracy in a second language setting.

CHAPTER 5: RESULTS AND ANALYSIS

In chapter 5, we set a goal to learn how students use language in mathematics in a French Immersion setting. Several secondary questions emerge from this broad objective. What are some aspects of language that allow students to express their learning? How do different learners manipulate language to suit their own needs/abilities? How does the second language factor affect the implementation of numeracy? In order to answer my research questions, my analysis comes in two stages: first, I examined the results of each individual task, based on each of the four criteria of mathematical concept, vocabulary, organization and reflection. Second, I analyzed the themes that emerged across the tasks. This chapter describes the first step of this analysis.

5.1 Task analysis

In the following paragraphs, I will describe how each of the problems encouraged students to use a diversity of skills to communicate in mathematics. As mentioned earlier, each piece of datum was rated according to four criteria on a scale of 1 to 4, a score of 4 corresponding to exceeding expectations and 1 being does not yet meet expectations. Out of a total of 27 students, the problems generated between 17 and 23 entries each. The missing entries were either due to absenteeism, students not handing in their work, or omissions due to a lack of useful data that could be gathered. In the following sections, each task will be followed by a table showing the frequency and average for each level of achievement, from not yet meeting expectations to exceeding expectations, and for each criterion. The frequency of a particular score have two interpretations. If there are many students scoring a high mark, it may be because that specific task made it easy to do well in that criterion. It may also be because the class had instructions that prompted them to focus on that aspect of the task, or they simply had more practice in that area. The average scores are also calculated to show central tendencies. I will mostly use these results as a springboard for discussion and interpret them case by case. The last column indicates the mean score, where 4 indicates a maximum total and 1, the minimum. For brevity, my discussion of these tables will focus on the more pertinent and interesting results, rather than on every aspect of each table. As a result, the discussion on each of these tasks varies in length according to the different kinds of data they produced. The tasks are described in order that they were given throughout the year, at approximately one to two month intervals. The only exception to this is the first two, which were given simultaneously to the Grade 6s and 7s respectively. Throughout the results and analysis sections, student work will be reproduced from the original without any corrections made on their typographical or grammatical errors. These are not to be considered transcription errors.

5.1.1 Camions

Although the two tasks *Camions* and *Rectangles* use similar skills and knowledge about ratios, the approaches required were different and they were intended to be completed by grade 6s and grade 7s respectively. Therefore, I will show the results on two tables and discuss them separately. That being said, the results of these two tasks are interesting to compare.

Table 2
Frequency and average scores in* Camions *task

Strands	NYM	MM	FM	EE	Ave
Vocabulary	1	4	1	0	2.00
Organization	0	2	1	3	3.29
Reflection	3	2	1	0	1.57
Math	1	1	4	0	2.29

Note. NYM = 'not yet meeting expectations' worth 1 point; MM = 'minimally meeting expectations' worth 2 points; FM = fully meeting expectations' worth 3 points; EE = 'exceeding expectations' worth 4 points.

According to Table 2, most students minimally met expectations in using the appropriate mathematical vocabulary. In this case, there is really only one important term involved in the problem: 'ratio'. However, the class had not explicitly learned it yet. The task was given as an introductory exploration of the concept. Nevertheless, they still met expectations because they were able to describe the simple process of placing the same number of boxes in each of the given trucks.

The second criterion of organization had a rather high average score of 3.29, indicating that it was not a difficult solution to organize. There were few steps involved and students were either able to use linking words to separate the various solutions or steps attempted, or they had such a simple solution that it did not require much organization. This might explain the distribution of scores between *exceeding* and *minimally meeting expectations*.

For the use of language for reflection, this task generated rather low scores, the average being 1.57 and the most of the individuals not yet meeting expectations. This is interesting with respect to the mathematical concept scores, in which most students fully met expectations. So, while the problem itself was not difficult, learners found it challenging to use language to reflect about it. In this case, it may be exactly because the task was not sufficiently challenging that there was little they could write. When the answer seemed obvious, they could not find words to explain their solution because it seemed self-explanatory. This was the sense gathered by several of the entries, including Craig who writes, "*J'ai diviser les boîtes et les camions, puis mis dans le groupes, et après sa j'était fini*". Despite the fact that this answer does not really explain how he distributed the boxes, this student was confident that it was sufficient. In trying to promote this mode of thinking through writing, the level of difficulty in the tasks provided is an important point of consideration.

5.1.2 Rectangles

In contrast with *Camions*, *Rectangles* challenged the grade 7s to think about the relationship between length and width of several shapes, whereas the grade 6 problem was more straightforward and could be completed without much

Table 3
***Frequency and average scores of* Rectangles**

Strands	NYM	MM	FM	EE	Ave
Vocabulary	1	5	5	0	2.30
Organization	1	1	5	4	3.10
Reflection	1	4	6	0	2.40
Math	0	8	2	1	2.40

reflection about the nature of the question. The distribution of vocabulary scores in grade 7 shows that more students were able to use effective mathematical terminology. To be fair, there were also more terms that could be used than in the *Camions* task, such as '*longueur'*, '*largeur'*, '*aire'*, etc. Many, instead of using mathematical language, opted to use common language by describing the shapes of the rectangles with the word "*forme*" without explaining what they meant by it or "*le nombre de carrés*" as referring to either the area or length of the shape. Although the mathematical terms should be very familiar to the grade 7 student, it may not be their lack of knowledge that compells them to use common language. Rather, it may be a question of setting up the expectation that using mathematical language is more highly valued in this context. Without this explicit instruction, students may feel they express themselves better using informal language. This supports research (Adler, 2001; Street, 2005; Barwell, 2005) that claims that learners need to use exploratory talk or informal language when in the process of working out their thinking. It may be interesting to observe what type of responses would result if the instructions were more explicit about the use of mathematical language. So, while the scores for vocabulary were higher in the grade 7 task, this was offset by a greater demand for specific terminology. In other words, the difference in scores had more to do with the nature of the two problems requiring more or less specialized vocabulary.

Not surprisingly, no student was able to make the connection between the task given and the concept of a ratio. Mathematically, this problem was much more challenging than the previous *Camions,* even if it was assigned to older children. There are some interesting differences between *Camions* and *Rectangles* in terms of the use of language for reflection and mathematical concept. Whereas it was difficult for many Grade 6s to write about their thinking process when the task was easy, the Grade 7s seem to find more to say when it was more challenging. In fact, few students actually found the anticipated answer to the question, but many of them came up with interesting strategies. In the process of attempting various approaches, the Grade 7s found they had more to write about, from the frustrations they felt at not finding the correct answer to trying to describe a procedure. In other words, having a challenging task helped them vocalize or write more about their thinking than having a simple task, particularly when they were encouraged to share their emotions about the process.

This was a useful pair of activities from the perspective of task choice. For the teacher who is trying to implement more problem-solving, *Camions* and *Rectangles* were excellent starting points because they revealed the importance of choosing sufficiently challenging tasks that could also be approached by learners at every level. Also, from the learners' perspective, it may be helpful to start with tasks that they can complete successfully, particularly with the Grade 6s. Although the Grade 7 students had difficulty finding the correct answer, they could obtain high scores by journalling their procedures and feelings appropriately. I found that problem-solving can be daunting for many learners that are uneasy with open-ended questions. Starting with easier tasks with clear writing criteria encouraged those who were more reluctant to volunteer their thinking processes in writing.

5.1.3 Olympiques

Initially, the *Olympiques* problem appeared to reveal little useful information because it seemed much easier than the others. In fact, compared to the other tasks, the average score across all four criteria was the highest for *Olympiques*, at 3.06.

Table 4
***Frequency and average scores for* Olympiques**

Strands	NYM	MM	FM	EE	Ave
Vocabulary	0	5	14	3	2.91
Organization	0	5	11	6	3.05
Reflection	0	4	13	5	3.05
Math	0	1	15	6	3.23

The average mathematical score was also the highest of all five tasks at 3.23, suggesting that it posed the least mathematical challenge. Most students had come up with similar solutions and there was little variety between responses. Interestingly, in all four criteria, a majority of students fully met expectations, supporting that there was a small range of answers. No one failed to meet expectations in any area. Twelve students decided on the winner of the Olympics by finding the total of all the medals won in each country, irrespective of their value. Ten of them chose to solve the problem by assigning a point value to each medal won by each country and adding the points to determine the country who should be considered the winner of the Olympics. Despite the narrow range of scores in each category, there was still the opportunity to expand in the area concerning the population. Students were free to decide whether this information was useful or not. Only four students factored in the country's respective populations and six of them explained why population should or should not be a factor Jack writes, "*Je pense cette système est la meilleur car un medaille est un medaille. Bronze argent ou or sais fais rien il est encore un des top 3 athelèthe à ce sport au monde*". He argues that there is no need to weigh the value of the medals, since having a medal in itself is a great accomplishment and all medals are equally valued. Alex explains why he chose to ignore the information on population in saying, "*ignore le population, car il y a probablement un nombre maximal d'athlète pour chaque pays*". One child who calculated the per capita value of a medal using the populations did not explain why this might have been a pertinent procedure. Both vocabulary and organizational requirements were minimal. There were no key mathematical terms and the task was simple enough that students could focus on the proper organization of the writing, by using clear tables and charts to illustrate their answers. The use of language to explain their thinking showed some range of responses, particularly to expand on their choice of methods used.

In *Olympiques*, weaker students were given a chance to demonstrate their mathematical writing skills, since the concept itself was easier. This was interesting because it showed that students' ability to use language was relative to the level of difficulty required in the mathematical problem. When the mathematical demand was low, even the weaker students were quite able to use language effectively. Unlike *Camions* or *Rectangles,* the task was neither too simple nor too difficult that the learners could not use language to share their thinking.

5.1.4 Rénovations

In this task, the mathematical challenge involved keeping track of every operation, while each step of calculation was quite simple in itself, requiring only the knowledge of the area of a rectangle. In terms of mathematical concept, this problem does not pose a large challenge.

Table 5
***Frequency and average scores for* Rénovations**

Strands	NYM	MM	FM	EE	Ave
Vocabulary	1	4	12	4	2.90
Organization	2	5	9	5	2.81
Reflection	6	6	5	4	2.33
Math	1	6	7	7	2.90

Only one student failed to meet expectations. However, the ability to apply a simple calculation in context without being told which formula to use should not be underestimated. Boaler (2002) notes the prevalence of situated learning in traditional settings. In her study, when students only had experience answering questions when the appropriate procedures were given, they were found to be helpless without guidance. Even though the concepts were familiar, they were unable to choose which skill to apply in a problem-solving setting. In *Rénovations,* the mathematical concepts were also not difficult, but those who did poorly in this category were most likely thrown off by the large quantity of details to take into account and to keep their work organized. All students were able to apply the formula of area to a certain extent. Those who scored poorly usually did not apply the formula to the right numbers or left out some measurements. In other words, their organizational skills affected the demonstration of their mathematical understanding. Despite this, the distribution of scores shows that a majority was successful in organizing their work. Several students seemed to struggle to keep their calculations in order. So, although organization posed the greatest challenge, those who met the challenge also succeeded well in this problem. There are many calculations to keep track of and without the proper signposts, such as units, labels and tables, it is very easy to get lost in one's own writing. Within the challenge of organizing the various calculations, students found different ways to communicate their thinking, using a combination of tables, diagrams, words and sentences. In this example, Terry (see Figure 10) clearly left out some of his calculations on another page and perhaps tried to keep track in his mind. The result is that his writing sample is hard to understand and assess.

Although the class scored the highest in mathematical concept and vocabulary on this task at an average of 2.90, they are certainly not the highest marks compared to other problems. The vocabulary in this problem is not overly complex either. However, there seems to be areas of challenge within vocabulary. First, there is mathematics-specific terminology and second, there is the non-mathematics-specific, everyday language required to communicate effectively. In the first category, there are those who had more difficulty using such mathematical terms such as "*multiplier*" or "*aire*" and opted instead for more informal language such as "*tu fais largeur et longueur*". This reflects a similar

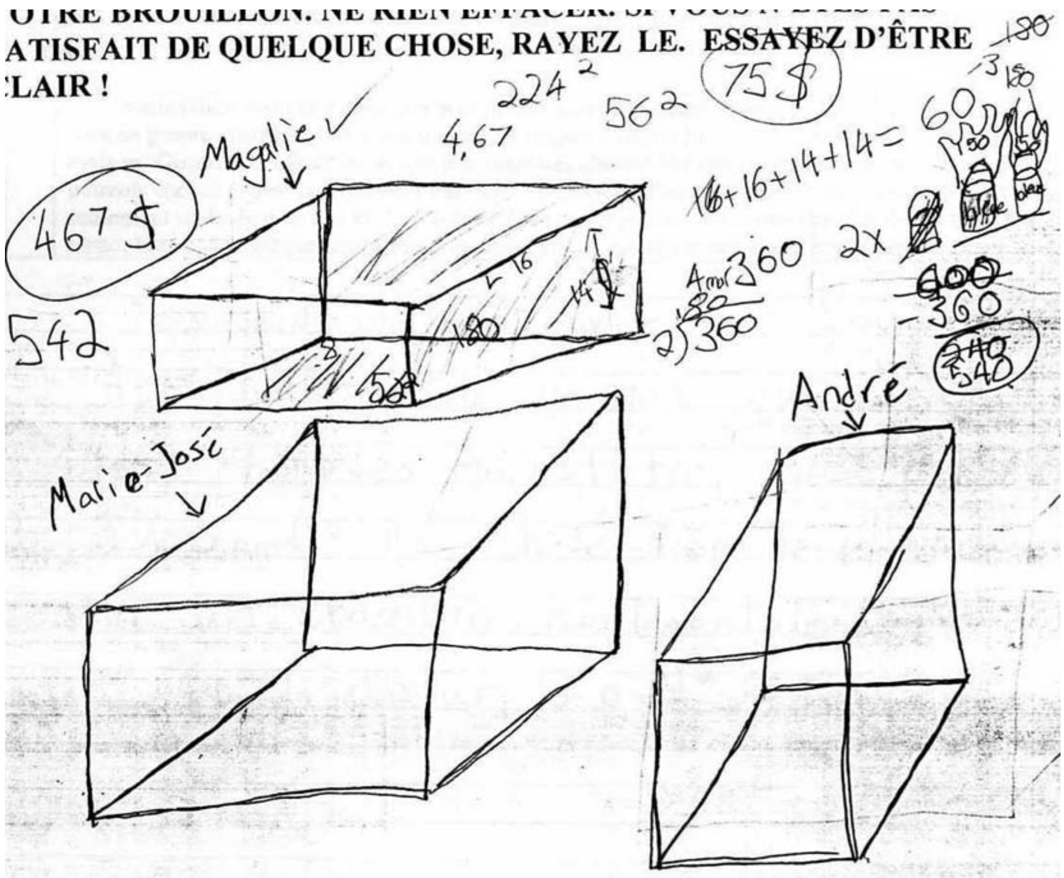


Figure 10. **Not yet meeting expectations in organization**

pattern as in the first *Camions et Rectangles* problem where students were more comfortable using informal than formal language in their exploration of a concept. In another example, Jennifer writes in *Rénovations*, "*En premier tu dois trouver la mesure de les plafonds, tapis et les murs. Sa c'est largeur X longeur En deuxièment j'ai trouver le total d'argent pour le plafond (…)*"*.* Her writing is a combination of formal mathematical terms and informal ways of expressing her calculations. The interesting question is whether her use of formal and informal language is a matter of choice or ability. It may be more natural for her to communicate this way, to get to an answer quickly. It may also be a question of cognitive overload. Having to find the proper terminology requires more cognitive function than using an informal word or symbol that can get the student to the answer quicker.

In addition to mathematics-related terminology, students must also consider using the appropriate everyday words to describe their process. Beth wrote this answer to the *Rénovations* task.

> *Pour Magalie, en premier j'ai écris combien d'argent j'avais. Ensuite, j'ai multiplier la largeur par la longeur pour voir l'aire du plafond. Après j'ai multiplier l'aire fois 2 car tu as besoin de 2 couches et puis je l'ai diviser par le nombre de pieds^2 dans un boîte. J'ai multiplier le nombres de boîte j'avais besoin par le prix d'un boîte. (pour plafond) Pour les murs, en premier j'ai multiplier 14 p^2 X 8 p^2 (long.X haut.) pour voir l'aire. Et puis j'ai multiplier par 4 murs et cette réponse par 3 couches. Cette réponse est l'aire totale. (…)*

The mathematical terms "*longueur*", "*largeur*", and "*aire*" were accurately used here. This is the basic expectation of mathematical vocabulary. Moreover, this student was able to use the words "*couches*", "*boîte*" properly, which were provided in the question, but may not have been words she was familiar with originally to refer to coats of paint and cans of paint. Being able to use these simple everyday words makes reading her work much easier. In comparing Beth's work with others, hers seems easier to understand because she takes the effort to use the appropriate French words, even for the non-mathematical ideas. Those who do not seem aware of this aspect of vocabulary produce journal entries that do not read as well. They may well have been written for the purpose of dialoguing with themselves rather than presenting their ideas to a reader.

Finally, in the category of reflection, the average score for *Rénovations* was the lowest of all four criteria at 2.33 and the scores were quite evenly spread between not yet meeting to exceeding expectations, with few people exceeding expectations. It was either challenging for learners to elaborate on a problem that appears mostly procedural, or, they were ineffective at doing so. The former seems more likely. There were several cases of students who, in the area reserved for explanations, simply stated their answers with no further details about their decision-making processes. Gillian writes,

> *Oui chaque enfant aura son choix qu'ils voulaient au début. Le prix du peinture de tous les plafonds est 100$. Le prix de tous les murs de tous les chambres est 405$. Le prix de les deux chambres qui on eu le tapis est 1384$. Le prix du plancher laminé du chambre de Marie-Josée est 420$. En totale, la somme est 2509$. Ils ont encore 41$ pour dépenser sur quelque vraiment petit meubles. En conclusion chaque enfant aura son choix.*

To be fair, because she found that every child can get their first choice, there is little discussion necessary. However, if the student finds an answer that does not fall within the budget, there is more reason to use language to explain decisions and describe one's thinking. In considering tasks that will encourage students to use language for reflection, the degree of ambiguity in the solution is an important factor. Problems with simple and objective answers may lend poorly to using natural language.

There were many examples of thinking that were conveyed through words. It happened in many cases that verbal language allowed them to share their methods of decision making, and to verbalize the problem as if in real life. Jack suggested that two of the children in the problem could swap rooms and then easily get their first choices. Danica observed that an extra 4$ could make everyone happy. In my view, whether this is true mathematically is less important than the nature of the observation. She noticed that the difference between getting their wish and not getting their wish was not a significant amount of money. This demonstrates the use of a mathematical judgment, which is in line with the definition of numeracy. Similarly, given the dilemma of having to make compromises, Helen relates to her own life by saying that in her family, the eldest usually gets first pick. Jack and Alex used words in order to clarify their diagrams and tables. In Arnold's case, his solution involved dividing the budget into three and allocating funds equally first, then redistributing the differences when needed. In this explanation, language was skillfully used to describe this complex, but effective solution.

In comparison with the other criteria, the use of language for thinking had the lowest score and mathematical concept and vocabulary, the highest. One possible interpretation is that students, in the face of a procedurally demanding problem, may be more inclined to take action by computing numbers rather than sit back to ponder the big picture. Despite lower scores in the reflection criterion, it is clear the opportunity to show the diverse solutions through language can be beneficial to fostering a sense of numeracy. This diversity may not have been apparent without the use of language. In a traditional word problem, only the solution and answers would have been the focus. In this case, allowing students to explain their thinking with language can help clear up more ambiguities or errors in their calculations. Even when the calculations are inaccurate, the words can help bring a better perspective to the child's thinking, and therefore, provide more insight for the teachers. So, while tables, diagrams, charts can help communicate a more procedural thinking, verbal language can help the writer communicate their awareness of the ambiguities of the task and of their thinking.

5.1.5 Quelle roche est plus grosse?

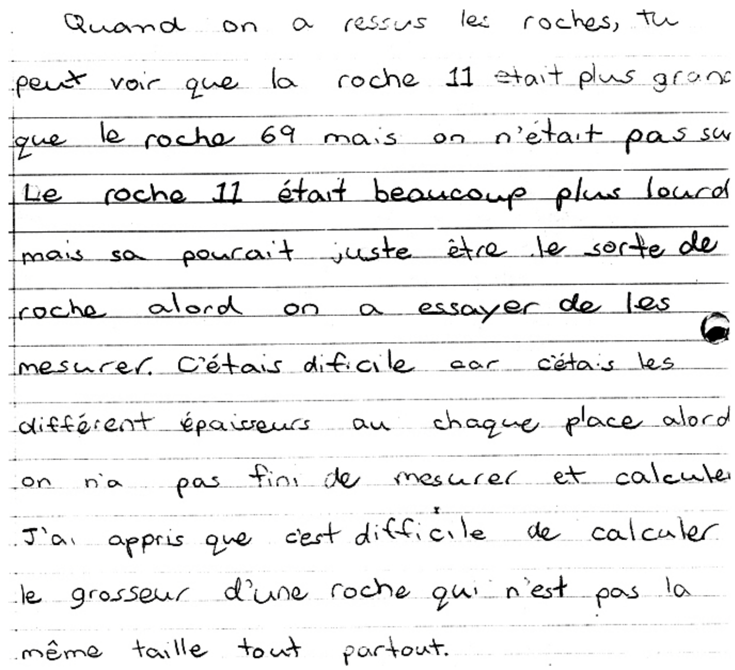
This problem demanded a little more writing than the previous two. Many students understood that size implied volume, although some others experimented with the concepts of mass and density, without necessarily recognizing it. Expectedly, the concepts of volume and mass were often

Table 6
Frequency and average scores for* Quelle roche est plus grosse*?

Strands	NYM	MM	FM	EE	Ave
Vocabulary	3	5	12	5	2.76
Organization	0	7	10	8	2.81
Reflection	2	2	14	7	3.00
Math	1	5	14	5	2.92

confused. The volume of an object is defined by how much space it occupies and the mass is the amount of matter it contains. This confusion between the two concepts is understandable, since they had not yet been taught in those terms. Although, most had had experience in making estimations and calculations of volume and mass, when given formula and exercises from textbooks. This ambiguity in vocabulary was a deliberate part of the design of the question. This can account for the lowest average score being in vocabulary at 2.76 and the distribution of scores being spread across the four levels of achievement. In evaluating this problem, it was challenging to distinguish a difficulty in mathematical concept from one of vocabulary. Researchers (Zazkis, 2000; Adler, 2001) would argue that they are related. If the student knew which calculations to use, even if he called it the mass instead of the volume, it was considered that he fully understood the concept, but not exceedingly well. Conversely, if the term chosen was accurate, but the calculations were not, then two outcomes are possible. First, neither terminology nor concept is sound, since calling the mass of a rock its volume is no more accurate when asked about the rock's size. Second, and a very unlikely possibility, the student may indeed understand the concept as it is associated with the correct term, but is unclear about which procedure is attached to that concept. Either way, mathematical concept and vocabulary seem inexorably tied.

On another aspect of mathematical concept, lower learners did not take notice of the fact that their volume calculations made use of approximations, and therefore estimations, since the rocks were irregular in shape. More advanced thinkers attempted to factor the irregularities in various ways. Arnold, for example, attempts to explain his understanding of the various “*épaisseurs*” of the rocks in figure 11.



Quand on a ressus les roches, tu
peut voir que la roche 11 était plus granc
que le roche 69 mais on n'était pas su
Le roche 11 était beaucoup plus lourd
mais sa pourait juste être le sorte de
roche alord on a essayer de les
mesurer. C'étais dificile car c'étais les
différent épaiseurs au chaque place alord
on n'a pas fini de mesurer et calculer
J'ai appris que c'est difficile de calculer
le grosseur d'une roche qui n'est pas la
même taille tout partout.

Figure 11. **Arnold's explanation of ambiguity of *Quelle roche***

The creation of tasks that incorporate ambiguity and the use of journalling as a means of communicating understanding can maximize the potential of problem-solving as an approach. A good task can allow advanced learners to demonstrate their skills while still allowing lower learners to complete the problem. Journalling allows the opportunity to demonstrate this savvy. This corresponds to the criteria of a high ceiling and a low floor (Liljedahl, 2006) as given for effective task design.

In terms of organization, this posed little difficulty, which may explain the high average of 3.04 and the absence of students not meeting expectations. Other than writing in paragraphs and separating each attempt with linking words, there were very few organizational challenges. Interestingly, when answers were written in a journal-like format on lined paper, like in *Olympiques* and *Quelle* *roche,* it seems as though students were more likely to organize their work effectively than if open space without lines was given. Perhaps this is because they are more familiar with the need to use linking words when writing paragraphs than when they are writing in this elusive language which is mathematics.

Finally, the use of language for reflection is interesting. Its average score was also quite high, at 3.00. Seven out of twenty-three people even exceeded expectations. Since the students worked in groups of four for this problem, it was very apparent that some students could only begin to attempt to retell what other members of the group had thought of doing without necessarily understanding the thinking behind it. The authors of those ideas had a much clearer explanation of their procedures. For example, one group designed a teeter-totter out of a ruler and an eraser, placed on top of a pencil sharpener. The followers could describe the set up, but the ones that understood could better explain how the heavier rock would tilt the ruler one way. This may be a good example of how cooperative learning can influence the learning of all the members of a group. The more advanced learners who come up with ideas get experience in explaining their thinking to others, while the lower ones still participate and have the chance to learn from the others. Either way, when they are individually accountable and given the opportunity to use open-ended answers, an authentic measurement of their final understanding can still be obtained.

As was mentioned earlier, one randomly selected half of the class was told to write their report in English and the other in French. While the sample chosen was not large enough to formulate any correlations, here are a few preliminary observations. In the English writers, none scored less than a three out of four when it came time to use language to communicate thinking. In other words, it could be interpreted that English writers were more likely to use the language to convey thinking. In the French writers, four out of a total of fourteen scored a 2 or a 1, including one participant who wrote so little that his report was discarded because it did not provide enough information to assess his understanding. On the other hand, when examining the incidents of codeswitching, I was surprised to see that both English writers and French writers alternated between the languages. The French writers were more likely to use English terms for non-specialized words. Helen writes,

> *En premier nous avons essaye de trouvé combien de poids c'était par les mettre dans nos mains et essaye de « feel » quelle paissez le plus, mais on ne pouver pas voir le diffence. Deuxième on a utilisé le band elastic de Annabelle pour voir combien c'était autour. On a utilisé sa band elastic parce-que on n'avais pas de « string ».*

In this passage, the use of English helps her overcome a communication obstacle. Without access to the English, she may have had more difficulty explaining her strategy, as suggested by Moore's notion of *bouées transcodiques* (1996). Interestingly, the terms she struggles with are not mathematical terms but everyday terminology. Similarly, another French-writing student felt compelled to use an English expression. He says, "*À ce point, je n'avais pas trops mit mes "hopes" sur cette façon*". It is clear in this passage that Kelvin did not need to use the particular expression to overcome a communication barrier, but rather to enrich his explanations. I wonder whether he would have chosen to codeswitch had the class not been told it was acceptable to do so.

The other half of the class revealed some interesting patterns as well. While the French writers codeswitched when they did not know everyday words, the opposite was true for some English writers. While there were far fewer incidents of codeswitching from English to French, when it did occur, it happened when the participant did not know how to use a mathematical term in English, probably because they had learned it in French. Catherine writes, *"… so we tried to find "l'aire" of the two rocks, so we mesured the length and height*". The cases of codeswitching described here support Grosjean's idea that bilingualism occurs in different degrees according to the areas of usage. The French Immersion students were more capable of using French mathematical terms than they were everyday non-specialized language. Also, even when learners did alternate languages, it was not always a matter of incompetence, but sometimes of choice. When Kelvin used the word "hopes" to translate the expression "put your hopes up", he was not demonstrating a lack of competence but rather, chose to codeswitch in order to fully express himself, or do so in a way that seems more natural to him.

These are important observations because they suggest that limiting the students' use of language may have more impact on their sense of freedom of expression and the fluidity of their expression in a mathematical setting. I would argue that these are very powerful incentives to use language to communicate and to learn mathematics.

5.1.6 Cultus Lake

The Cultus Lake Water Slide problem is a well-rounded challenge. The mathematical concept at play is the division of students into groups according to the policies of both the bus company and the waterslide park. Most students understood the basic challenge, but those who saw beyond the division problem

Table 7
***Frequency and average scores for* Cultus Lake**

Strands	NYM	MM	FM	EE	Ave
Vocabulary	1	2	15	2	2.90
Organization	1	7	9	3	2.70
Reflection	5	8	4	3	2.25
Math	1	7	10	2	2.65

also factored in the reality that the last bus to be filled would have very few students and many adult supervisors. To make it more "fair", many of them suggested to distribute the students from the more crowded buses into the last one to make the groups more evenly distributed. The distribution of scores indicate that the majority of writers either minimally or fully met expectations. Few did not meet or exceeded expectations.

The vocabulary in Cultus Lake requires a little attention. According to table 7, a large majority of the class fully met expectations in this area. Only two students exceeded expectations, suggesting that it was easy to do well, but some terminology was consistently left out by many individuals. The term "*arrondir*" for example, while familiar to learners at this stage, was rarely used, even though the procedure is required when the quotient is not a whole number. Also, explaining the procedures involved in the basic calculations seemed to pose a challenge to many. This may be a good example of providing situations for producing comprehensible output. That is, if the process of explaining a solution was so simple that all students did so with exceeding success, then they would not be in the optimal situation for developing their French language skills. However, problem-solving can naturally offer opportunities to practice comprehensible output, by requiring the use of sophisticated and accurate language.

The low average and distribution of scores in reflection indicate that *Cultus Lake* challenged many learners to write about their thinking process. In more than a few cases, learners did not in fact convey much of their thinking through words, but expected that their calculations spoke for themselves (see Figure 12). In the space allotted for explaining their solution, a few students simply wrote out their answer, rather than justify their thinking. Without having had much practice sharing their thinking, some children can be reluctant to open up in their writing, particularly if they are not confident about it. In reading the students' mathematical writing, it seemed they knew more than they were telling and may have been holding back for various reasons, whether it was self-doubt in the validity of their thinking, or a lack of words to express what they knew instinctively, or inexperience in being detailed in their writing.

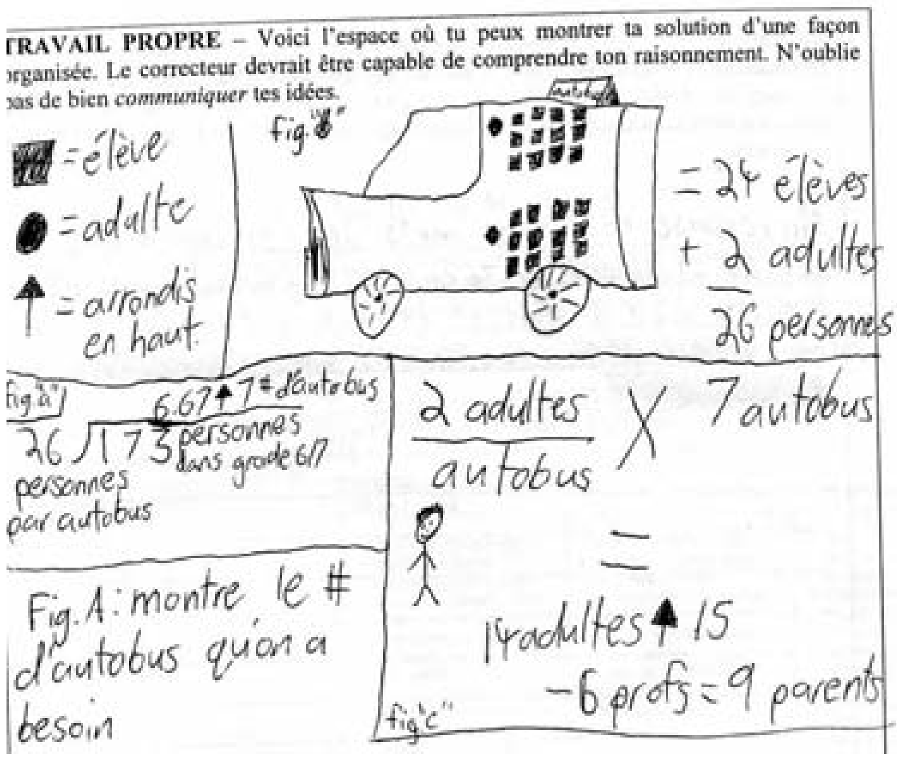


Figure 12. **Example of good understanding with very little writing**

Others, in contrast, might be very wordy in their explanations without necessarily being clear. For example, Helen writes,

> *Combiens de groupe est-que il y a? Je veux que, je peux diviser par 24 élèves alors je vais mettre le nombre à 168÷24 élèves par autobus = 7. On a bessoin 7 autobus. Mais maintenant tu as 5 élèves qui reste. Mets un élève dans 5 autobus et mets un parent dans un sixème. Après c'est 5 élève vont être dans un groupe et la parent va être un superviseur.*

In this explanation, it is unclear where the number 168 came from. Either way, experience in writing mathematically cannot be assumed. Within the same task, two students chose to represent their understanding in different ways, suggesting that there are many ways to use language in mathematics.

5.2 Initial conclusions

The criteria of vocabulary, organization and reflection led to many observations on numeracy and second language learning. The six tasks described begin to paint a picture towards answering the question “in what ways do French Immersion students use language to communicate about mathematics?” This mosaic of language use is so diverse that it is difficult to answer this research question simply and elegantly. In looking at the analysis criteria, a few statements can be made. First, the option of writing an explanation offers the possibility of justifying a unique strategy in problem-solving that may not have been given credit otherwise. Second, the amount of writing may be related to the prompts, instructions and the level of ambiguity in the task. I have not studied the first two here, but I can say that the level of ambiguity contained within a task is certainly important. We saw that in *Rénovations*, some students wrote very little because they found that all three children’s desires could be met within the budget, therefore there was little ambiguity to explain. In contrast, *Olympiques* required a decision to be made about the importance of the population in choosing a winner. Third, the ability to use appropriate vocabulary and be effective at organizing information seem to be more likely tied to mathematical understanding. Although, it is not clear at this point which preceeds the other. Is having the correct vocabulary an indicator of good understanding, or does having the correct vocabulary enrich understanding? Either way, having the proper words and an efficient organization of ideas are certainly important elements of effective communication. Also, the choice of words, whether formal and informal, French or English, seems to be a factor in the individuality of written work.

Although these criteria have helped in deconstructing the elements of communication, they do not completely address the question, “in what ways do French Immersion students use language in mathematics?” They may, however, have helped in defining what constitutes language in mathematics. In order to answer the research question, we may need to look at how individuals use combinations of words, sentences, symbols, tables, and their organization to express their thinking in the particular contexts of problem-solving and second language. In the following chapter, I look for emergent themes across all tasks and within the individual learners.

CHAPTER 6: EMERGENT THEMES

The process of analysis involved sorting the data according to trends that emerged across all the tasks. When a pattern appeared, the data was recoded with new themes in mind. Several iterations later, these themes evolve and the important ones remained as the focus. In examining the data collected starting with the original criteria, two major themes emerged with respect to the question, “In what ways do French Immersion students use language in mathematics?”. In this section, I am particularly interested in the juxtaposition of mathematics communication in a second language and the notion of these means of expression as multiple languages. First, students express themselves along a continuum between symbolic and narrative writing styles. Second, their relationship with French colours their mathematical communications, situating them somewhere within the broad spectrum of bilingualism, and perhaps even plurilingualism. In between the two extremes, the majority of students practice some form of codeswitching. The examination of this second language strategy reveals some of the motivations behind its use. Furthermore, it shows how the bilingual learner uses multiple strategies and languages in complex but specific ways, echoing the views of Grosjean (1982) and Lüdi & Py (2003).

6.1 Mathematical writing styles

If language is used to negotiate meaning, it follows that different types of learners use language differently. In the process of analyzing the data collected, two particular profiles were highlighted, which I have named the symbolic and narrative styles of mathematical writing. These occurred when individual scores in the area of mathematical concept and language for reflection were widely inconsistent. Other than the discrepancy in scores, the writing also revealed different areas of comfort and skill. In each writing style, it is possible to be an effective communicator. Although these two profiles were singled out for closer examination, they really represent exceptions within the group. They represent a minority of students who have demonstrated a strong propensity for either the use of narrative or the use of symbols in exclusion of the other. The majority of writers in fact tend to use both styles of writing to varying extents. However, they are interesting cases because they highlight extremes within a spectrum of writing styles. After showing examples of the two extreme profiles, I will also show the range of variation within the norm of the group. When I use the term language in this section, I usually refer to the use of words in sentences or verbal language, rather than elements of the mathematical language including symbols, numbers, diagrams, equations, etc. Although there are researchers who argue that mathematics is a language itself (see for example Pimm, 1987) and there exists an area called semiotics (see for example Sáenz-Ludlow & Presmeg, 2006) which examines the concept of signs and language as they are used, for example, in mathematics, I have avoided the discussion of these theories and fields in this thesis. I use the term mathematical language only loosely and in specific contrast with the use of words in sentences and phrases.

6.1.1 Symbolic style

This style of writing is characterized by fewer words and more mathematical symbols, including numbers. First, I describe two extreme cases here, the first one demonstrating an effective use of this style and the second, an ineffective one. Next, I will use some journal entries to highlight the role that symbolic language can have in mathematics.

Peter is an example of a student who seems to have a preference for showing his explanations mathematically rather than through words. In two of the five problems, he had a lower score in Reflection than in mathematical concept. However, he can still be successful overall. In two of the five problems, he scored a 4 in mathematical concept, showing that a lower language score did not prevent him from demonstrating his outstanding understanding. In other words, Peter found ways to convey his understanding other than with long paragraphs of words. Indeed, he also uses good vocabulary and his work is organized. However, even in the one case in which he scored a 4 for Reflection in *Olympiques* task, he preferred a very short explanation for how he used the population in his solution. He says "*je vais mettre le pays avec le moins de personnes premier. Parceque ils ont moins de chances de gagne*". Although he has technically justified his choice, the reader must make some effort to interpret his meaning. When he does use sentences, they are usually used one at a time, rather than in a paragraph. Sometimes, his sentences seem incomplete, only because he finishes them off with a calculation. It is as though he is saying what he has written. In doing so, he makes assumptions that his calculations or numbers speak for themselves. In *Rectangles*, his text is clear. He does refer to a small drawing which could easily be replaced by words. His sentence begins "*Ma deuxieme solution etait que tous les rectangles qui ont deux boites sur la cote*  *sont mis dans un categorie*". What he means to say is that the rectangle with a height of two squares will form a category. It is unlikely that the appropriate words are beyond his abilities. However, this suggests that the drawing is a choice, a preference in the method of communication. Interestingly, in journalling situations, such as in *Olympiques* or *Quelle roche est plus grosse?*, where sentences are expected to be the main vehicle for ideas, Peter does fairly well. He writes,

> *Nous pensons que le roche noire est le plus grosse parce que nous avons fait deux expériences pour prouver que la roche noire est plus grosse. (…) Plusieur fois, on a faire tomber les roches, mais la réponse était toujours le même: la roche noire tombe premier. Donc, la roche noire est plus lourdre, et je suis pas mal sûre que c'est aussi, le plus grosse.*

With the exception of a few gender and spelling mistakes, his sentences are generally well constructed; there are many linking words to guide the reader through his work; and his ideas are easy to follow. Yet, in problems that are more reliant upon calculations and numbers, Peter seems to prefer a briefer writing style. In most cases, he succeeds. For students like Peter, it may be difficult to convince them that they need to use more words, since they are successful and comfortable using mathematical symbols and diagrams to convey their understanding. Moreover, their solutions are complete. The use of additional words may add little to the quality of their thinking or explanations. However, a key component of Peter's success is in the effective use of a minimum of words. His solution does use some words when needed. The numbers in his writing are labelled with units and the diagrams are properly explained. This economic use of language, I suspect, is more innately acquired rather than taught explicitly, given the fact that a majority of students are not able to make such judicious choices in the language used.

Peter’s successful use of minimal language may suggest that not everyone needs long paragraphs to be an effective communicator in mathematics. However, his success may also be limited by the level of difficulty of the tasks assigned or even the audience with which he is communicating at the time. Moreover, it also demonstrates his skill as a user of the French language. Although he still makes minor mistakes in grammatical structure, it is not an obstacle to his communication.

Taylor is similar to Peter in that she prefers to express her understanding with symbols, numbers and diagrams, rather than using sentences. Unlike Peter, her solution is difficult to understand. In many of the problems, I can make reasonable guesses about her thinking only because I saw her perform the solution, but otherwise, Taylor's explanations can sometimes be too vague. Unlike Peter, she is not always successful at communicating the depth of her thinking. This should not be confused with someone who has difficulty understanding a mathematical concept or not able to find an appropriate solution. Taylor would be considered "good at math". In objective-type questions, she is likely to do well. However, it is in the assessment of her communication skills in mathematics that she falls short. In *Rénovations*, she uses numbers and diagrams to show the calculations she has done. For the most part, they are self-explanatory. However, her explanations are limited to one vague sentence "*j'ai fait tous mes calculs pour tous les objets qu'ils voulaient*". Her use of language does not add to the clarity of her solutions. Despite her lack of verbal explanation, her calculations are well organized and have the appropriate units, use self-evident symbols and use clear diagrams. In some cases, when mathematical symbols suffice, her lack of words does not impede the communication of her understanding, although the reader must make some effort to read through the calculations. In one example of her work, however, the thinking is not transparent. In her journal on *Quelle roche est plus grosse?*, Taylor (see Figure 13) wrote only two sentences followed by some diagrams she referred to as *'calculs'* but in fact contain no calculation.

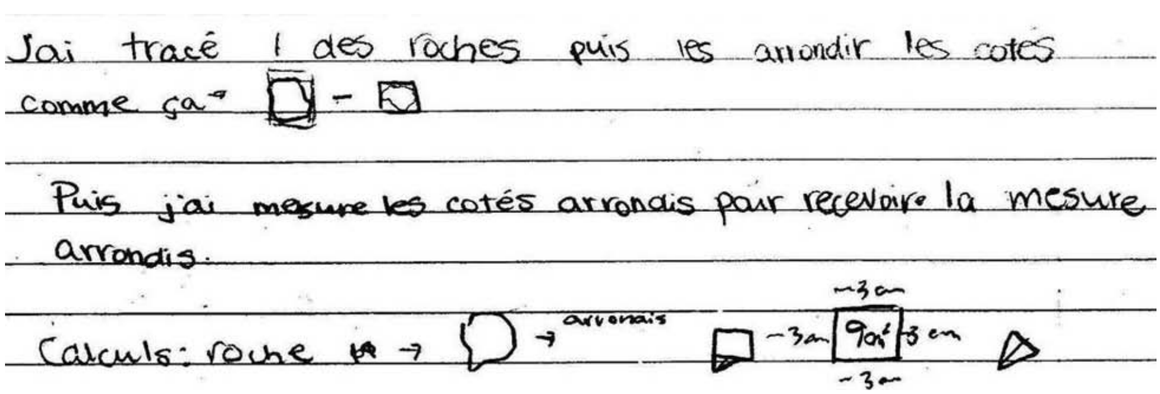


Figure 13. **Taylor's vague explanations**

One can guess that she drew a regular shape around the irregular rock and measured the square box. However, the reader can get no sense of whether she attempted to arrive at a more accurate estimate by subtracting the empty space from the larger box, or whether she only measured the regular shape and took that to be the answer. This would be the difference between getting 4 points rather than the 3 points she earned. Furthermore, she does not actually answer the simple question of "which rock is bigger?". The use of French language is also highly problematic. Only one of the two sentences is grammatically correct and the meaning of the other is unclear. In other words, the language used, with its grammatical flaws and poor choices of words, prevents the reader from ascertaining her level of understanding.

Taylor "understands" mathematics, and it is likely that teachers in the past forgave her overly brief explanations because she indeed is capable of having good ideas. This may also have perpetuated her habit of leaving details out of her explanations. This is a good example of how language is needed in order for mathematical understanding to be recognized. In *Rénovations,* language did not serve as crucial a role, but nevertheless, Taylor loses out when the complexity of the concepts cannot be explained only with numbers and diagrams. This may be a good example of the importance of comprehensible output opportunities and providing sufficiently challenging tasks so as to create the need for clearer language. When tasks are simple and solutions can be self-explanatory with a minimal use of words, the need for effective language is not as pressing. In these cases, Taylor can probably be successful. However, when the problem involves more ambiguity and requires more subtlety to solve, she may not be as able to convey the full extent of her understanding. Moreover, without the practice of having to explain complex solutions, she may never develop the skills to communicate properly. In this sense, teachers may be shortchanging learners when they limit the opportunities for practicing communication skills in mathematics.

After examining the case of individuals who show a strong preference for symbolic writing, the following examples highlight how this mathematical style of writing can benefit learners. The first is a student who falls back on a form of symbolic writing when he is reluctant to use verbal language to communicate his original thoughts (see figure 14). Unlike Peter or Taylor, who have chosen symbolic writing as their strongest means of expression, Chris relies on symbolic language, and only marginally well, to express what he is incapable of communicating in words. The only time he uses full sentences or even short phrases is when he states his answer. However, his entry would have been much stronger had he described his thinking. Without using verbal language, his reasoning is a little opaque and could easily be misinterpreted as being weaker than it really is. The need for verbal language is more pronounced in this case because Chris demonstrates an unusual way of representing his arithmetic in the Cultus Lake task. In organizing his calculations into a table, he is using symbolic language to convey his unique way of thinking and processing arithmetic.

When told to use tables to organize information, he constructs what he considers a logical table. For the reader, there is some interpretation required, because he does not always choose efficient titles. One column does not have a title at all. Interestingly, Chris uses this unnamed column to perform his simple additions, even though this is not indicated. Every subsequent row in this column is the cumulative sum of students he has ‘placed’ in the buses, thus, possibly avoiding a complex addition. In the final column which he entitles “*Groupe*”, the numbers represent the size of each group, once they have arrived at the water slide park. In doing so, he is dividing the students of each bus into groups rather

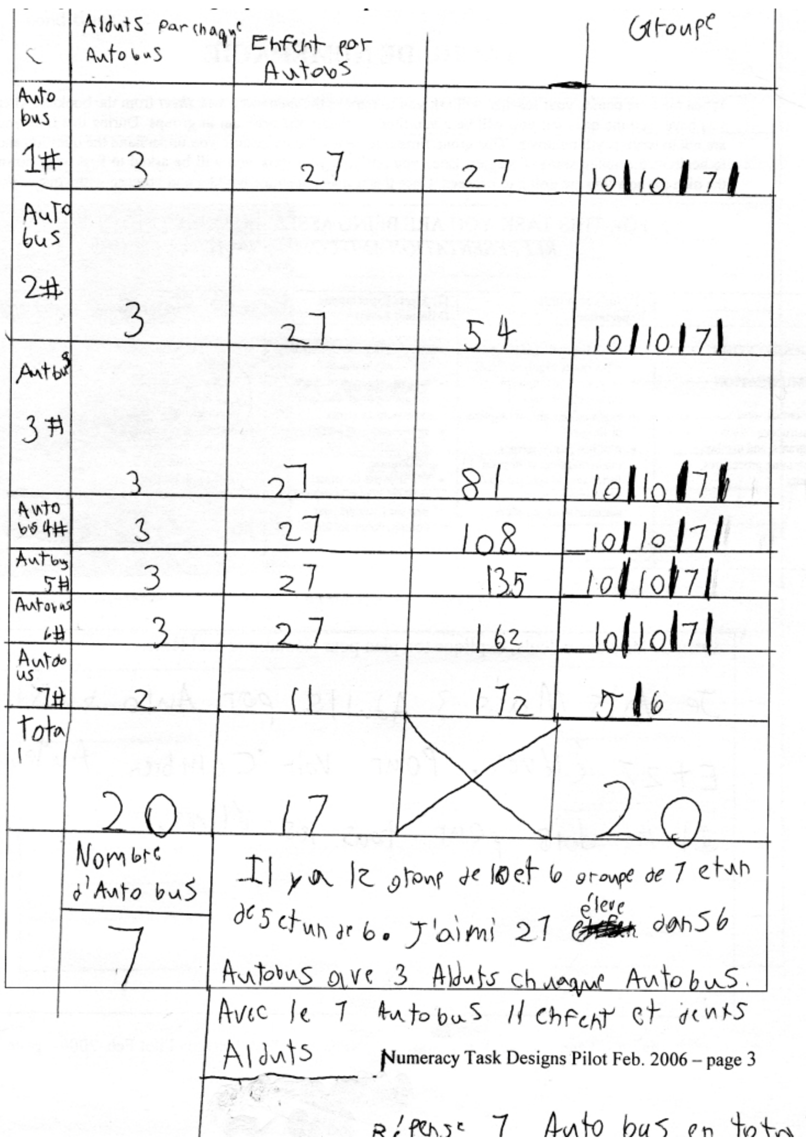
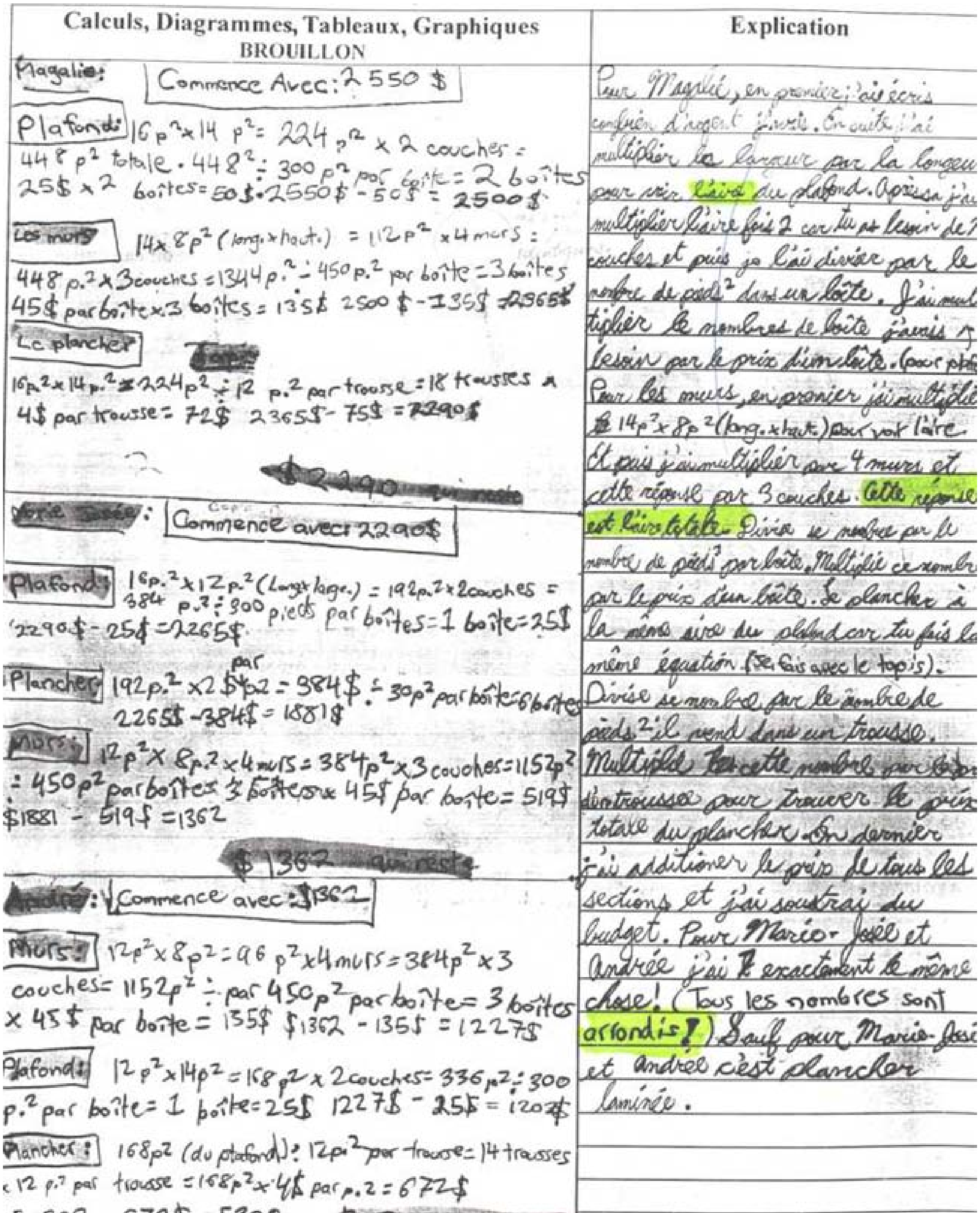


Figure 14. **Chris’ unusual representation of arithmetic**

than create new groups from the whole population. Chris’ work is certainly inferior in terms of mathematical reasoning because he has not shown an awareness of other, more effective ways of solving the problem. Also, his organization also minimally meets expectations since his table lacks the proper headings. Thirdly, his method of addition is a little unusual, and may suggest lack of ease in doing addition or using division. Seeing as I also taught him other subjects, it is my observation that Chris struggled to use language in other areas of his work as well. When instructed to use tables and to explain his work, he made an effort to break down his solution into steps in a somewhat logical fashion. For him, this was a tremendous improvement on his previous work. But still, without the use of verbal language, Chris' reasoning will never be quite as transparent as it could be. This example shows that for some students, particularly those who have more creative strategies, verbal language might help them be understood. When verbal language is not easily accesible, symbolic language can help provide some cues to their insight. In other words, symbolic and narrative languages become tools that individuals can choose from, depending on the task, the ability and the personal preference.

Along the same idea of organization as communication, but in contrast with the previous case, here is an example of efficient communication through organization. When done well, organized work hardly requires verbal explanation. Beth's writing in Figure 15 is an example of that. The numbers and calculations are accompanied by units and short one word clarifications in such a way that the narrative explanation becomes redundant. On top of labelling her calculations efficiently, she also uses titles, boxes and similar spacing between each of the three rooms to show the parallel information. She even adds colour



Calculs, Diagrammes, Tableaux, Graphiques BROUILLON	Explication
Magalie: Commence Avec: 2 550 \$ Plafond: 16 p² x 14 p² = 224 p² x 2 couches = 448 p² totale. 448² ÷ 300 p² par boite = 2 boites 25\$ x 2 boites = 50\$. 2550\$ - 50\$ = 2500\$ Les murs 14 x 8 p² (long. x haut.) = 112 p² x 4 murs : 448 p.² x 3 couches = 1344 p.² ÷ 450 p.² par boite = 3 boites 45\$ par boite x 3 boites = 135\$ 2500 \$ - 135\$ = 2365\$ Le plancher Tapis 16 p.² x 14 p.² = 224 p² ÷ 12 p.² par trousse = 18 trousses x 4\$ par trousse = 72\$ 2365\$ - 75\$ = 2290\$ \$2290 qui reste	Pour Magalie, en premier j'ai écris combien d'argent j'avais. Ensuite j'ai multiplier la largeur par la longeur pour voir l'aire du plafond. Après ça j'ai multiplier l'aire fois 2 car tu as besoin de 2 couches et puis je l'ai diviser par le nombre de pieds² dans un boîte. J'ai multiplier le nombres de boîte j'avais besoin par le prix d'un boîte. (pour plafond) Pour les murs, en premier j'ai multiplier 14 p² x 8 p² (long. x haut.) pour voir l'aire. Et puis j'ai multiplier par 4 murs et cette réponse par 3 couches. Cette réponse est l'aire totale. Divise ce nombre par le nombre de pieds² par boîte. Multiplie ce nombre par le prix d'un boîte. Le plancher a la même aire du plafond car tu fais la même équation (se fais avec le tapis). Divise ce nombre par le nombre de pieds² il vend dans un trousse. Multiplie cette nombre par le prix d'un trousse pour trouver le prix totale du plancher. En dernier j'ai additioner le prix de tous les sections et j'ai soustrai du budget. Pour [redacted] et [redacted] j'ai exactement le même chose! (Tous les nombres sont arrondis!) Sauf pour [redacted] et [redacted] c'est plancher laminée.
[redacted]: Commence avec: 2290\$ Plafond: 16 p.² x 12 p.² (Long x lage.) = 192 p.² x 2 couches = 384 p.² ÷ 300 pieds par boites = 1 boite = 25\$ 2290\$ - 25\$ = 2265\$ Plancher: 192 p.² x 2 \$ par p2 = 384 \$ ÷ 30 p² par boite = 6 boites 2265\$ - 384\$ = 1881\$ murs: 12 p² x 8 p.² x 4 murs = 384 p² x 3 couches = 1152 p² ÷ 450 p² par boites 3 boites x 45\$ par boite = 519\$ \$1881 - 519\$ = 1362 \$1362 qui reste	
[redacted]: Commence avec: \$1362 Murs: 12 p² x 8 p² = 96 p² x 4 murs = 384 p² x 3 couches = 1152 p² ÷ par 450 p² par boite = 3 boites x 45\$ par boite = 135\$ \$1362 - 135\$ = 1227\$ Plafond: 12 p² x 14 p² = 168 p² x 2 couches = 336 p² ÷ 300 p.² par boite = 1 boite = 25\$ 1227\$ - 25\$ = 1202\$ Plancher: 168 p² (du plafond) ÷ 12 p² par trousse = 14 trousses x 12 p.² par trousse = 168 p² x 4\$ par p.² = 672\$ \$1202 - 672\$ = 530\$ \$530 qui reste	

Figure 15. **Beth's organized writing**

(visible in the original) to highlight the titles further. Like Chris, the very structure of her organizations communicates her ideas. Unlike him, however, she hardly needs additional words to add to the symbolic information given through the table.

In these examples of symbolic writing, Peter and Taylor demonstrate their preference for using less words in their mathematical writing. While Peter does so effectively, Taylor does not. When using very few words to explain their thinking, it becomes clear that how they use everything else: symbols, organization, diagrams, tables, etc. becomes crucial. Peter knows how to economize but can still convey his ideas clearly, whereas Taylor seems to avoid the use of a skill she is struggling with. Similarly, Beth's neat organization of ideas echoes the importance of economy through proper use of tables. Chris, on the other hand, benefits marginally from the construction of a table. His unorthodox methods minimally succeed in demonstrating his understanding. In other words, economy in writing may be seen as a virtue in mathematics, but when the student has not learned effective ways of organizing ideas into tables, or explaining diagrams and symbols, language seems essential in bridging the gap. Students like Taylor and Chris need to have access to stronger language skills to help communicate their mathematics.

6.1.2 Narrative writing style

In contrast to symbolic writers, there are students who employ a more narrative style in their mathematical writing. Helen and Arnold are two individuals who tend to use more words to explain their thinking than numbers and symbols. However, they do so in different ways. I chose Helen as a first case because language seems to have an important role in her mathematics. Her style of writing could be described as narrative. It seems almost as though she is writing as she speaks to herself; she is sharing her internal dialogue with the reader. She often asks questions which are later answered in her writing and even tells us about the obstacles she encounters in her thinking. For example, she writes,

> *On a encore un problem. On achete un piece de tapis 4X12 mais on a encore 2X4 de espace qui n'ai pas couvere de tapis. Pour simplifier on peut achete un tapis de 1X8 coupe en deux et voila! 2X4 de tapis.*

Unlike Taylor and Peter, Helen's writing seems to occur at the same time as her thoughts, whereas in the previous cases they appear to have done the thinking first, and then found the words to describe what they did. The result is that Helen's work may be more difficult to follow as it is not necessarily organized logically, but rather chronologically. Despite this apparent challenge for the reader, the advantage is that her writing reveals more of the process of her thinking.

Interestingly, although her French is not perfect, Helen handles subordinate clauses with ease. This may be a factor in explaining why she is more comfortable using language as a medium to convey and develop understanding. In contrast, Taylor struggled to create two sentences that were easy to understand. Indeed, if she were to write more sentences, it may even further obscure our assessment of her thinking. Employing more numbers and symbols to convey her meaning may be a way of avoiding what she knows is a weakness. Another interesting observation in Helen's work is the relationship between her language skills and her mathematical concepts. In two of the four problems she completed, her mathematical concept mark was as low as a 2, while the other two problems earned 4s. Overall, and according to traditional criteria as those used in objective testing, Helen is considered a weaker student in mathematics. However, in both cases where the concepts were challenging to her, the narrative she describes appears to enhance her work. Although difficult to show, in these two tasks, her symbolic writing, that is, calculations, charts, etc. are hard to read. However, she follows her rough work with many long paragraphs of writing, which, although they are written chronologically, show much more clarity in her thoughts. By the end of her paragraphs, she has a reasonable answer. This suggests that allowing Helen to verbalize her thinking may indeed help her improve her mathematical concept and the teachers' assessment of her understanding. Furthermore, the role of verbal language is quite crucial for Helen. Without allowing her to verbalize on paper, she would not be able to demonstrate her thinking quite as well. This supports the importance of providing these opportunities for communication in various forms.

Traditionally, much mathematics is assessed summatively. That is, a singular skill will be taught and evaluated shortly after. Following the assessment, teachers move on to the next skill and do not often give a chance to build on the feedback given. In this model, learners have a short time-frame in which to develop their understanding. Once the assessment has passed, there is little chance to try again and improve upon the previous conceptual understanding. It is no surprise to me that Helen was not considered a "good Math student" in this context, as Taylor or Peter may have been. Using her narrative style of learning in mathematics, however, I would imagine that Helen could improve on her own more readily than Taylor, for example. The reason for such a theory is that Taylor's writing probably did not help her gain insight on her own thinking, whereas Helen used language almost as a means of conversing with herself. These written conversations are more likely to help clarify her own thinking.

Arnold is another intriguing case. It was easy to miss noticing his work because he seems like a well-rounded mathematician. However, closer examination of his writing reveals some important qualities. Arnold's communication is very succinct, even in the case of complex ideas. Arnold scored 4s in the *Quelle roche est plus grosse* and the *Renovations* problems respectively in the category of language. His mathematical concept is also strong and can therefore make the contrast difficult to see. However, in the case of the *Quelle roche* problem, Arnold seemed to prefer to describe his thinking using language at the expense of showing the mathematical operations performed. He writes,

> *Le roche 11 etait beaucoup plus lourd mais sa pourait juste etre le sorte de roche alord on a essayer de les mesurer. C'etais dificile car c'etais les different epaisseurs au chaque place alord on n'a pas fini de mesurer et calculer.*

He explains how being heavy may not mean that it is the largest, as the type of rock may be, presumably, denser. However, he does not show any calculations. In the *Rénovations* task, Arnold does show some but not all of his mathematical work. And again, the explanation of his thinking is very clear. Even though he has chosen a unique approach, his words convey it perfectly.

> *Au début, j'ai calculé combien sa couterait pour peinturer leurs plafonds. S'était $100 en total pour les plafonds. J'ai enlèver cette $100 de la total puis j'ai diviser le total en 3 pour trouver combien ils recoiveraient chaque ($816). J'ai commencer avec André. Avec touce ces premièr choix il y avait $54 qui reste. J'ai continuer avec la chambre de Marie-Josée. Avec ses premiers choix, elle lui restait $342 Je savais que la chambre de Magalie coutera le plus alors j'ai ajouter le $54 de André qui reste et le $342 qui reste de Marie-Josée alord le buget pour Magalie étais $1212. Elle a aussi resoive se qu'elle voulait.*

Unlike Helen though, Arnold's style does not suggest that his thinking is simultaneous with his writing. Rather, he has found an effective way of using words to review what he has done mathematically. In fact, if the reader of his work were to look only at the mathematical part, his thinking would not necessarily be evident. In this sense, his words add significant depth to his mathematical answer.

What strikes me about Arnold's case is that he has repeatedly chosen to use words over mathematical symbols to convey very insightful ideas. Unlike Helen, this choice does not stem from lacking in mathematical understanding but simply a personal preference, since Arnold also does well in objective tests in which only calculations are required. In other words, when given the choice, he will use words to show the full extent of his insight. He does so very effectively. When not given the choice, he is still able to convey his understanding with numbers and symbols. The reason why I described Arnold's case is to show that the reasons and ways learners use language can vary widely. Some students use less language because they are more skilled in communicating symbolically. Others use less language because it may be to the detriment of conveying their understanding. Others yet use language because it is the most effective way of conveying and working through a problem. And finally, there are those who have the choice and flexibility to adapt to each situation with skill. This pattern emerges again in the next theme of second language learning.

Although I have chosen a few extreme cases to describe, narrative and symbolic writing are not to be considered dichotomies. In mathematical journalling, every writer makes choices about how to convey his thoughts: how much symbolic writing and how much narrative writing best suits his thinking process or even a particular task. Each of the two styles makes different contributions to the overall picture. The option of journalling allows for a much wider range of responses and in many cases, the inclusion of narrative writing can add significant depth to what the writer can communicate.

In many writing samples, it seems that the thoughts students have evolve as they find their words. In a sense, there is a negotiation between the words found and the thoughts in their mind. With each attempt, the two come closer to each other and help the student reach a clearer, more reasonable and complete answer. In this way, they are constructing their own knowledge from existing representations. As discussed in the earlier chapters, the construction of knowledge happens when learners have the opportunity to create and challenge their representations of mathematics concepts (Maher & Davis, 1990). Allowing the use of language for this purpose not only validates the process but allows the teacher to enter the mind of the child, thus getting feedback about the progress of her learners. Maher and Davis believed that this was very important in the promotion of a student-centred pedagogy (1990). In one example, Sarah writes at the beginning of her journal, in English,

> *At first I was completely lost, I pretty much forgot I was in math. Karl and Catherine were mesering both rocks and I rememberd that when I was coming into class Mme had two weights [on the table]. So I asked her if the rock that is larger and has more volume would weight more and vise-versa.*

This journal is a record of the understanding that she gained in her group discussion, as they were required to write their thoughts after a period of collaborative dialogue with three other students. Two observations struck me in this passage. First, she pointed out that having to measure rocks seemed unusual for the typical mathematics class. Perhaps she was more accustomed to working with pen and paper; or perhaps she believed that the rocks we were studying in science class only pertained to science. These types of comments shed light on how children's ideas evolve with every new experience. It was clear that for this student, the measurement they were practicing in this task was different from the measurement chapter of the textbook. This experience was an opportunity for her to understand that measurement can occur with irregular and three-dimensional shapes, even ones borrowed from the Science lesson. The second observation from her journal had to do with questioning the relationship between weight and volume. This inquiry also framed her group's teeter-totter strategy, described earlier. Without elaborating on the point, her group concluded that indeed, weight and volume are synonymous. Although this is conceptually wrong, it demonstrates her ability to formulate a hypothesis and to apply it to her thinking, a much more demanding skill than simply applying the formula for volume. In the short one-page journal, Sarah demonstrated her representation of the nature of mathematics class and had an opportunity to formulate a question regarding her understanding of weight and volume, without the direct prompting of the teacher.

Using words to explain mathematical understanding also provides the opportunity to defend solutions that the teacher may not have anticipated. In traditional objective testing, answers are either right or wrong. Even in problem-solving, teachers often have an anticipated answer that they look for. If the solution matches the standard one, it usually earns the points. However, it was demonstrated several times in the data that individuals can find perfectly valid non-standard strategies. Once again, I use Arnold's example when he was able to explain his original approach in *Rénovations*.

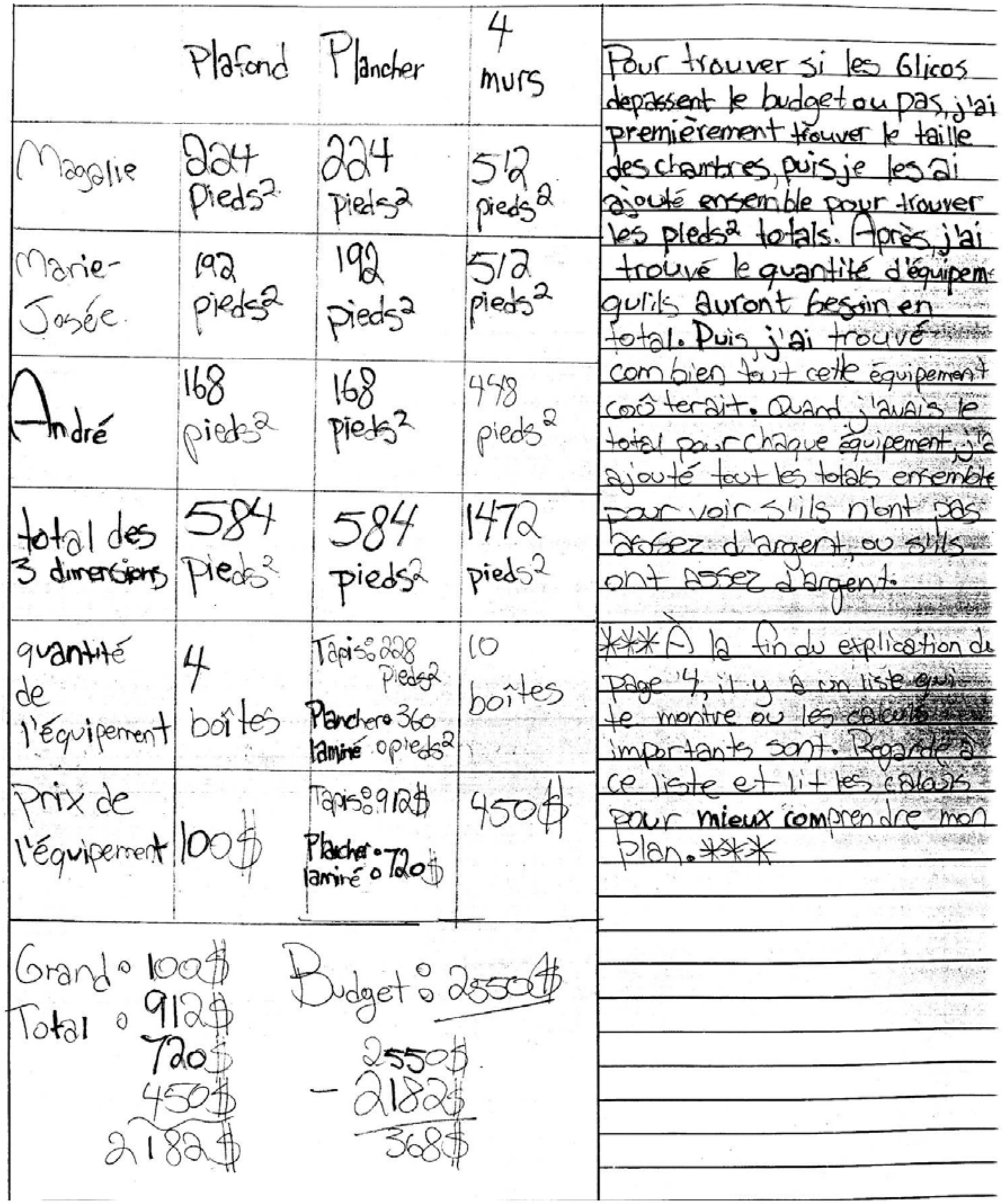
> *Au début, j'ai calculé combien sa couterait pour peinturer leurs plafonds. S'était $100 en total pour les plafonds. J'ai enlèver cette $100 de la total puis j'ai diviser le total en 3 pour trouver combien ils recoiveraient chaque ($816). J'ai commencer avec André. Avec touce ces premièr choix il y avait $54 qui reste. J'ai continuer avec la chambre de Marie-Josée. Avec ses premiers choix, elle lui restait $342 Je savais que la chambre de Magalie coutera le plus alors j'ai ajouter le $54 de André qui reste et le $342 qui reste de Marie-Josée alord le buget pour Magalie étais $1212. Elle a aussi resoive se qu'elle voulait.*

Arnold's strategy involved splitting the budget equally first. Then he calculated the cost of each child's wishes and figured out how much was left from their individual allowance for the job. When one child's wishes were more expensive, he redistributed the funds leftover from the two other bedrooms' renovations. This approach, although unusual, was perfectly sound and well executed.

Without the opportunity to explain and defend themselves, students like Arnold do not get credit for their original thinking and others like Chris get even less credit for their creative approaches. With journal-based problem-solving, learners are free to explore a wider range of solutions and justify them in their words, rather than try to find the single correct answer. Most of the time, real-life mathematics problems do not necessarily have one correct answer or solution. Allowing individuals to explain and justify their own understanding can make for a more genuine and meaningful learning. It also allows the teacher to see the variety of thinking styles that make up a class. As in the case of Chris, the open space on his page allowed him to begin the process of learning how to present his ideas logically.

6.1.3 Other cases

Although I have highlighted a few cases where the narrative and symbolic styles are striking, the majority of the class combined the two styles in their writing. However, there are differences in the way their narration and symbols worked together to convey their ideas. The following section describes five different ways in which students combine these two styles. There are three individuals that I will discuss here that demonstrate flexibility and the ability to use both. The two other are cases where neither narrative nor symbolic styles are helpful to the writer. However, rather than describe the first three in order, I chose to alternate the examples to highlight their differences. The first example is from Alex in figure 16. The blend of narration and symbols seemed highly effective and efficient, neither being used excessively nor too sparingly. Every number is explained, either with units or a one-word descriptor. The table he draws in figure 16 is a summary of his calculations. His verbal description uses linking words, proper mathematical and non-mathematical vocabulary and shows how he proceeded to solve the problem from beginning to end. On second glance, Alex's writing is interesting because it is so concise. In problem-solving, there is rarely a situation when the solver will move seamlessly from the beginning to the end of a problem without having to go back to earlier calculations, or to revise a procedure, unless the task was very simple. In some other students' writing, these revisions or transitional thoughts are evident in their narration, such as Helen. Unlike Helen, Alex' writing suggests that he has either solved the whole problem elsewhere and written the narrative after he reached a solution, or else, was able to picture the whole solution before writing it down. Either way, the seamless and economical writing suggests that his thinking preceded his words. He may have edited his thoughts several times before writing them down. Liljedahl (2007) explains that when this occurs, it obscures the true process in favour of the product. Somehow, Alex understands that journalling should describe the neat and tidy version of events, with the messy detours and impasses edited out. Ironically, it is often the skilled writers, those who can edit both their mathematical work and their linguistic communication, who practice this. It is ironic because the very skill being encouraged – effective and efficient



	Plafond	Plancher	4 murs
Magalie	224 pieds²	224 pieds²	512 pieds²
Marie-Josée	192 pieds²	192 pieds²	512 pieds²
André	168 pieds²	168 pieds²	448 pieds²
total des 3 dimensions	584 pieds²	584 pieds²	1472 pieds²
quantité de l'équipement	4 boîtes	Tapis: 228 pieds² Plancher laminé: 360 pieds²	10 boîtes
Prix de l'équipement	100$	Tapis: 912$ Plancher laminé: 720$	450$

Grand Total: 100$ 912$ 720$ 450$ 2182$

Budget: 2550$

2550$ − 2182$ = 368$

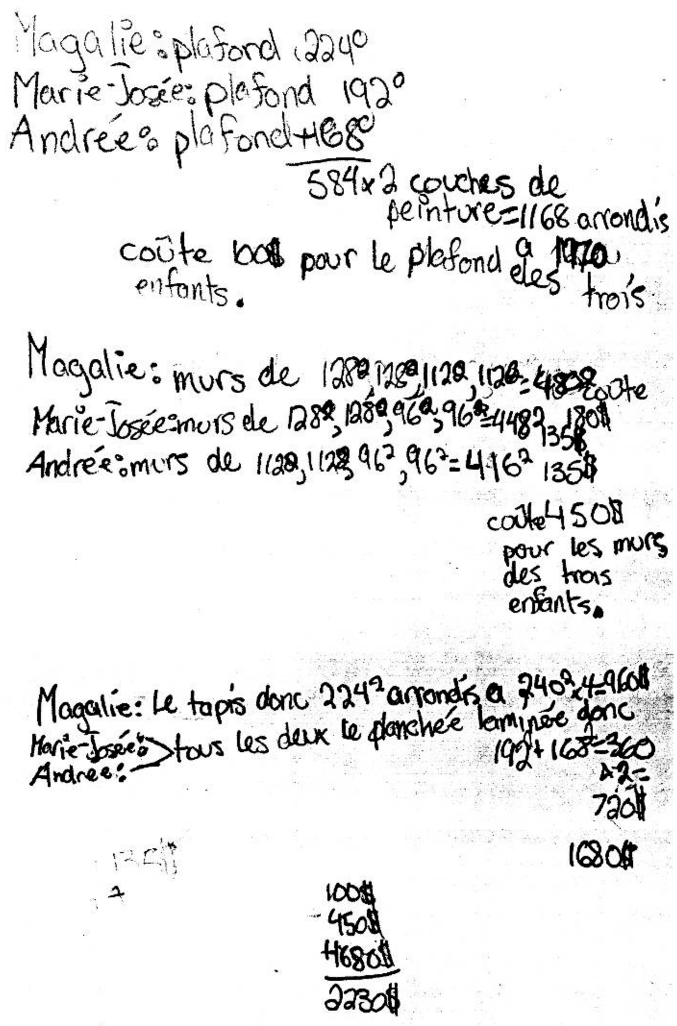
Pour trouver si les Glicos dépassent le budget ou pas, j'ai premièrement trouver le taille des chambres, puis je les ai ajouté ensemble pour trouver les pieds² totals. Après, j'ai trouvé le quantité d'équipement qu'ils auront besoin en total. Puis, j'ai trouvé combien tout cette équipement coûterait. Quand j'avais le total pour chaque équipement, j'ai ajouté tout les totals ensemble pour voir s'ils n'ont pas assez d'argent, ou s'ils ont assez d'argent.

*** À la fin du explication de page 4, il y a un liste qui te montre où les calculs importants sont. Regarde à ce liste et lit les calculs pour mieux comprendre mon plan. ***

Figure 16. **Alex' product-based journal**

communication – eclipses an important objective of journal-writing, which is to describe the thinking process.

In contrast with Alex, whose writing is easy for his readers to follow, Isaac (see figure 17) shows little awareness of his audience. Both forms of writing, narrative and symbolic, are inefficient. His words hardly paint a logical narrative.


Magalie: plafond 224²
Marie-Josée: plafond 192²
Andrée: plafond +168²
584×2 couches de peinture=1168 arrondis
coûte 100$ pour le plafond des trois enfants.

Magalie: murs de 128², 128², 112², 112² = 480² coûte
Marie-Josée: murs de 128², 128², 96², 96² = 448² 135$
Andrée: murs de 112², 112², 96², 96² = 416² 135$
coûte 450$ pour les murs des trois enfants.

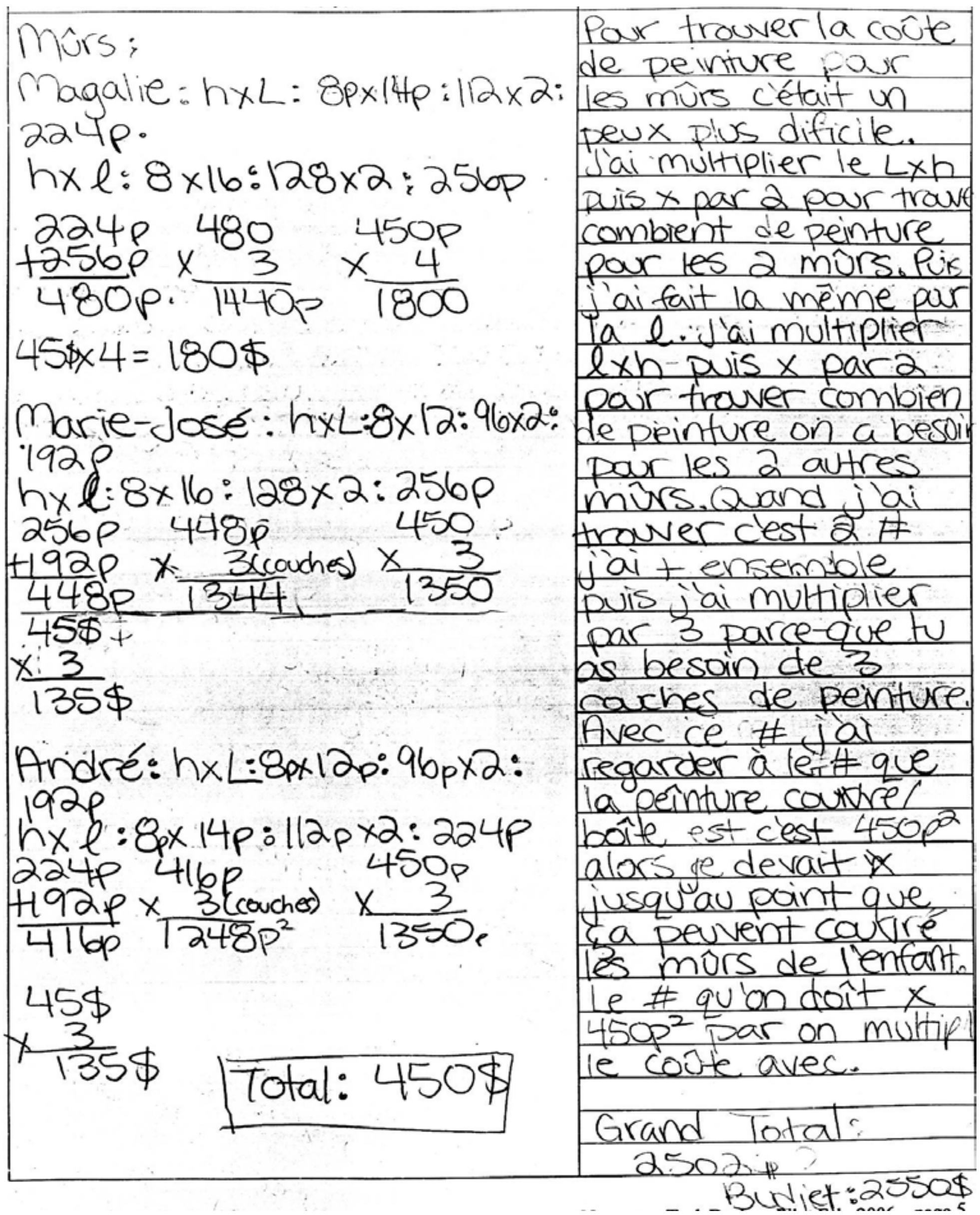
Magalie: Le tapis donc 224² arrondis a 240² × 4 = 960$
Marie-Josée: } tous les deux le planchée laminée donc
Andree: 192 + 168 = 360 × 2 = 720$
1680$

100$
450$
+1680$
2230$

Figure 17. **Isaac's lack of audience awareness**

There are no full sentences other than in his answer and his symbols are often incorrect. However, there seems to be a unique form involved in his writing. His writing is not grammatical but there seems to be a certain unit of communication, which we can call neither a sentence nor a mathematical phrase. For example, at the bottom of his page, he writes, "*Magalie: le tapis donc* 224^2 *arrondis a* 240^2 *X 4 = 960$*". There is no verb and the "2" in superscript was probably meant to be squared meter units rather than an exponent. Moreover, he rounds 224 up to 240, which of course, is grossly incorrect. Mathematical issues aside, this unit of communication, for lack of a better term, is difficult to decipher. Despite this, it leads Isaac to find an answer, suggesting that this writing is intended primarily for himself. Unlike Alex who was skilled at communicating his ideas logically, Isaac uses the journalling opportunity to keep notes for himself and possibly to dialogue with himself in an abbreviated form of language, one without verbs and with incorrect symbols, but one that allows him to reach a satisfying answer. In other words, Isaac uses language to think on paper whereas Alex uses it to communicate with the reader. It is likely that Isaac's thinking occurs simultaneously with his words and symbols. It reveals some of the process he has undergone, but unfortunately, it is opaque because it was not written with an audience in mind. Here is the dilemma: Alex's lucid writing obscures process, while Isaac's process-driven but indecipherable work is hard to follow. Clearly, different learners interpret different purposes for language, and hence use it differently.

The following example shows a case of flexibility. While Alex is capable of offering a good balance between narrative and symbolic writing, Annabelle (figure 18) integrates mathematical symbols into sentences, suggesting that for her, symbols are quicker ways of naming things that could be spelled out in words. And although she chooses to incorporate mathematical symbols into her writing, she demonstrates on the left side of the page that she is also capable of manipulating numbers, symbols and units effectively, despite a few misuses of the colon. For Annabelle, the distinction between symbolic and narrative



Mûrs ;
Magalie : hxL : 8px14p : 112x2 : 224p.
hxℓ : 8x16 : 128x2 : 256p
224p + 256p = 480p. 480 x 3 = 1440p 450p x 4 = 1800
45$x4 = 180$
Marie-José : hxL : 8x12 : 96x2 : 192p
hxℓ : 8x16 : 128x2 : 256p
256p + 192p = 448p 448p x 3(couches) = 1344 450 x 3 = 1350
45$ x 3 = 135$
André : hxL : 8px12p : 96px2 : 192p
hxℓ : 8px14p : 112px2 : 224p
224p + 192p = 416p 416p x 3(couches) = 1248p² 450p x 3 = 1350
45$ x 3 = 135$
Total : 450$

Pour trouver la coûte de peinture pour les mûrs c'était un peux plus dificile. J'ai multiplier le Lxh puis x par 2 pour trouver combient de peinture pour les 2 mûrs. Puis j'ai fait la même pour la ℓ. J'ai multiplier ℓxh puis x par 2 pour trouver combien de peinture on a besoin pour les 2 autres mûrs. Quand j'ai trouver c'est 2 # j'ai + ensemble puis j'ai multiplier par 3 parce-que tu as besoin de 3 couches de peinture. Avec ce # j'ai regarder à le # que la peinture couvre/boîte, est c'est 450p² alors je devait x jusqu'au point que ça peuvent couvré les mûrs de l'enfant. Le # qu'on doît x 450p² par on multipl le coûte avec.
Grand Total :
2502 ?
Budjet : 2550$

Figure 18. **Annabelle's integration of symbols within sentences**

language is not as significant. For her, both forms are accessible and interchangeable. It could be argued that she is 'fluent' in both forms of expression. This reveals an interesting notion that how we define language may not be clear cut. For Annabelle, symbols and words can coexist. And when one is skilled, the movement between ‘languages’ is fluid and easy. Along with evidence that languages are not compartmentalized in the brain (Cook, 2001), there may be an argument to be made for mathematics as a language. Without entering into

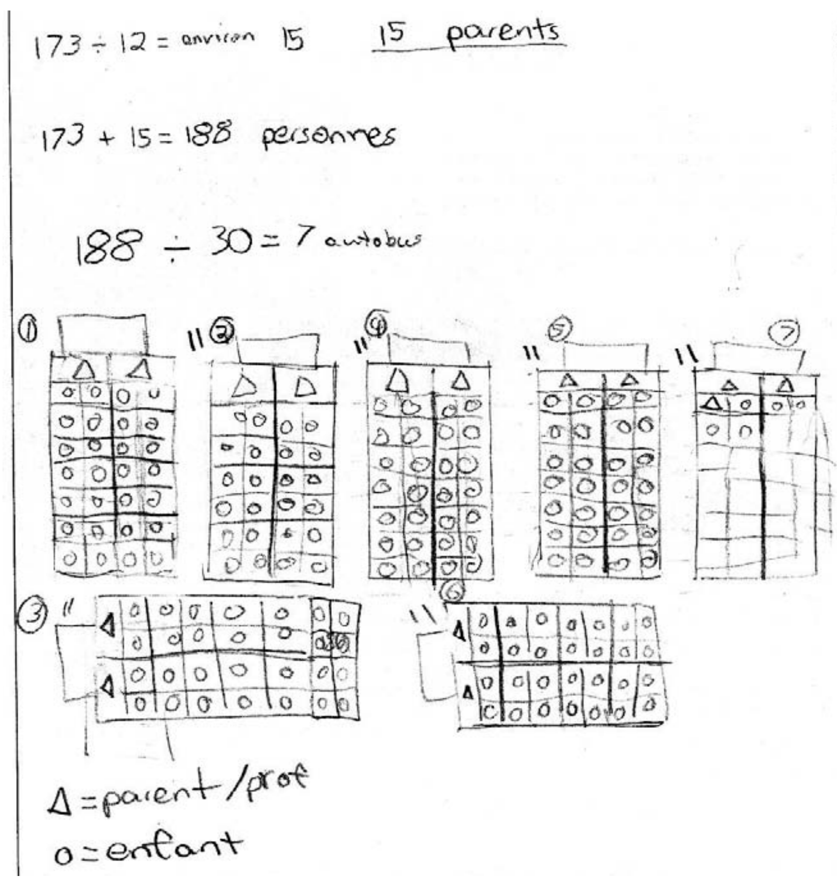


Figure 19. **Arnold’s use of symbols**

this debate, it is interesting that we may be more open to the idea that fluid movement between mathematical and verbal languages is acceptable and a demonstration of skill than the fluid and grammatical movement between English and French. This bias underlines the difficulty we face in accepting what it means to be bilingual.

Similarly to Annabelle's linguistic flexibility, we have seen Arnold's work before and how it demonstrated an ease with narrative language. In the previous example, he preferred to describe his work in long sentences, rather than showing his calculations. In this entry, he demonstrates his flexibility by opting for more symbolic language. In the Cultus Lake task (figure 19), Arnold's solution uses symbols exclusively. He chooses to represent adults and children with pictograms to solve the problem visually, rather than with calculations. Because of his previous work demonstrating his ability to communicate his understanding in sentences, I can conclude that it is the particular task that invites this form of writing, rather than ability. In fact, the Cultus Lake problem seems to have generated more use of symbolic writing than the others. Because most students combine symbolic with narrative writing, it is difficult to quantify this tendency. However, Arnold and Annabelle show that the use of either styles of language can be a matter of choice. This choice may depend on the particular task or the comfort level of the writer. They are able to alternate between the two styles at whim and they do so fluently and eloquently.

Not everyone is as flexible and versatile as Annabelle and Arnold, however. Terri struggles with both narrative and symbolic language (figure 20). He begins solving the Cultus Lake task by performing a division in the upper left corner of the page, which he recognizes eventually as being incorrect and crosses off. Next, he creates a drawing to represent busloads of children and adults. However, while his drawing has led to him to find a number, he misinterprets what that number means. His mathematical logic was not totally wrong. It was probably not the most effective way of dividing 173 by 12, but because his use of symbols was flawed, and because he did not supplement his drawing with words to explain his thinking, it appears as though Terri has done poorly in solving the problem.

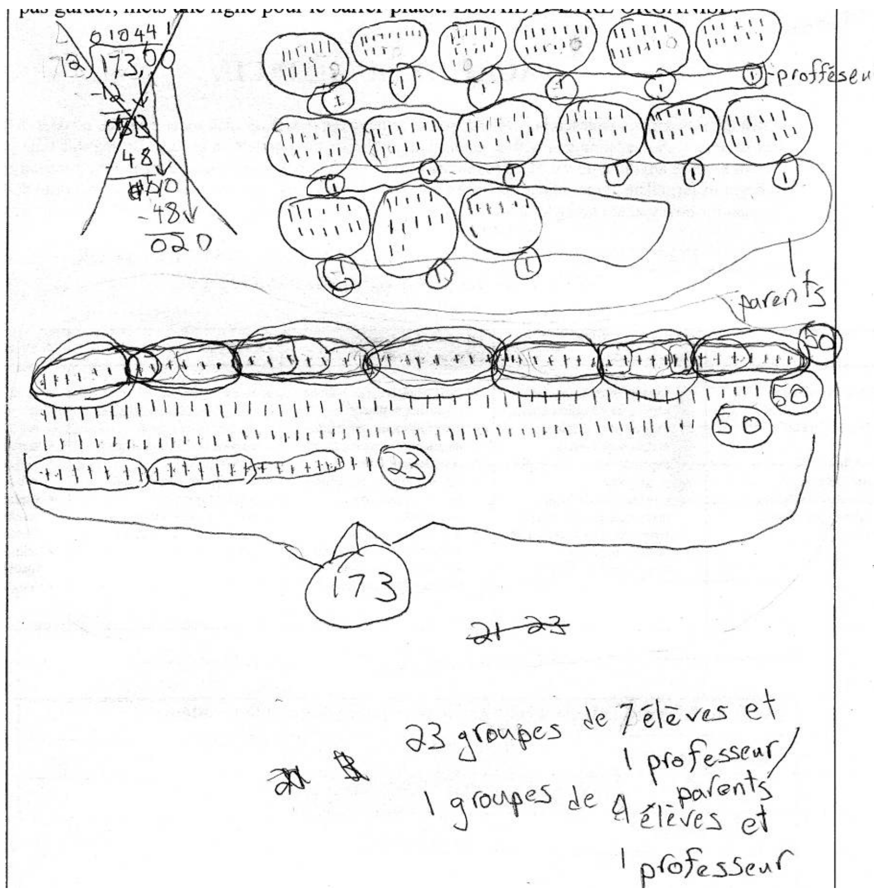


Figure 20. **Terri's difficulty with both narrative and symbolic language**

Furthermore, he does not use the space alloted to explain his final answer. This may be because he ran out of time, or because he was not confident of his answer and chose to leave it blank. At the bottom of his page, he indicates the answer he has found, even though he does not actually state how many buses are required. Terri does not manipulate language easily, neither symbolic nor narrative language.

The only occasion for him to demonstrate his narrative language skills was on the space reserved for planning (figure 21).

Avant de faire des calculs, explique ton plan pour résoudre le problème.
- Je vais diviser les élèves en des groupes.
- Je mêtre des différents groupes dans différent autobus.
- Je vais caculer des personnes dans tous les autobus.

Figure 21. **Terri's use of narrative language**

In this space, his sentences do not really say much about his approach. Despite his difficulty with both symbolic and narrative languages, his solution is not a complete failure. It is apparent that he knows what he needs to find the answer. He recognizes the need to divide or distribute the students into buses, but he fails to accomplish this and fails to communicate what he does know. Without these language skills, it is very difficult for Terri to get credit for the little that he masters.

The purpose of showing these examples is not to make generalizations about how language is used in particular, but rather to show the broad spectrum of ways in which language is used. Not only is this dependent upon individual styles and preferences, but it may also be a factor of their abilities and limitations in using various forms of language. Moreover, we perceive the role of language differently and use it accordingly. For Alex, the purpose of using writing is to present the final product of his problem-solving. For Annabelle and Arnold, the alternation between symbolic and narrative languages between and within tasks is a matter of choice. In contrast, for Terri and Chris, can only begin to marginally demonstrate their mathematical thinking with the combination of both symbolic and narrative languages. In following this idea that individuals need, use and manipulate language differently, the second theme which emerged from this data is how the communication of mathematics and the context of French as a second language interact.

6.2 Use of languages to communicate about mathematics

Although Grosjean's definition of bilingualism (1982) steps away from a quantitative assessment of mastery, towards a qualitative description of usage, it does not imply that everyone is bilingual in the same way. The following section shows some of the ways in which learners demonstrate their bilingualism in writing about mathematics. I examine three general categories within this theme. First, there are those for whom having to communicate in French is a major obstacle, to the extent that it becomes an impasse for the communication of mathematics. Although they are bilinguals under Grosjean's definition, arguably, the language skills of these participants hinder them from demonstrating their mathematical knowledge. Second, there are interesting subtleties within the practice of codeswitching that will shed light here on this controversial second language practice. Finally, there are those who demonstrate ease in using

French to communicate about mathematics. This fluency is not synonymous with grammatical and lexical accuracy, but certainly suggests that French language is a tool and not merely a task for some Immersion students. In looking at the use of French in mathematics, it is essential to discuss how English, the first language, fits into this complex relationship and interaction.

6.2.1 French language as an impediment

In the first category of students, French language is a huge impediment. However, these cases are not necessarily easy to identify. Some students who have very poor French skills adapt, particularly in the context of mathematical writing, by simply using less verbal language. In the context of mathematics, this is possible, since they can choose to use more symbols than sentences. However, this does not always help them communicate better, depending on their ability to use symbols. In some cases, the only evidence that points to this linguistic hurdle is the occasional sentence used for planning purposes at the beginning of a large task or from journals that require mostly narrative rather than symbolic writing, such as *Quelle roche* or *Camions*. In the following example, which was used earlier to demonstrate her choice of symbolic language, Taylor (figure 22) shows her limited French skills. Rather than trying to explain her strategy of drawing a rectangle around an irregular shape and estimating its area or volume, she draws the picture of an irregularly shaped rock and a rectangle around it, hoping that it will suffice to communicate her strategy to the reader. And even though her mathematical ability could be marginally ascertained, her writing skills are clearly an impediment to communicating the full extent of her understanding. So, an implication for the educator is that language is important in content. Even if it may appear negative, from a pedagogical perspective, it shows us as educator we also need to focus on language when we teach mathematics.

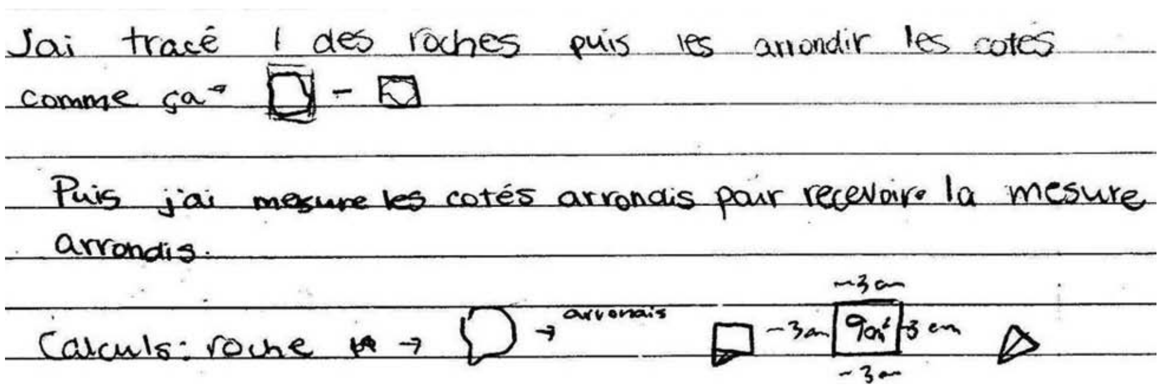

Jai tracé 1 des roches puis les arrondir les cotes
comme ça →
Puis j'ai mesure les cotés arrondis pour recevoire la mesure
arrondis.
Calculs: roche → arronais ~3 cm 9cm² 3 cm ~3cm

Figure 22. **Taylor's difficulty with French**

In another example of weak French skills, Craig's journal (figure 23) on the Camions task is unusually empty, compared to other students.

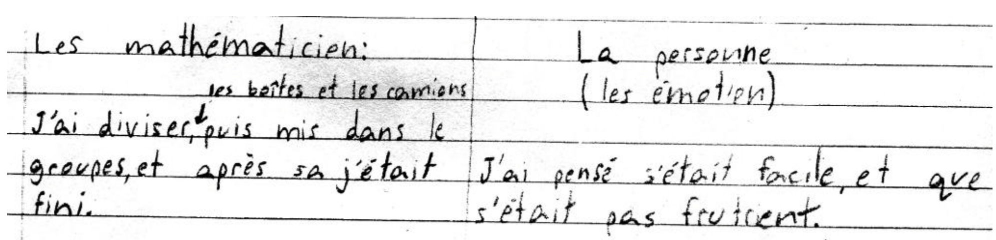

Les mathématicien: les boîtes et les camions
J'ai diviser, puis mis dans le groupes, et après sa j'était fini.
La personne (les émotion)
J'ai pensé s'était facile, et que s'était pas frutient.

Figure 23. **Craig's lack of writing**

In fact, I had initially discarded this entry because it seemed too brief to contribute anything interesting. Alone, the lack of words to describe a relatively simple exercise is not necessarily meaningful. However, Craig, like a few others, had only turned in two out of the five writing tasks, each containing very little writing. It turns out, out of twenty-seven students, there are five who have only handed in two out of five tasks; in each case, the amount of writing was critically low to the point that the mathematical understanding was very difficult to ascertain.

This is an interesting outcome. When students struggle with language, they will not necessarily codeswitch to English to help themselves, despite the fact that codeswitching can be a bilingual communication strategy (Lüdi & Py, 2003). Interestingly, in all of the students mentioned above, which do not hand in their work and do not use much French verbal language, there is zero instance of codeswitching. Instead, they will simply use very little verbal language. This may be attributed to the culture of French Immersion, where the long-established and unwritten rule is that English is not allowed. The irony is that with less verbal language, it is difficult to identify these cases. Moreover, this difficulty of assessment is compounded by the fact that this group of learners feels compelled to not turn in their work, perhaps preferring the consequence of work not done than the consequence of exposing their weakness. These language learners are unlikely to use what they know to be insufficient skills. So, the previous notion of codeswitching used exclusively by those who have minimal language skills does not seem to apply. Moreover, these students do not seem to believe that it is acceptable to codeswitch in order to improve their French skills. This outcome questions the usefulness of disallowing English at all costs.

6.2.2 Codeswitching for various purposes and functions

More likely, and this is the case for a greater portion of the class, there are those who fit in a second category. These students will occasionally codeswitch to English in their journals. What is interesting about this group is the variety of reasons for which they codeswitch. Traditionally, it is believed that this second language habit occurs because of incompetence and lack of knowledge of the target language. However, the previous group demonstrated that difficulty with French may not result in codeswitching. I examine here two different reasons for codeswitching. In *Quelle roche*, half the class journalled in English and the other in French. Interestingly, the instances of codeswitching occurred in both groups. When invited to use English words as needed within their French journal on Quelle roche, many did refer to English words in quotation marks, suggesting that they had come to certain terminological impasses. More importantly, those who were asked to write in English did not necessarily find it easier to speak about mathematics. In fact, they were more comfortable using mathematical terminology in the language that they learned it, suggesting that vocabulary difficulties are not only tied to mathematical understanding but also their areas of bilingualism. French writers would codeswitch on everyday words such as '*string'* and '*feel'*, whereas English writers would prefer to switch to French to refer to '*aire'* or '*volume'*. Even when the English equivalent was identical, students would still use the French article "*le*", suggesting that they felt they were using a French word.

The second discovery about codeswitching is rather surprising. There were cases of students who took advantage of the opportunity to use English words to express more of their individual personalities in their writing. So, whereas some learners codeswitch to overcome linguistic obstacles, others do so to improve the fidelity of their expression. Given the negative connotations of this practice, I was surprised to see students codeswitch unapologetically. My initial expectation was to see codeswitching in cases of incompetence, or as Moore (1996) calls it, as a 'bouée transcodique', to keep one afloat in the flow of communication. However, in Robert's journal, his use of English is not tinged with any sort of embarassment. In fact, much of his rough work is completely done in English. See figure 22. Based on the rest of his writing, Robert is not incapable of using French. In fact, codeswitching here would require more work to translate back to English since the terms "*plafond*" and "*mur*" are already offered in the question. It seems possible that Robert does not consider using English in his rough work to be problematic. To be sure, there are few others who have joined him in doing so. He is an unusual case that seems comfortable handing in a full page of work in English. So, unlike his teachers and his peers, who take the notion of codeswitching very seriously, Robert's unapologetic switching could be attributed to laziness but may also be an indication of how he understands English and French to simply be linguistic choices, both accesible, albeit to varying degrees.

Moreover, on the next page, he combines the two languages in an interesting way. (See figure 23) He seems to be applying French vocabulary to

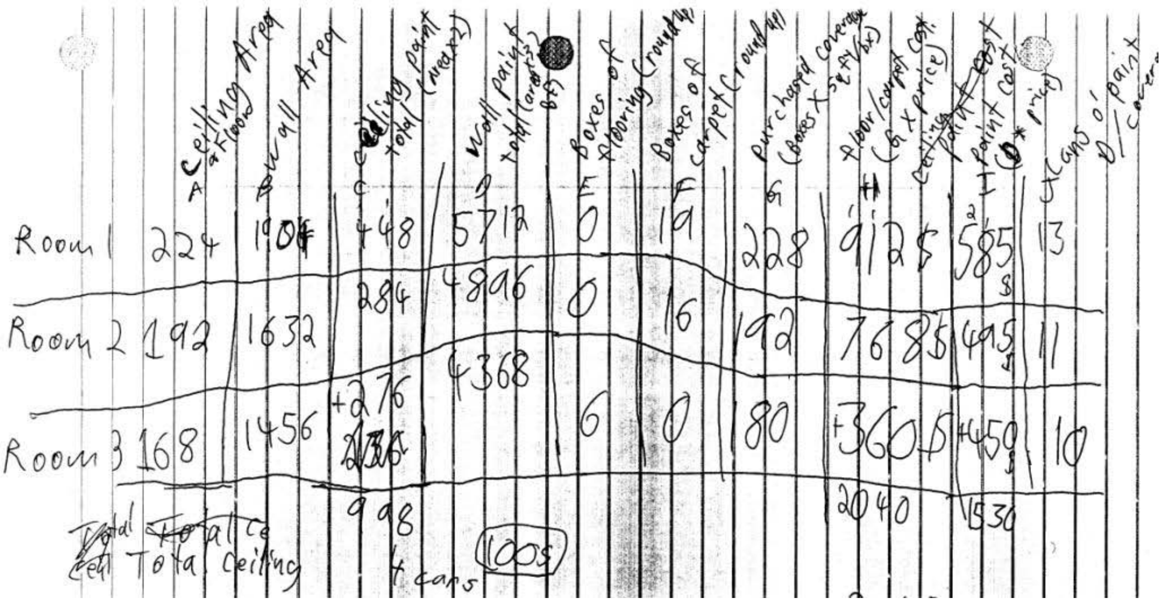


Figure 24. **Robert's use of English**

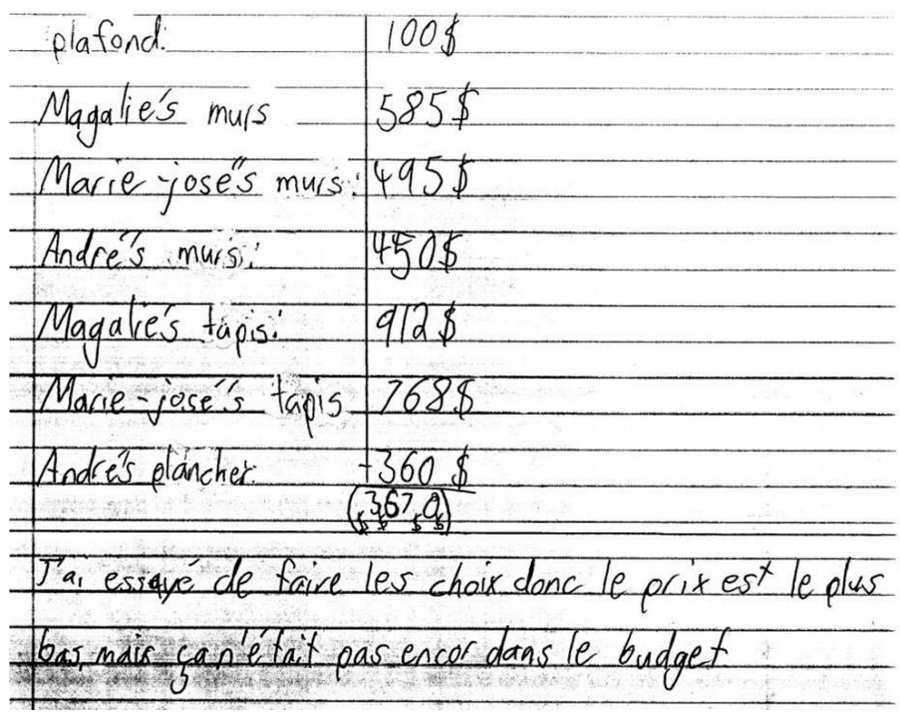


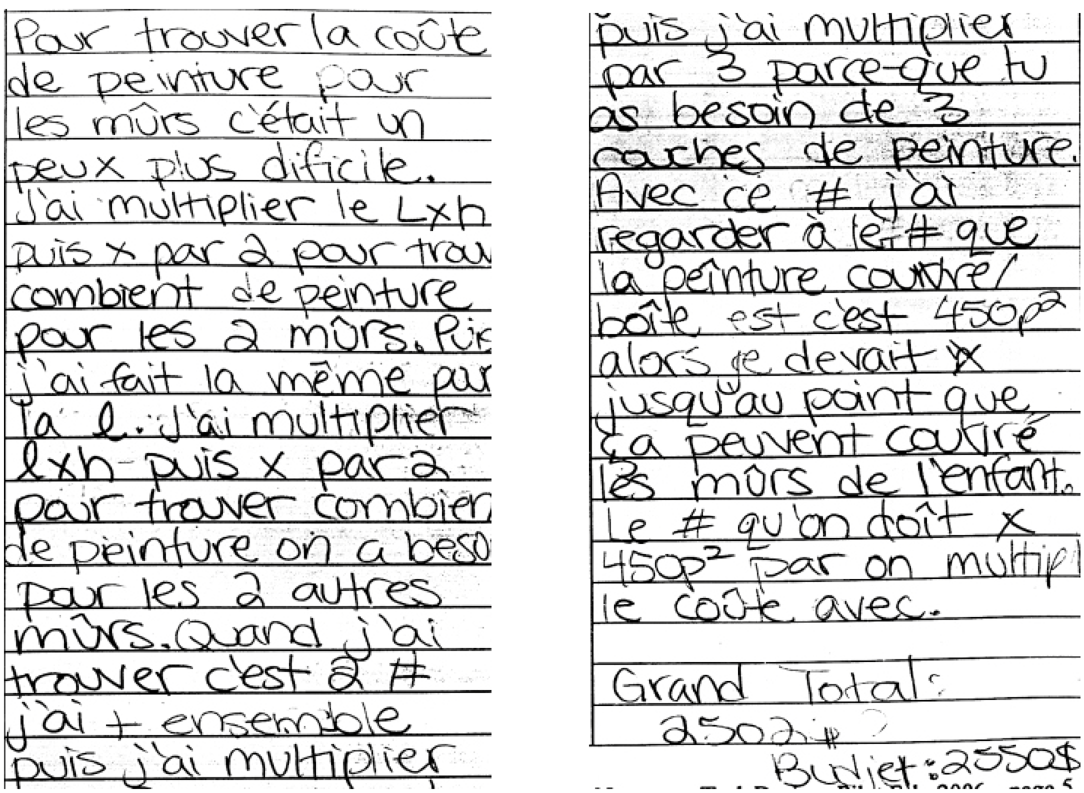
Figure 25. **Robert's English grammar applied to French vocabulary**

English grammar rules in writing “*Magalie’s murs*”. As an aside, after making this observation, it strikes me that one of the commonest features of French Immersion writing is the application of French vocabulary upon English sentence structures. In Robert’s case, his skillful combination of French and English within a sentence demonstrates a certain playfulness in his writing. Lüdi and Py would qualify this as an example of *parler bilingue,* as one of many linguistic practices unique to the multilingual person (2003).

This would not be the only instance of codeswitching being used playfully. In the case of Kelvin, his French writing skills are quite proficient but yet chooses to codeswitch. He writes, “*À ce point, je n’avais pas trops mit mes “hopes” sur cette façon*”. Judging from the rest of his writing sample, he is very capable of expressing himself in general but chose to attempt a translation of the expression “put your hopes up”, rather than using other words. Somehow, he decided that switching to English would enrich his journal, perhaps injecting it with more personality. In both these cases, students of immersion chose to use English words to improve their communication. Conversely, earlier, we saw others who did not use any English, but were clearly facing linguistic hurdles. So, codeswitching does not seem to be used when students struggle to communicate. Instead, it is used when the particular area of communication is unfamiliar, or else, when the writer feels comfortable to be playful in his use of English. Because they both belong to his linguistic and communicative repertoire, which is not compartmentalized, Robert uses one language, the other, or both, according to his communicative needs as bilinguals do “in the real” world.

6.2.3 Language as a tool

The final example of codeswitching is not between English and French, but rather between narrative and symbolic styles. Although I have already discussed Annabelle's writing earlier to show how students combine narrative and symbolic styles, I would like to return to it with a new perspective. There is something distinctively fluid about her use of languages. In alternating between mathematical symbols and French words, she seems comfortable with the two,


Pour trouver la coûte
de peinture pour
les mûrs c'était un
peux plus dificile.
J'ai multiplier le Lxh
puis x par 2 pour trou
combient de peinture
pour les 2 mûrs. Puis
j'ai fait la même par
la l. J'ai multiplier
lxh puis x par 2
pour trouver combien
de peinture on a beso
pour les 2 autres
mûrs. Quand j'ai
trouver c'est 2 #
j'ai + ensemble
puis j'ai multiplier
puis j'ai multiplier
par 3 parce-que tu
as besoin de 3
couches de peinture.
Avec ce # j'ai
regarder à le # que
la peinture couvre/
boîte est c'est 450p²
alors je devait x
jusqu'au point que
ça peuvent couvré
les mûrs de l'enfant.
Le # qu'on doît x
450p² par on multipl
le coûte avec.

Grand Total:
2502
Budjet: 2550$

Figure 26. **Annabelle's fluid use of French and mathematical languages**

almost as though symbols are really abbreviations for her. Moreover, her writing does not suffer as much of the blatant and clumsy application of French words upon English sentence structures that others practice. To be sure, her French writing is not perfect by normative standards. She makes spelling and grammar mistakes. However, what distinguishes her writing from many others is the fluidity. She uses language as a tool and if it is also a task for her, she does not betray it. Possibly, Annabelle might fit Grosjean's functional definition of bilingualism, since she can use it to learn and communicate mathematics. There are others like Annabelle who navigate the French language well enough that it is used seemlessly with the mathematics. Earlier, I commented on Alex's writing and how he used it to present a polished product of his thinking. Clearly, he is able to use French language to serve his needs as a communicator of sophisticated mathematical ideas. Again, his fluidity does not require grammatical perfection. Here then, are two examples of students for whom French language is a tool with which they can discuss and learn mathematics. More importantly, the ease with which they can manoeuvre and juggle their mathematical ideas, along with the mathematical and French languages is impressive. It demonstrates a polyglot quality to their communication that is aligned with both the goals of numeracy and those of French Immersion.

CHAPTER 7: CONCLUSIONS

The tendency to isolate the two areas of language and content oversimplifies the importance of their partnership in learning. This final chapter addresses my research question about students' use of language in mathematics and then elaborates further on the conditions in which language and content work best together: first, within the French Immersion program, then, by examining other second language education programs around the world. Finally, this chapter will outline a few recommendations following these findings.

7.1 Answers to research question

In looking at how French Immersion students use language to communicate about mathematics, two perspectives emerge, that ultimately, converge. These do not describe a range of proficiency in any quantitative way, but rather paint a qualitative rainbow of language use. Through these two perspectives, we learn that students use language in many different ways which are determined by personal style, ability and limitation, and the perception of what is expected and accepted as mathematical writing in a French immersion setting. Students vary in how they balance their use of symbolic and narrative language within and across the numeracy tasks. Their communications are also affected by their understanding of which forms of language: mathematical, French and English are allowed and valued.

The form of language most naturally associated with mathematics is the symbolic writing characterized by the use of diagrams, symbols, numbers and tables to convey mathematical understanding. Peter and Taylor both prefer it over the use of lengthy sentences and paragraphs. They choose this style, either because they feel more comfortable using a concise, economical language, or because they struggle to construct effective sentences in French. For Chris, who is not skilled in verbal language, the use of a table barely succeeds to show his understanding. On the other hand, students such as Beth have learned that organizing information into a table can sometimes convey more than a narrative. In doing so, the somewhat postmodern notion of organization as language is evoked. When a weaker student such as Chris used a table to convey his understanding without the use of narrative explanations, we were able to see how even poorly organized information can communicate some of his thinking. Despite seeing many samples of flawed symbolic writing, this style reflects the perception that mathematics is an aesthetic language consisting of rules of usage. When used properly, its beauty comes from its neatness and a lack of redundancy. For students like Peter, there seems to be little need to include narrative. Every calculation, symbol, diagram seems self-explanatory. However, learners are not all adept at manipulating this system of symbols. When done poorly, the numbers, diagrams, tables can lead to much confusion, a confusion easily remedied by short explanations in sentences or words. Moreover, the exclusive use of symbolic language can also limit the range of what is communicated. For the vast majority of students, the means of communication of mathematical ideas cannot be taken for granted. Despite Peter's proficiency with the symbolic language and his lack of need for narrative, his is a very rare case.

In contrast with this symbolic style, there are those who prefer to use narrative sentences to explain their thinking. Although less concise and less economical, narrative writing serves several purposes that symbolic writing often cannot. Students like Helen and Isaac use narrative as a medium of self-dialogue. This is apparent because, in Helen's case, the text shows the progression of her thoughts from one step to the next. She will even write about the challenges and questions she asks herself along the way. For her, narrative writing helps her think on paper. Despite the fact that the narrative is in French, Helen does not seem impeded by the language barrier. In fact, without these long unedited explanations, her mathematical symbols and calculations are difficult to understand. While the experience for the reader may not always be an easy and smooth one, Helen and Isaac have found a tool to help them think. When using narrative, they also reveal a lot more of their thinking processes. Particularly, those students who have creative solutions, such as Arnold, enjoy using this means of communication because it offers them the opportunity to justify their convoluted or non-standard ideas. In addition, the open-ended nature of journalling sometimes encourages writers to think on paper about questions that have not been posed. Sarah, for example, took the occasion to muse about the connections between science and math classes. Alex wondered whether there is a method to determine the exact volume of an irregularly shaped object. The valuing of process, these connections with real life experience and the overall communication of mathematical ideas form the essential parts of developing numeracy. However, in acknowledging these goals of numeracy, there is also an acceptance of the messiness of verbal language. Its sentences are not as neat and tidy as those of symbolic mathematical language. The ideas they convey are not as objective and easy to categorize. Rather, and particularly in the case of learners, verbal language is sometimes clumsy and roundabout. But without it, a whole dimension to communication is muted.

Although narrative and symbolic writing seem like dichotomies, they should not be considered discrete options. Most individuals combine the two styles in their own unique ways. Each of these ways suggests a different relationship with the reader or conversely, a different degree of privacy. We see that some combine narrative and symbolic languages so seemlessly that the process of their thinking can be lost. Unlike Helen, who describes every turn and move in her thinking exhaustively, Alex edits out all the detours and superfluous explanations so that the reader gets a logical presentation of his thinking. It is ironic, because while his awareness of the audience and writing skill allow him to present a neat and smooth explanation, they also obscure his process. Even though Helen shows a thoroughness in her descriptions, her self-dialogue can still be accessible to the reader. Isaac, on the other hand, performs a monologue that seems to be meant for him alone. His system of writing is neither narrative nor effectively symbolic, but it succeeds in bringing him to a satisfying answer. His writing, then, is much more private than Alex’s or Helen’s.

Another interesting discovery in the combination of narrative and symbolic styles is the relatively fluidity in the alternation between the two and the reliance upon the two to create a coherent message. Annabelle and Arnold showed that when they are comfortable using both styles, either within one task or across several, they demonstrate a bilingualism of sorts. They alternate freely and fluently. Both ‘languages’ are accessible and serve as tools they can access at whim: certain elements of each language are more appropriate for certain tasks or purposes of communication. In contrast, students such as Terri and Chris struggle with both forms of communication. In their case, only by desperately attempting a little of both can they minimally manage to convey part of their thinking. Without access to both, surely, the assessment of their mathematical skills would be compromised. In other words, when symbolic and narrative writing are both accessible, they can amplify what is conveyed. When neither form alone is sufficiently functional, then it is the combination of the two that allows the reader to piece together the story. So, while some students opt for narrative writing because it is a strength, it can also be a means of avoiding a weakness in expressing their thoughts otherwise. By giving the option of choosing the means of expression, individuals knowing their weaknesses can maximize their assets. Either way, the partnership of symbols and organization with narrative writing produces a powerful result.

The second perspective focuses on the unique overlap of French as a second language with mathematics. It highlights moments of communication in which second language can impede, amplify or, alternatively, shape and colour the learning of content. One particular type of moment is characterized by the practice of codeswitching. Typically conceived as a moment of weakness, we see in this research that in fact, codeswitching does not necessarily relate to incompetence, but instead, it exposes a complex learner who employs sophisticated tools and demonstrates a linguistic saavy unique to the French Immersion context. Interestingly, despite the perception that students codeswitch to survive, those that completed very little of the mathematical tasks never switched to English and do not perceive their first language to be a tool that is useful or even allowed in the French Immersion class. This perception may stem from the culture of French Immersion in which the use of English is denied and considered a symptom of reliance and failure, rather than acknowledged as the starting point of a bilingual learner and as such, a tremendous influence on the acquisition of a second language. Finally, we discover that when our teacher lenses are attuned to mistakes and deviations from grammatical accuracy, we overlook the cases in which young learners are capable of using and manipulating French well enough to discuss, discover and learn about mathematics. Ultimately, this should be the measure of success of the program.

It is interesting for the educator to uncover the motivations behind each student choice. Some learners can benefit from the freedom of choice, when they are equipped with many tools, both linguistic and mathematical. Others benefit from choice because they are limited by their lack of skill, either linguistically or mathematically. In these cases, choice allows them to survive better than in the context of objective testing. One factor of these choices, which until now, has only been mentioned casually, is the individual perception of what constitutes language. While this is not intended to be a forum for linguistics, the variety of ways in which students use narrative, symbolic, French and English languages suggest that they have differing ideas of what means of communication exist and in what forms. Earlier, I hinted at the idea that the very way we organize information could arguably be a form of language. Similarly, I categorize the use of symbols, numbers, diagrams as a language distinct from words in sentences. And while students have demonstrated an ability and propensity at using effective combinations of these languages, there is a strong denial of the role of the first language, English. This is largely attributed to the culture of French Immersion, which I shall discuss in the final section on Recommendations. For now, suffice it to acknowledge that language is messy. In order to fully embrace the benefits of learning a second language in a numeracy setting, and also, the benefits of learning to become numerate in a second language, we must allow individuals the freedom to use it in multiple ways while providing tools and support to expand on their ways of thinking. We must acknowledge the full ramifications of being multilingual.

7.2 Language and content as partners

Although the premise of my thesis argues that language and content can be effective partners, it does not assume that this partnership is a given. Learning language through content extends logically from Grosjean's functional definition of bilingualism (1982). Unlike learning language in a language-only course, the focus can easily end up being on the structures rather than on the function. At best, language courses may include opportunities to communicate for a variety of purposes. Instead, using other subjects as the object of communication can be a very effective use of time to acquire both language, and more specifically, communication skills, while being exposed to other subjects. In numeracy, problem-solving allows the chance to explain one's thinking and creates authentic opportunities to use language. In this sense, Immersion can be a very productive and realistic context to learn a second language and content simultaneously (Snow, Met, & Genesee, 1989). However, although this can work well theoretically, it does not describe how teaching in Immersion can pose a unique challenge. If other subjects such as mathematics, science and social studies are to become opportunities to learn language, then the Immersion teacher has, at all times, a dual focus (Tardif & Weber, 1987; Snow et al., 1989). This is an important distinction between Immersion and non-Immersion teaching. Unfortunately, it has been observed that Immersion teachers in the content areas often assume that their students are fully competent in the second language, and leave the linguistic component of content teaching aside (Lapkin et al., 1990; Plazaola Giger, 2003). Researchers suggest that "investigations need to be made of the language (vocabulary, grammar, discourse) typically used to talk about/teach a particular subject, and then that language needs to be built into language lessons" (Lapkin et al., 1990, p. 652). Another way of explaining this is that curriculum development must include an awareness of what Snow et al. (1989) call content-obligatory and content-compatible language needs. In other words, both content and language courses need to work in concert. Lyster (2007) proposes that "Content-based and form-focused instructional options need to be counterbalanced to promote shifts in learners' attentional focus through activities that interweave balanced opportunities for input, production, and negotiation" (p. 134). This may include assisting students like Taylor in creating more coherent sentences to explain what her symbolic language attempts to do so. Or conversely, it may involve comparing Chris' idea of a logical table with a more effective one. Another example of cross-discipline focus is working with weak writers such as Craig or Terri to show them how to begin describing mathematical work using words and sentences. In other words, there needs to be time put aside to focus on the linguistic challenges of writing in mathematics which ultimately benefits both French and mathematics learning.

Balancing content and language is not as simple as devoting equal amounts of time in the practice of each. Some researchers argue that the amount of integration between content and language may vary according to the subjects or level (Lapkin et al., 1990; Cavalli, 2005). For example, some believe that science and mathematics should not be courses taught in a second language at upper levels of secondary school (Cavalli, 2005; Lapkin et al., 1990). At this level, there is a concern for the loss of depth in processing upper level material in a second language. On the other hand, others argue that the use of a second language should not be an obstacle to higher order thinking and in fact, can force the learners to activate more cognitive competencies (Cavalli, 2005). For example, a student struggling to make sense of a new concept may need to learn ways to organize her thoughts by using graphic organizers, or by accessing a form of metacognitive analysis. In other words, the challenge of capturing a new concept in a second language forces learners to use more sophisticated cognitive tools and thus, can make them better thinkers. This, of course, includes the occasional codeswitching, which I will address shortly. According to Day and Shapson's survey (1996) of French Immersion teachers, although it was reported that conceptual complexity was important as a goal, they did not all feel qualified or supported in finding ways to make it happen in the second language.

On the question of first language, overzealous policies about limiting its use can lead to students feeling so constrained and helpless that they may be unable to express any understanding of the content, even if the content was not necessarily the obstacle. Effective second language programs take the cognitive load into account when considering language use policies. Similarly, restricting the use of mathematical terminology to formal language may have similar effects on learners. Although researchers (Zazkis, 2000) maintain the importance of learning formal mathematical language, it is also vital to acknowledge the usefulness of some codeswitching, whether it is between formal and informal language in mathematics, or between first and second languages. Adler maintains, that “it is not a matter of whether or not to code-switch, nor whether or not to model mathematical language, but rather, when, how, and for what purposes” (2001, p. 85). Despite the tradition of segregating language outcomes from content outcomes, it is in the recognition of their areas of overlap and influence on one another that the Immersion program – that is, a method of learning language – can thrive.

7.3 The language and content balance in other programs

While the Immersion program is known as a Canadian legacy, other parts of the world have adopted different ways of offering multilingual education programs. Gajo (2001) categorizes Immersion as the first generation of second language education, while the second generation employs Content and Language Integrated Learning (CLIL), thus making a distinction between Immersion from CLIL. And then, he considers a third generation, called Enseignement d'une Matière par l'Intégration d'une Langue Étrangère (EMILE). He explains the difference between the second and third generations by saying:

> L'intégration doit 'profiter' à la langue et elle s'inscrit ainsi dans une didactique prioritairement linguistique. Il s'agit de mettre plus de langue dans un enseignement en langue. Dans la troisième génération, elle doit 'profiter' à la discipline et s'inscrit alors dans une didactique prioritairement disciplinaire, ou dans une didactique articulée ou transversale. (Gajo, 2001, p. 11)

In other words, Gajo suggests that current approaches focus too much on the language component of an integrated program. Instead, he proposes that a third generation should emphasize the content subjects and perhaps offer more strategic juxtapositions between content and language.

In the Aosta Valley in Italy, schools have been experimenting with a system in which the aim is to find balance between Italian and French (Cavalli, 2005). In this bi/plurilingual education experiment, teaching is done in both languages. An attempt is made to integrate content with language by coordinating the efforts of content teachers with those of the language teachers. Within the courses, teachers and learners may alternate between languages at both microscopic and macroscopic levels. By this, it is explained that a microalternation consists of using the more familiar language as a way of helping a learner make the transition towards the target language when there is an immediate obstacle. Conversely, the macroalternation involves choosing the more appropriate method of communicating according to the cognitive demand of the subject in question. In other words, parts of some topics may be taught in the more familiar language because of the level of difficulty involved. And yet, there is an acknowledgement of the importance of the vernacular involved in acquiring new knowledge. Most importantly, the switching between familiar and target languages is not random nor is it subject to the whim of a teacher. The choice of language is deliberate and is made an integral part of the curricular design. Furthermore, the values inherent in the bi/plurilingual education system in Italy emphasize metalinguistic and cross-curricular abilities in language. Whereas the Immersion program may have started out prioritizing language learning over content, the bi/plurilingual education system in Italy attempts to find a better balance between language and content.

The theories behind the teaching in the Val d'Aosta have much in common with principles of communicative language teaching, a functional view of bilingualism and communicative competence. However, the major difference between French Immersion in Canada and the bi/plurilingual education in Italy is the acknowledgement of the role of the L1 in trying to balance the learning of a second language with that of content subjects. Having said that, the political circumstances in the Italian region are very different from those of the 60s in

Québec (Cavalli, 2005), and even more so those of British Columbia in 2008. The existing tensions and relationship between English and French in Canada cannot be ignored when examining the influences on second language education.

7.4 Recommendations

The effective juxtaposition of French Immersion and mathematics does depend, as mentioned, on pedagogical approaches. However, some challenges to this combination stem from wider, culturally-established beliefs about mathematics and language learning. In order to maximize the benefits of this partnership, we must challenge these biases. This final section puts on the stand two strong beliefs about mathematics and second language learning. To begin with, the very notion of questioning the importance of language in a mathematics class can seem like a contradiction. One secondary school program that has chosen to teach mathematics in French defends that the language in this course is not problematic because it is simply not a factor, mathematics being exempt from language. This statement reveals the kind of values perpetuated in some mathematics departments: accuracy, speed and isolated skills never to be applied to real situations. Verbal language would add disorder to this peace and should be kept to a minimum. This speaks to a broader question about how society and even educators perceive mathematics as an elegant, concise and objective science where ambiguity and subjectivity are lesser versions of a noble science. This is unfortunate because in the case of Helen and Arnold, the option to adopt a more narrative style of writing in mathematics clearly suits them better.

Without an acknowledgement of language, others like Taylor never get the opportunity to develop new and more flexible tools. This suggests then, that educators should consider broadening their idea of effective mathematical communication to include both narrative and symbolic forms.

Similarly, in the language area, there are also long-standing biases against codeswitching which deny the role of the first language and impede the progress of such programs and philosophies as French Immersion and numeracy. The culture of French Immersion harbours very strong feelings about the role of English. It is perceived as detrimental and disallowed. Even those who struggle to minimally meet expectations do not feel free to use English as a last resort. But yet, others who are not concerned with demonstrating minimal abilities feel free to use English in playful ways, suggesting that there is an implicit shame and association between English and only the most desperate attempts to stay afloat and survive. The discussion of these features of interlanguage causes fear in many educators that it will open the floodgates and students will never choose to speak French and will never learn to do so properly. In order to recognize what it means to be bilingual, teachers need to face the reality that their learners use English, French and mathematical languages, as their needs, abilities and situations demand them. According to Cummins (2000), the rigid barriers between languages need to be broken in order to promote a better awareness of linguistic operations used by bilinguals. Even within the progress of analysis of the data, my own perspective has shifted. In the earlier analyses, I began by sorting those students according to their skills in mathematics and French. Later, it became clear that these multiple languages: symbolic, narrative, French, English and mathematical, cannot be easily dissociated. Instead, in the bi/plurilingual view of language, these multiple languages interact. They depend upon and complete each other. Unlike the monolingual learner who is limited, the bi/plurilingual person enjoys the advantage of flexibility, the pleasure and freedom of multiple forms of expression. The French Immersion educator of the 21st century needs to be aware of these plurilingual interactions and allow them to maximize the potential of the multilingual experience.

The examples from these case studies show that the way we choose to use language is both personal and dependent upon our strengths and weaknesses. We tend to use strategies that best enhance our skills and avoid those that highlight our weaknesses. In other words, plurilingual mathematicians communicate as best they can, free to employ any and all the tools they possess, mathematical and linguistic. Despite the messiness, language serves a tremendous role in numeracy.

APPENDIX A: ETHICS APPROVAL LETTER

APPENDIX B: QUAND EST-CE QUE L’ANGLAIS EST MOINS DÉFENDU?

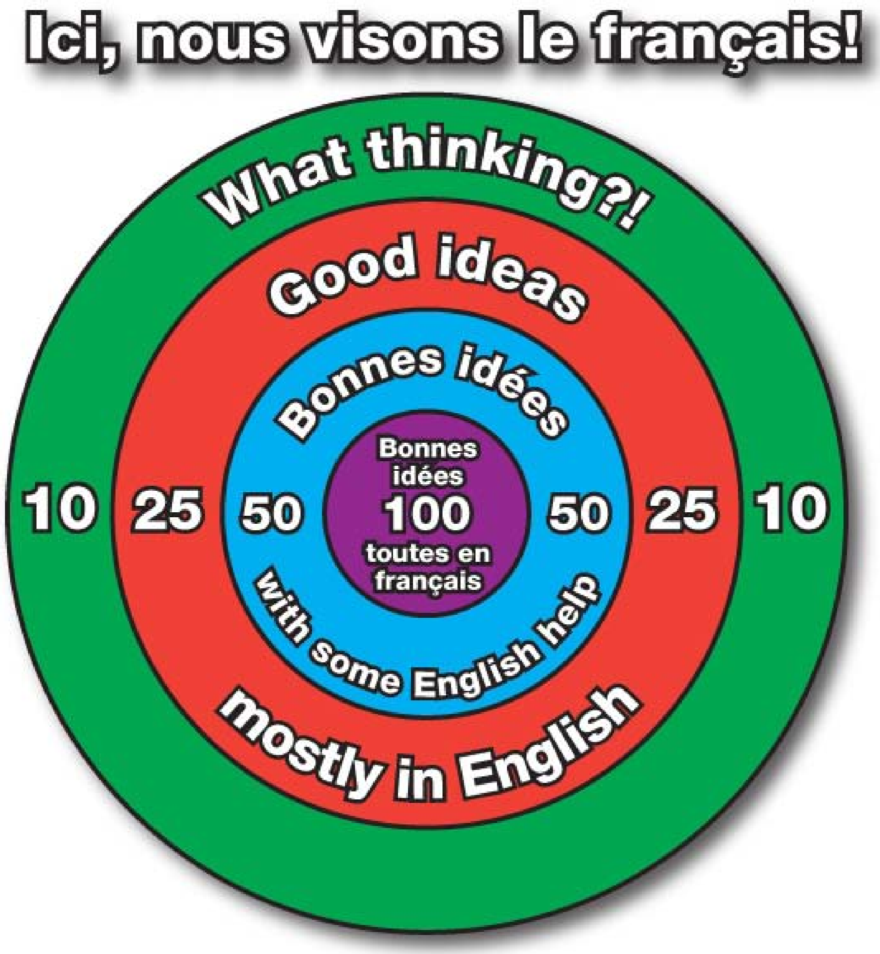
Quand est-ce que l’anglais est MOINS défendu?

Cours de	☹	😐	☺	☺☺
Français	- anglais lors d’une discussion en groupe	- qq mots anglais lors d’une discussion de groupe quand on ne sait pas le bon mot	- tout en français	- tout en français et bonne grammaire
Maths **Sc. Nat.** **Sc. Hum.**	- anglais dans une discussion qui n’est pas complexe	- anglais lors d’une discussion complexe*	- quelques mots anglais lors d’une discussion complexe	- tout en français
En tout temps	- poser des questions en anglais au prof. dans un cours qui est donné en français -utiliser l’anglais par paresse	- quelques mots difficiles en anglais pour poser une question	- s’exprimer le mieux possible en français	- utiliser un bon français qui intègre les leçons qu’on a apprises.

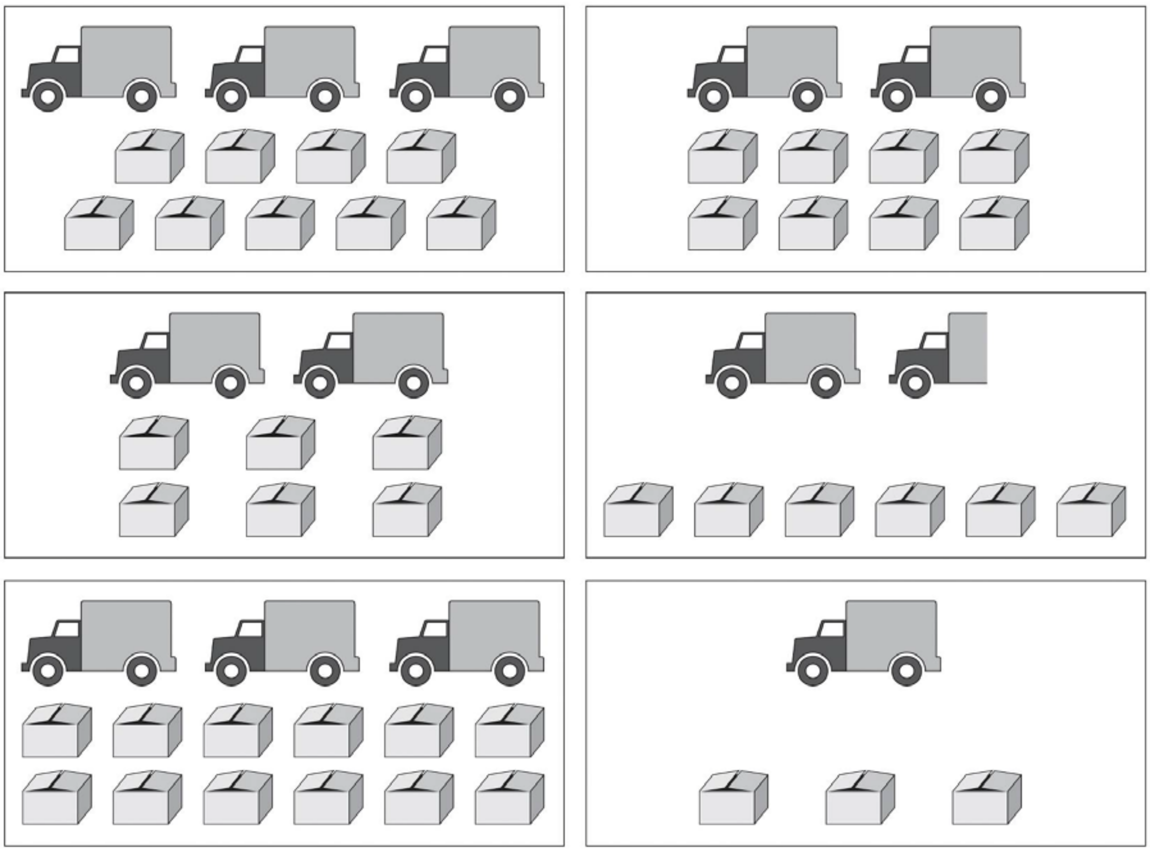
* lors d’une discussion d’un sujet difficile/nouveau, c’est acceptable d’utiliser de l’anglais pour mieux comprendre, mais tu dois par la suite apprendre comment exprimer tes idées en français.

Créée par Monica Tang

APPENDIX C: TARGET METAPHOR

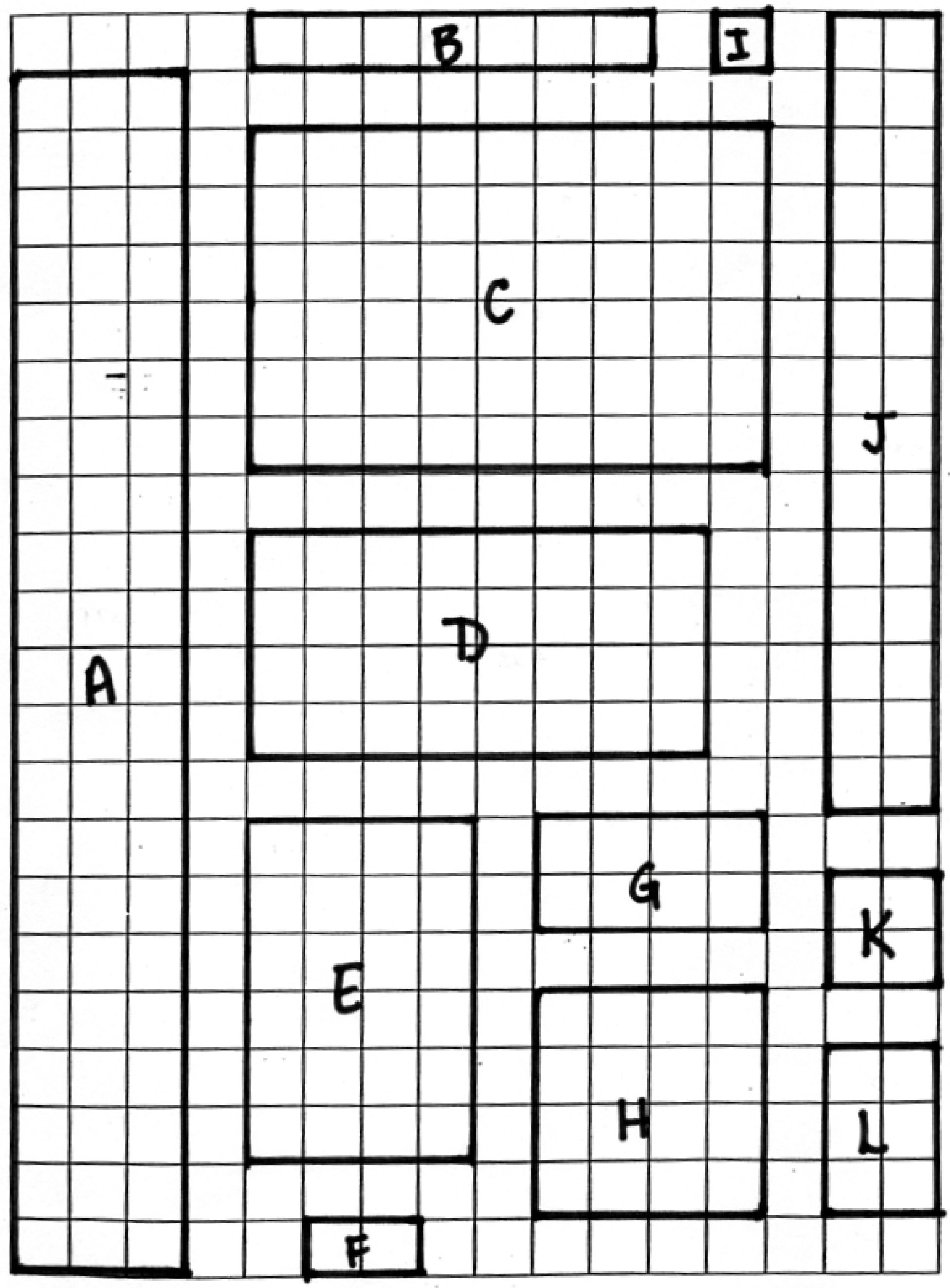


APPENDIX D: *CAMIONS* TASK



On which cards is the ratio of trucks to boxes the same?

APPENDIX E: *RECTANGLES*



APPENDIX F: OLYMPIQUES

CLASSEMENT DES PAYS AUX OLYMPIQUES

Aux Olympiques de Turin, en Italie, les athlètes canadiens ont bien réussi. Mais la question est, à quel degré ont-ils réussi? Classe les pays ci-dessous en ordre de succès. Tu peux utiliser l'information donnée ci-dessous. Justifie ta méthode de classement.

PAYS	OR	ARGENT	BRONZE	POPULATION
l'Autriche (Austria)	9	7	7	8 150 835
la Norvège (Norway)	2	8	9	4 525 116
la Corée du Sud (South Korea)	6	3	2	48 422 644
l'Allemagne (Germany)	11	12	6	82 431 390
la Suède (Sweden)	7	2	5	9 001 774
le Canada	7	10	7	32 805 041
la Suisse (Switzerland)	5	4	5	7 489 370
les États-Unis (USA)	9	9	7	295 734 134
l'Italie (Italy)	5	0	6	58 103 033
la Russie (Russia)	8	6	8	143 420 309

APPENDIX G: *RÉNOVATIONS*

LES RÉNOVATIONS

La famille Glico vient tout juste de s'acheter une nouvelle maison, et pour la première fois chaque enfant aura sa propre chambre. Puisque la maison est un peu vieille, M. et Mme. Glico veulent redécorer la maison. Les enfants voulaient tous choisir comment redécorer leurs chambres. Les parents ont accepté mais ont dit aux enfants qu'ils avaient seulement un budget de $2550. Une autre condition était que le plafond de chaque chambre devait être peint en blanc.

Magalie, la plus vieille, a demandé que sa chambre soit peinte en bleu clair. Elle veut avoir du tapis car elle a toujours froid au pieds. Son deuxième choix de couleur serait une peinture violet claire.

Marie-Josée veut que sa chambre soit peinte violet clair. Elle veut du plancher laminé et ne veut pas de tapis. Si nécessaire, elle serait contente avec des murs jaunes.

André veut que sa chambre soit peinte en jaune. Il aimerait avoir un tapis jaune/orange, mais il serait prêt à avoir du plancher laminé.

Les prix du matériel

Peinture du plafond	$25/ boîte	Couvre 300 pieds^2	2 couches nécessaires
Peinture murs	$45/ boîte	Couvre 450 pieds^2	3 couches nécessaires
Tapis	$4/pied^2	Est vendu en largeur de 12 pieds	
Plancher laminé	$2/pied^2	Boîte couvre 30 p^2	

La taille des chambres

Chambre	Hauteur	Largeur	Longueur
Magalie	8 pieds	16 pieds	14 pieds
Marie-Josée	8 pieds	16 pieds	12 pieds
André	8 pieds	14 pieds	12 pieds

Est-ce que chaque enfant aura son choix et si non, quelles décisions prendras-tu et pourquoi?

APPENDIX H: *QUELLE ROCHE EST PLUS GROSSE?*

QUELLE ROCHE EST PLUS GROSSE?

Vous avez deux roches de forme irrégulière. Vous devez décider laquelle des deux roches est plus grosse mais vous n’avez qu’une règle et les outils dans votre étui à crayons pour vous aider.

Travaillez en équipe de quatre et trouvez deux (2) moyens de résoudre votre dilemme

APPENDIX I: *CULTUS LAKE*

SORTIE AUX GLISSADES D’EAU

Les 173 élèves en 6è/7è année de ton école intermédiaire vont à Cultus Lake pour une journée de glissades d’eau. Vous allez vous y rendre en autobus.

Le parc de glissades d’eau demande qu’il doit y avoir minimum un adulte pour superviser chaque groupe de 12 élèves à la sortie.

La compagnie d’autobus et la commission scolaire demande qu’il y ait au minimum 2 adultes par autobus.

Il y a 6 professeurs qui y vont. Les autres adultes seront des parents bénévoles.

Chaque autobus peut contenir 30 passagers.

COMMENT VA-T-ON AMENER TOUT LE MONDE AUX GLISSADES D’EAU?

Explique ta réponse en donnant des détails spécifiques sur le nombre d’autobus qu’il te faut et combien de personnes seront sur chaque autobus. Montre que ta solution peut satisfaire TOUTES les conditions.

APPENDIX J: PERFORMANCE STANDARDS IN WRITING

APPENDIX K: PERFORMANCE STANDARDS IN NUMERACY

APPENDIX L: RUBRIC FOR ANALYSIS

	Not yet meeting expectations 1	Minimally meeting expectations 2	Fully meeting expectations 3	Exceeding expectations 4
Vocabulary	- Words chosen are often vague or incorrect - Frequent use of informal and incorrect French and/or mathematical language	- Some use of clear vocabulary, may be vague at times - Frequent use of informal or incorrect French or mathematical language	- Use of somewhat clear and concise mathematical vocabulary - May use some informal language	- Use of clear, concrete, concise, and precise mathematical vocabulary
Organization	- Few or no linking words used and various steps of problem are not clear - Work is often confusing, with key information omitted - Often omits required charts, diagrams, or graphs, or makes major errors	- Minimal use of linking words and various steps of the problem are not always clear - Most work is clear; may omit some needed information - Creates required charts, diagrams, or graphs; some features may be inaccurate or incomplete	- Use of some linking words to connect different parts of problem - Work is generally clear and easy to follow - Uses required charts, diagrams, or graphs appropriately; may have minor errors or flaws	- Effective use of linking words to connect different parts of problem - Work is clear, detailed and logically organized - Uses required charts, diagrams, or graphs effectively and accurately
Reflection	- No attempt to show insight or reasoning process	- Few attempts or inefficient attempt to use words or sentences to show insight or reasoning process	- Uses some words and sentences somewhat effectively to show insight into problem or reasoning	- Uses words and sentences effectively to show insight into problem, un/tested theories or reasoning process
Mathematical understanding	- Unable to identify mathematical concepts or procedures needed - Does not apply relevant mathematical concepts and skills; major errors or omissions	- Identifies most mathematical concepts and procedures needed - Applies most relevant mathematical concepts and skills appropriately; some errors or omissions	- Identifies mathematical concepts and procedures needed - Applies mathematical concepts and skills appropriately; may be inefficient, make minor errors or omissions	- Identifies mathematical concepts and procedures needed; may offer alternatives - Applies mathematical concepts and skills accurately and efficiently; thorough

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