

Relationship between Mindful Teaching Methods and Student Perception of their Retention of Mathematical Knowledge

by

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Thesis Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

in the

Secondary Mathematics Education Program
Faculty of Education

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SIMON FRASER UNIVERSITY

Fall 2018

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Abstract

Although little research has been done on what students perceive increases their retention of mathematical content, studies show that how a memory is acquired has a direct impact on how strong that memory is. By increasing student engagement through the use of teaching tools found in Liljedahl's *thinking classroom*, along with digital technology, an increase in student retention in a mathematics classroom could occur. This research study focuses on if there is a relationship between students' perception of their retention of mathematical knowledge and the use of engaging teaching methods such as vertical, non-permanent surfaces, visibly random groupings, mindful notes, and digital technology. Results were gathered through student surveys and interviews, although staggered assessments were also analyzed to see if variance in results occurred with the implementation of new teaching methods. Results showed that the implementation of engaging teaching methods have a positive impact on students' perception of their retention of mathematical content.

Keywords: engagement; relational understanding; retention; *Thinking Classroom*; digital technology

- For Sonia, my angel in the stars.

- For Meher, Jaitin, and Anita, my angels here.

Acknowledgements

My inspiration for undertaking this thesis study was, Dr. Peter Liljedahl. Your research on the *thinking classroom* has transformed my mathematics classroom, and will continue to do so. Thank you for all of your endless support, patience and guidance throughout this journey.

I would also like to thank Dr. Sean Chorney for his critical analysis of my multiple drafts of writing. Because of you, I believe I have become a better writer, and see writing through a different lens.

I was lucky enough to be part of an incredible Master's program at SFU. Thank you to all of my professors for showing me the different aspects of mathematics education and my cohort members for making it such an enjoyable program.

On a personal side, I have had a strong network of support from my family and friends during this endeavour, but one person who I must thank personally is my brother Kubby. You have been a pillar of support in this process and you must be very happy that I am finished!

I would finally like to thank my parents. You continue to inspire me to work harder and follow my dreams. I am where I am because of you.

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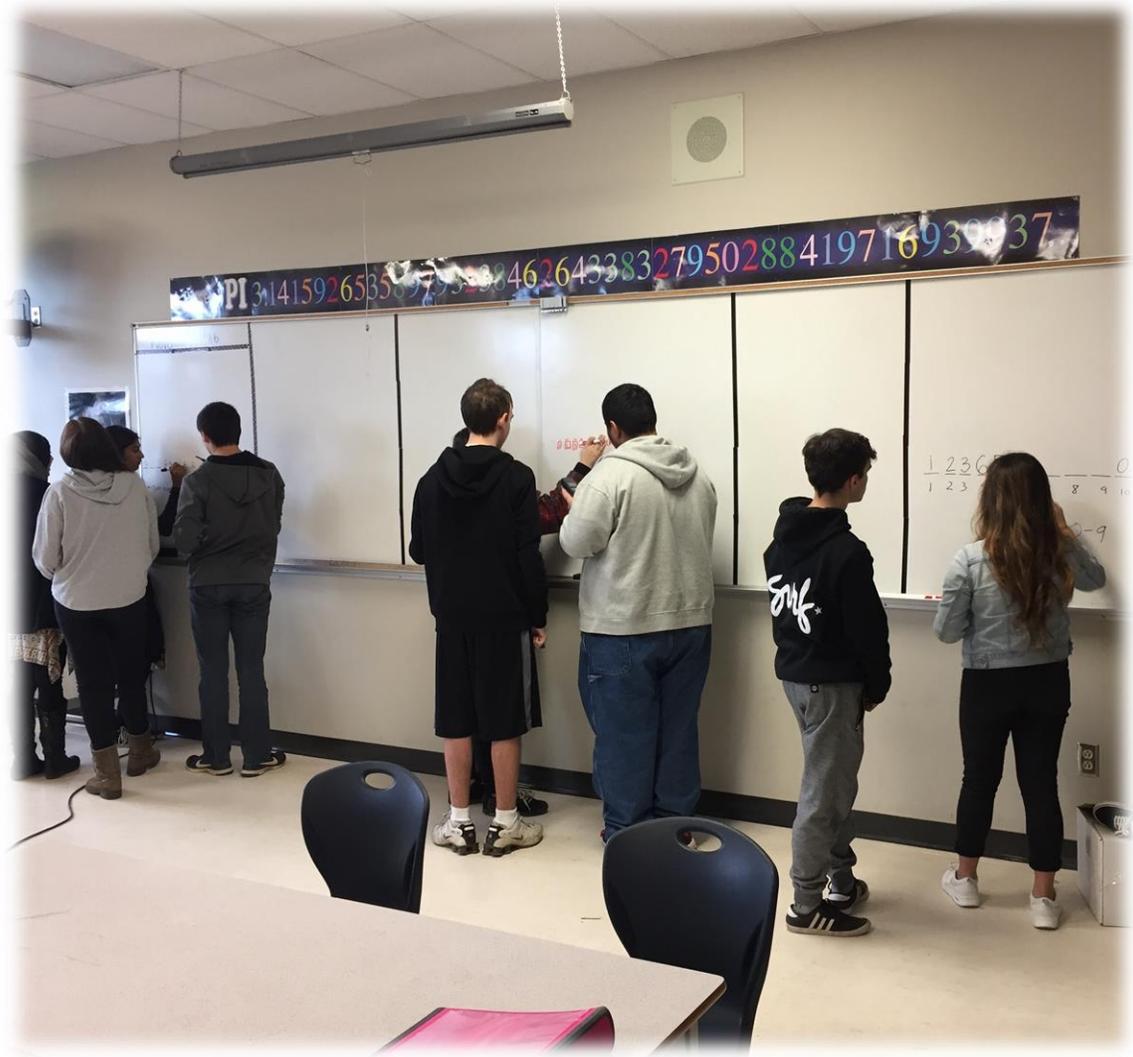
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List of Acronyms

MEd	Master of Education
MSc	Master of Science
PDP	Professional Development Program
SFU	Simon Fraser University
VNPS	Vertical, Non-Permanent Surfaces
VRG	Visibly Random Groupings



Chapter 1.

Introduction

“Education is what remains after one has forgotten what one has learned in school.”

- Albert Einstein

“What does it mean to be human?” I was asked this question first by Dr. Robin Barrow in my Philosophy of Education class during my undergraduate studies in mathematics at Simon Fraser University (SFU). I remember thinking in class, “That’s a simple question, humans have the capacity to think critically. What other species have the ability to do that?” However, the question was not as simple as I initially, naively assumed. Throughout the course, Dr. Barrow explained the work of key philosophers in education, and my initial response to this question changed. On my final paper for the class I wrote:

The question of what it means to be human is one that evokes much discussion. To live to one’s human potential it is extremely important that one develops their mind through education, whether it be through formal schooling or life experiences, ultimately acquiring worthwhile knowledge and understanding.

Dr. Barrow spent a considerable amount of time in his class explaining the terms knowledge and understanding. I never distinguished between these terms before taking this class, and his definitions resonated with me. As defined in Barrow and Milburn’s (1990) *A Critical Dictionary of Educational Concepts*, knowledge is “the acquisition or possession of information in an inert form” (p. 320). Understanding, “implies that the information in question is imbibed in such a way that it cannot lie inert” (Barrow & Milburn, 1990, p. 320). Thus, knowledge is about attaining information, which consists of facts, and understanding is the actual application of those facts. By the end of Dr. Barrow’s class, I made a connection that obtaining an education, and with that attaining knowledge and understanding, were important components to developing a human mind. As I thought back to my own high school education, I realized there were so much I had forgotten from most of my classes. It made me question, how much understanding did I actually get from my education if I remembered so little? I also started questioning

whether our schools were helping us realize our full human potentials by creating environments where we could make transitions from knowledge to understanding.

After, completing my undergraduate degree in mathematics I decided to pursue teacher education at SFU. Being passionate about mathematics, I wanted students of mathematics to enjoy the subject as much as I did. Although I always did well in mathematics until grade 12, where my teacher's highly structured methodology made me not want to engage with the subject, I truly did not enjoy the subject until university. I found mathematics classes were always the same in every high school class I attended: notes then homework. I was never one who could just memorize concepts, and the lack of explanation teachers gave behind why things worked mathematically was at times frustrating. I found that I would have to explore concepts I didn't understand on my own, as my classes rarely allowed this opportunity. As I thought back to Dr. Barrow's class during my Professional Development Program (PDP) classes, I knew that I wanted students in my class to gain understanding, more specifically I wanted to be a mathematics teacher that focussed on reasoned understanding rather than mechanical understanding. As defined by Barrow and Wood (1988), mechanical understanding is the "knowing-what-to-do kind of understanding" (p. 54). In mathematics, when a student is asked to find the hypotenuse of a triangle, he or she may demonstrate mechanical understanding if they successfully use the Pythagorean formula, $a^2 + b^2 = c^2$, to obtain the hypotenuse. Reasoned understanding involves using both theory and rationale to explain the "why" behind certain situations (Barrow & Woods, 1988). In the case of finding out the length of the hypotenuse, a person who understands why the Pythagorean formula allows one to find the hypotenuse, would have reached reasoned understanding. I felt optimistic during PDP that this was something I would be able to achieve.

In our teaching methods portion of the teacher PDP, we were given the skills to create our classrooms. Things such as unit plans, lesson plans, and classroom management were examined. Although one of my faculty associates originally taught in an elementary school, and the other was a social studies major, they both stressed the importance of having students explore the "why" in mathematics and engaging our students through activities and explorations in our classrooms. This was further reinforced by Dr. Peter Liljedahl, who taught one of my secondary mathematics methodology classes during PDP. I felt Dr. Liljedahl's class was one which impacted my

teaching the most as I started my journey as a teacher partially because of my experiences in his class. Dr. Liljedahl's class was unique in the sense that we, as potential teachers, were given the opportunity to explore mathematics through various engaging means, which we, in turn, could implement in our classrooms. Everything was taught in such a way that we had an opportunity to experience what it would be like to be an actual student in a mathematics classroom, allowing us to see the mathematics classroom from a student perspective. This structure had a huge impact on me as I was used to having most mathematics classes in lecture format and I finally had an opportunity to explore a different form of instruction by actually experiencing it.

I was excited as I went into my practicum school, but when I started my mathematics classroom observations, I was surprised with how things had changed very little compared to when I was in high school. It appeared to still be all notes and homework and again I questioned how this was helping create an educated individual who should gain understanding through their education. I couldn't help but feel a bit discouraged. Nonetheless, I reasoned that just because things did not appear different in the mathematics classes I observed, that didn't mean that things could not change. I kept that in mind as I successfully completed my practicum. I used explorations, and had students work on questions cooperatively during the notes rather than me just telling students what to do. However, two things still bothered me. First, how difficult it was in practice to involve all students in a classroom of different skill sets in the explorations, and secondly, how quickly students were forgetting the knowledge that I was hoping was to turn to understanding. Why was the retention of mathematics so low?

As I started to teach full-time, along with focusing on the transition of mathematical knowledge to understanding, I explored how long it seemed my students were retaining the knowledge that I was hoping they gain from my classes. If students were not retaining the knowledge for longer than a few days I felt that this meant that they were not gaining complete understanding of the concepts I was teaching. Meaning, as a teacher, I was not creating an environment where this could be done. As I observed students each year in my classes, I felt that for many of them the sole purpose of taking my class was to memorize mathematical concepts long enough to do well on the assessments. Marks were what many students were focusing on and being motivated by. This feeling was reinforced by the amount of time both my colleagues and I were spending on review each year. There seemed to be an endless cycle of reviewing and

teaching by the teacher each year and memorizing and forgetting by the student. I felt that regardless of the professional development I was receiving, I was not implementing anything in my classes that was helping break this cycle. It was at this point I decided to pursue my masters at SFU. The master of education (MEd) program with a focus in secondary mathematics education was something I was always interested in taking. This was partially because Dr. Liljedahl had recommended this program upon our graduation from PDP. I felt that by taking Dr. Liljedahl's advice and pursuing this particular Master's program it could potentially help me grow further as a teacher and perhaps help answer my many questions on helping create a mathematics classroom where students valued understanding over memorization.

The Master's program consisted of six key courses, each of which brought a variety of positive changes to my teaching practice. Although I had not initially intended, halfway through the program I decided to change pathways to obtain a master of science (MSc) rather than a MEd, giving me the opportunity to do a research study within my classroom. The main reason for this change was I thought this would be a good opportunity to explore what kind of teaching practices I could implement in my classroom to help increase retention of mathematical knowledge for students and again have them move away from memorizing mathematics. I felt that my struggle in my teaching practices of increasing student retention of mathematical knowledge and hence, attaining true understanding in my classes over just mathematical knowledge was linked to the teaching model I was utilizing. I knew that engagement was important, but up to this point although students were listening and participating in my classes, they were not truly engaged with the mathematics. I made this connection in my second class in the program, Teaching and Learning Mathematics. Taught again by Dr. Liljedahl, this was my first experience with vertical, non-permanent surfaces (VNPS) and visibly random groupings (VRG). I still remember how Dr. Liljedahl had us engaged in the class right from the start. There was no review of the course syllabus or the expectations of the day. Instead we were immediately divided into random groups through playing cards and set to work on mathematics problems on the boards. Before I knew it, the class was almost over and it was at this point Dr. Liljedahl had us reflect on what we learned from our experiences in the class that day. I was immediately drawn to this process because of not only how engaged I felt, but the fact that the experience remained with me. I wanted to explore whether a similar experience could be recreated for the students in

my mathematics classes and they too would feel like this would help them retain the material for a longer period of time.

Einstein stated, "Education is what remains after one has forgotten what one has learned in school." Hence, what really remains with us after all our schooling are the experiences where we felt we were actually learning something of value. Important questions arise from this thinking: as a classroom teacher how can we create an environment where students not only value learning for the sake of gaining knowledge and understanding but also provide opportunities for them to be involved in meaningful experiences, which affect student perceptions of the retention of their learning? Here is my journey as I explore these questions with the help of my own education at SFU in the MSc program.

Chapter 2.

Literature

“Teaching is a strategic act of engagement.”

- James Bellanca

A mathematics teacher’s perceptions of teaching methods that promote student engagement might differ depending on their personal definition of the term “engagement”. For some, student engagement with material might be demonstrated by seeing students sitting at their desks and quietly working on an activity or an assignment. How quiet the room is, is taken as an indicator of the engagement level of students with the content. For others, student engagement is seen through the active discussions about the material they have with their peers; hence, now, a quiet classroom might indicate a class that would be disengaged with the material. Along with engagement, retention is another term that may cause some variance in definition among educators. For some, retention can be demonstrated by students through correctly answering a series of questions on a test. Other educators might view student retention as their ability to apply prior knowledge to new ideas they learn, as they believe retention is demonstrated through understanding the material and if a student understands the material they should be able to apply it to new concepts.

Along with the inconsistent definitions of engagement and retention, the connection between student engagement and retention has not been directly established through research. Nonetheless, many studies have been done to show the positive effects of having students engaged in the classroom. In this chapter I will first examine literature on what having students engaged in the classroom setting entails, looking particularly at the tools and models used to create student engagement in the classroom. I will then look at the positive effects of acquiring mathematical knowledge in a manner that students find engaging and how this may increase student perception of their retention of mathematical content.

2.1. “Engagement” in the Classroom

Michael Corso, Matthew Bundick, Russell Quaglia, and Dawn Haywood (2013) define engagement in the classroom by focussing on the interactions of students, teachers, and subject content. Using their Student Engagement Core Model (Figure 1), Corso et al. (2013) reason that there are four key interactions in the classroom: student – teacher, student – content, teacher – content, and student – teacher – content where optimal student engagement is achieved. The student – teacher relationship refers to how a student relates to their teacher. To have a positive relationship a student must feel a teacher is “available, concerned, impartial, and respectful” (p. 56). Hence, it is important a student feels connected in the culture created by the teacher in the classroom, and feel that their teacher genuinely wants to help them achieve success, not just in their particular class but with their outside goals as well. Furthermore, students should feel comfortable getting help from their teacher on course content they are finding difficult or ask them for clarification during a lesson without feeling they will be ridiculed. The student – content relationship refers to how relevant a student perceives a course’s content to their personal interests, goals, or identity. If a student feels a course is related to something they are interested in outside of class, or is something that will help them in succeeding in achieving future goals, they will be more engaged in the material. The teacher – content relationship refers to the teacher’s expertise in the course content, but more specifically how they deliver this course content. Being an expert in a field is not sufficient in having a strong teacher – content relationship. Content must be presented in a way where students can make their own personal connection. More specifically Corso et al. (2013) suggest that:

higher engagement in thought, feeling, and action in the classroom are supported by a teacher’s ability to deliver quality instruction; create a caring structured learning environment; have high expectations of students; involve students in meaningful tasks with real-world implications; and allow students to share knowledge with each other. (p. 57)

Finally, the student – teacher – content relationship, which is at the centre of the Venn diagram, where student engagement is optimal, is attained only when students have a positive relationship with their teacher, someone who they believe is an expert in the content of the class, which they know is relevant to the pursuit of their personal goals. If all the interactions of the engagement core model are strong, students will likely engage in classroom activities and put in the effort to do well in the class.

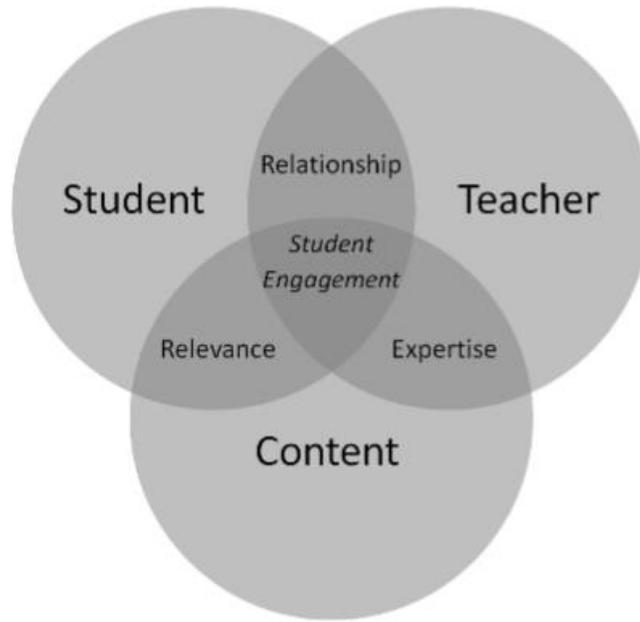


Figure 1. The Student Engagement Core Model (Corso et al., 2013, p.55)

Note: Used with permission of Carl Walley, Acting Editor, American Secondary Education

2.2. Tools Used to Increase Student Engagement in Mathematics Classroom

Now that a definition of classroom engagement has been established, it is important to look at how engagement can be increased in the mathematics classroom. I will next look at strategies found in literature that have shown a positive correlation with student engagement in the mathematics classroom.

2.2.1. Thinking Classrooms

Peter Liljedahl has done extensive research on how to build *thinking classrooms* for mathematics. As defined by Liljedahl (2016), a *thinking classroom* “is not only conducive to thinking, but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together and constructing knowledge and understanding through activity and discussion” (p.362). Therefore, for a mathematics classroom to foster a thinking environment it is important that both the classroom teacher has a belief in their students’ ability to engage in mathematical thinking activities as well as for a student to have belief in their own capabilities. In such a space both a teacher and student should be able to learn and grow from each other. Students should also be able to construct their mathematical knowledge by making

personal connections with the material through a social environment based on discussion and collaboration. Thus, the role of the teacher becomes one of using tools that will give students this engaging learning environment, enhancing the teacher-content relationship outlined by Corso et al. (2013) engagement model. Liljedahl (2016) not only examined his own teaching practices when determining the key components of a *thinking classroom*, but he also examined the classrooms of over 40 other mathematics teachers. From his research he found nine practices that played an important part in creating a *thinking classroom*. Three of the tools Liljedahl suggests for the creation of this environment are mindful notes, vertical, non-permanent surfaces (VNPS), and visibly random groupings (VRG).

Mindful Notes

The classroom teacher plays an important role in creating a *thinking classroom* environment through the class structure implemented and the activities utilized with students. The type of tasks that are used and when and how they are given to students helps distinguish between a *thinking classroom* and one that is not. During his research, Liljedahl (2016) worked with Ms. Ahn, a grade seven teacher, in creating a *thinking classroom* environment. Initially, Liljedahl provided Ms. Ahn with a set of problems to engage her entire class so that students would start getting involved in discussion as they worked through the problem-solving process. They were “low-floor, high-ceiling problems” which gave all students, regardless of mathematical skill level, an opportunity to enter the problem and eventually find success. Such problems allowed students to have an opportunity to think about the problem for themselves and the underlying mathematical content found within it. However, despite using such problems in her class, Ms. Ahn’s students would not engage with the problem-solving process. After Ms. Ahn’s negative results, Liljedahl came to observe her regular classroom environment and made an interesting discovery. Liljedahl (2016), was able to conclude from observing Ms. Ahn’s class for a period of three days that the issue came from the fact “that Ms. Ahn’s teaching was predicated on an assumption that the students either could not or would not think” (p. 362).

In a non-thinking classroom Liljedahl (2016) states practice work is given once the teacher has worked through practice examples with the students. These examples are either copied by students straight from the textbook or from the teacher exemplar on

the board. Ms. Ahn's students were used to engaging in this process, and now that an opportunity came where they were to think for themselves, they found making the transition to be difficult. Notes given in mathematics classrooms where students are simply told what to do can consequently have negative effects on students' ability to think for themselves when they are challenged to do so. As Liljedahl (2016) writes, "under such conditions, it was unreasonable to expect that students were going to spontaneously engage in problem-solving enough to get stuck and then persist through being stuck enough to have an AHA! Experience" (p. 362). By simply changing the classroom environment so that notes become more mindful, can help shift the classroom environment to become one that thinking is encouraged. More specifically, by changing student workspace can help in creating an environment where any notes given can become more mindful.

Vertical, Non-Permanent Surfaces

In most traditional mathematics classrooms, students work at their desks using notebooks or lined paper and writing utensils as their main tools for learning. In his research study taking place in five high school classrooms ranging from grade ten to twelve, Liljedahl (2016) explored whether changes made to this environment would lead to changes in the thinking environment found in the classroom. Instead of students working individually at their desks, students worked in groups of two to four, with one writing tool, in five different work environments: at the walls using poster boards or flip chart paper, at VNPS surfaces such as a blackboards or whiteboards, at their desks using smaller whiteboards, at their desks using flipchart paper, or at their desk using their own notebook.

To measure levels of effectiveness of the different work spaces, Liljedahl (2016) utilized "proxies for engagement – observable and measurable (either qualitatively or quantitatively) student behaviours" (p.366). The proxies were measured by a team of three to five people, which gave the study an objective perspective as observations were not made by only one person. The proxies for engagement that Liljedahl (2016) and his team looked at consisted of: how long did it take for the group to begin to discuss the task once it was given, the amount of time it took for students to write something mathematical on their work surface, how enthusiastic the group was to start the task, the richness of group discussion during the task, how well all group members participated in

the task, how determined a group was to complete the task given, how organized the work was on the work surface, and whether groups interacted with other groups in the classroom to attain an answer. Thus, through this study, one has an opportunity to see at which working surface students are most engaged with the mathematics.

The results of the study were quite informative for mathematics teachers as, unlike what they may have suspected, the most traditional setting of students being at their desk was found not to be the most engaging setting. Liljedahl (2016) found that out of the five selected settings, students were most engaged at the whiteboards or blackboards. It was at these vertical surfaces in comparison to horizontal surfaces, students were more involved in discussion, had more participation from all its group members, were more relentless to finish the task and had their work scattered as opposed to linear. Furthermore, the non-permanent surfaces scored higher for engagement compared to the more permanent surfaces given. As concluded by Liljedahl (2016), these vertical, non-permanent surfaces offered students a safe place to take risks and easily depict the task using multiple representations. It is while standing and utilizing this tool that students will be more willing to participate and start the problem quickly as “sitting, even while working at the whiteboard, still gives students the opportunity to become anonymous, to hide and to not participate” (Liljedahl, 2016, p. 370).

Visibly Random Groupings

Jo Boaler (2016), who has done some leading research of the use of collaborative work in the mathematics classroom, writes about the importance of group and whole class discussions:

Not only are they the greatest aid to understanding – as students rarely understand ideas without talking through them – and not only do they enliven the subject and engage students, but they teach students to reason and to critique each other’s reasoning. (p. 29)

Thus, the affordances of having students work collaboratively in the mathematics classroom are that it gives students an opportunity to verbalize their thoughts, engage with the mathematics, and concretize their understanding through discussing and evaluating their peers’ views. These are qualities of an engaged, *thinking classroom*. Liljedahl’s (2016) research also showed that having students work at VNPS such as

whiteboards, blackboards, and windows in groups, increased engagement levels in the mathematics classroom. Nonetheless, this can be only done if groups are created and utilized keeping certain factors in mind. How the students were grouped in Liljedahl's research will now be examined.

In Liljedahl's (2014) research in a grade 10 classroom, groups were formed daily using visibly random groupings rather than strategic grouping. Strategic grouping is used by the classroom teacher to form groups so that certain students, based on academic or behavioural reasons, are working together (or not together). With visibly random grouping, the grouping process is completely random, not based on teacher preferences and students are well aware that they have not been placed in a particular group for a certain reason. According to Liljedahl, to ensure that the grouping process is visibly random for students, teachers can create the groups with students present using a variety of randomized grouping processes. For example, giving students shuffled playing cards and then having students with the same number form a group, or use a digital program which can be projected in the classroom so students can see that the computer is choosing groups with no set goal.

Within three to four weeks of implementing visibly random groupings, he observed several key changes in a grade 10 mathematics class. With randomization, students had the opportunity to work with most of their classmates which led students to be more willing to work with any of their peers rather than only their friends. With students being more willing to collaborate, this led to students to rely less on the teacher to attain an answer, and more on their own group or their surrounding groups. With this increased collaboration within the classroom comes a sense of community where students rely on each other for a deeper understanding in mathematics and, consequently, increased engagement. As he concluded, "... the class coalesced into a community... and their reliance on themselves and each other increased. Their enjoyment of mathematics (the class, if not necessarily the subject) increased as well as their engagement" (p. 19).

2.2.2. Flipped Classroom

Another model, which can also be utilized in collaboration with Liljedahl's *thinking classroom* model (2016), to create an engaged mathematics classroom is a flipped

classroom. Unlike a traditional mathematics classroom, where teacher directed instruction takes place during class time, and practice through homework and assignments occur outside of class, the flipped classroom environment reverses these two fundamental roles. Technology is used as a tool to deliver pre-recorded lessons to students outside of class time, and class time is meant to be used to develop a deeper understanding of the mathematics by working collaboratively on mathematical tasks with ones' peers. Liljedahl's framework of a *thinking classroom* can be implemented with the flipped classroom model. VNPS and VRG can be used during class time when students attempt to understand the mathematics they viewed at home. As Kevin Clark (2015) writes, "such use of class time could potentially give students the opportunity to learn to think for themselves by being actively engaged in the mathematics content" (p. 94).

Clark undertook a research study to see whether, as he suspected, a flipped classroom framework in comparison to a traditional framework leads to positive changes in student engagement and performance in secondary mathematics. Although his study was done in two Algebra I classes with students between the ages of 13 and 15 in the United States, his findings are applicable to Canadian classrooms as the classes he researched contained students with a variety of mathematical abilities and needs, similar to what can be found in Canadian classrooms. What Clark discovered was that, although there was not a significant change in student academic performance between a traditional classroom and a flipped classroom, there was an increase in student engagement and communication of mathematics based on his classroom observations, student surveys and interview results. Students in these classes also found that quality of instruction and use of class time improved under the flipped classroom model. He found that students felt that in the traditional classroom setting time is not always effectively used, especially when they cannot focus on teacher instruction: "Some students commented how they were pretending to be involved during the lectures, but were really daydreaming about after school football practice" (p. 104).

One of the ways students in Clark's study found the flipped classroom model fostered an engaged learning environment was this model encouraged student classroom participation compared to the traditional environment, which was based on teacher instruction and allowed for little communication to occur between students: "during the flipped classroom, the students witnessed an increase in their classroom participation and communication, thus promoting a student-centered classroom

environment conducive to learning and success” (p. 103). Working collaboratively in groups helped students engage with the mathematics and enhance understanding. Furthermore, having the ability to view lessons ahead of class and join groups based on their current level of understanding of the mathematical content, allowed students the ability to work at their own pace with the material.

2.2.3. Digital Technology

Boaler (2015) writes, “when we open mathematics, we broaden the number and range of students who engage and do well” (p. 208). One of the things she is referring to is opening mathematics through the use of digital technology, another tool that can be utilized within the mathematics classroom to foster student engagement. Consisting of such things as calculators and computers, digital technology can be imbedded into the mathematics classroom to enhance mathematical content. Virginia Fraser and Joe Garofalo (2015) conducted a research study on fourteen graduates of a technology-rich mathematics teacher education program to see what the affordances of implementing technology in the mathematics classroom were. Participants were interviewed at the beginning and end of the study to see why they were utilizing technology to teach mathematics. One of their findings was that teachers were utilizing technology “to help promote student engagement, questioning, generalization, and conceptual understanding,” (p. 29) while learning mathematics. For instance, dynamic software packages were considered to be fun to use by students and enabled students to be responsible for their own learning. Teachers also felt both students and teachers got timely feedback on student understanding of the mathematical content in this dynamic environment.

Another affordance of the use of digital technology was that it promoted student engagement through multiple representations. Fraser and Garofalo (2015) write, “technology use can help broaden students’ mathematical thinking processes, since it can be used to easily generate multiple representations of mathematical concepts” (p. 29). Having the ability to view mathematical content under multiple representations allows for a variety of learners found in a mathematics classroom to make their own personal connections with the material and connect it to previously learned mathematics: “Technology emphasizes the main qualities of each representation and aids students to more easily draw connections among them (p. 30). This can further help in the problem-

solving process found in the *thinking classroom* as students have further tools to understand the problem for themselves. Such an environment also allows for a new range of problems which may not have been possible to explore in the paper-and-pencil environment. As Goldenberg (2000) clarifies, “with technology what changes is the pool of problems to choose among and the ways they can be presented” (p. 1). For instance, Boaler (2015) discusses how Geometry Pad for iPad and GeoGebra to investigate complex geometric ideas such as trigonometric ratios which may be extremely difficult to do in a non-digital environment.

2.3. Relationship between Engaging Classroom Experiences, Relational Understanding and Student Perception of their Retention of Mathematical Knowledge

The affordances of creating a mathematics classroom environment focused on student engagement are many. According to Boaler (2015), “Studies have shown that active engagement with mathematics increases student interest as well as high achievement and persistence in the discipline” (p. 180). Therefore, by utilizing methods found in Liljedahl’s *thinking classroom* model, the structure of a flipped classroom, and/or digital technology within a mathematics classroom, teachers can not only help students attain higher marks, but can also help them build skills so they are not likely to give up if they don’t understand something the first time. With student interest in mathematics comes the desire for learning for the sake of learning rather than memorizing facts and, consequently, this can lead to a greater understanding and retention of this material. However, how exactly do these engaging tools lead to changes in student perception of their mathematical retention, and what is meant by retention of mathematical knowledge? How these transitions are made will now be examined.

2.3.1. Student Engagement in Classroom → Relational Understanding

In some mathematics classes, students are under the misconception that the ability to mimic teacher solutions will lead them to finding success in the class, as their mathematics class may consist primarily of teacher led note taking. As Boaler (2016) elaborates: “In North America... classroom environments more typically involve a teacher presenting examples while students are expected to sit quietly, watch, and listen, before practicing similar problems” (p.180). At times, it may do so, especially if the

assessments mimic notes delivered by the classroom teacher. However, high achievement on such assessments does not necessarily indicate understanding of the mathematics. Jean Piaget, who is known for his theory of cognitive development, was a Swiss psychologist in the early 20th century. His work in childhood development is still of great value today and he strongly believed that learning did not consist of memorizing procedures. As Boaler (2016) writes of Piaget, “he pointed out that true learning depends on an understanding of how ideas fit together” (p. 18). Piaget accounted students learning new material as the process of going from disequilibrium to equilibrium (Boaler, 2016). He believed that students have unique “mental maps,” which show them how ideas connect together, and when they can clearly understand these connections, they are in a state of equilibrium. However, when they see new material which must be incorporated into their mental maps, they may reach a state of disequilibrium until they can see how this material fits into their current map or until they make changes to their current map to piece in this new information. According to Piaget, it is during this process of going from disequilibrium to equilibrium that is essentially changing knowledge given by a teacher to understanding by a student.

Boaler (2016) writes that to help students go from disequilibrium to equilibrium it does not help to simplify the mathematics and give students repetitive work so they can demonstrate understanding of simple mathematics. This process is memorizing and does not allow students to make deeper connections with the material. Richard Skemp (1978) further distinguishes between two forms of understanding that can be found in a mathematics classroom: instrumental understanding and relational understanding. According to Skemp (1978), instrumental understanding can be demonstrated by students in the mathematics classroom when they can demonstrate “rules without reason” (p. 20). For example, a student that knows that the square root of nine is three, but cannot explain why it is three has demonstrated instrumental understanding. Relational understanding is an expansion of instrumental understanding and can be shown through “knowing what to do and why” (p.20). Going back to the previous example, a student who can explain that the square root of 9 is 3 using such tools as area models of a square has reached relational understanding as they understand the reasoning behind the answer, and not just the answer itself.]

John Van de Walle (2019) elaborates by stating that to go from basic instrumental understanding, where few or no connections have been made, to relational understanding, where there are a magnitude of connections, it is important to make as many connections as possible with prior knowledge that is understood: "Understanding is a measure of quality and quantity of connections that a new idea has with existing ideas. The greater the number of connections to a network of ideas, the better the understanding" (p. 31). To make deeper connections with mathematics, students must be given learning settings which foster risk-taking and making mistakes. Boaler (2016) argues that students need to value making mistakes in mathematics as it is during this process that the mind grows as the student has the opportunity to re-examine the material again and once again enhance their mental map. Furthermore, students need to realize that there is not only one right answer or one correct approach in mathematics, and it is reasonable to have multiple answers to some conceptual questions given in mathematics. Utilizing engaging learning setting in mathematics such as Liljedahl's *thinking classroom*, a flipped classroom setting, or digital technology fosters opportunities for mistakes and ambiguity necessary for learning mathematics for understanding. As Boaler (2016) writes, "This is a big change for many teachers who currently plan the tasks given in mathematics classrooms to ensure student success... they are not getting sufficient opportunities to learn and to grow their brains" (p. 19).

2.3.2. Relational Understanding → Retention

Equilibrium for students is a direct result of using methodology which enables students to engage with mathematics, allowing for more connections to be made with prior knowledge, consequently promoting more relational understanding compared to traditional settings. Van de Walle (2019) states that "connected information is more likely than disconnected information to be retained over time" (p. 32). What retention refers to in mathematical education and how it is enhanced through attaining relational understanding will now be examined.

What is Retention in Mathematics Education?

Before looking at how student perception of their retention of mathematical knowledge changes, it is important to understand what is meant by the term retain. According to the Merriam-Webster online dictionary, one of the definitions given for retain is "to keep in

mind or memory: REMEMBER.” When clicking on REMEMBER for a more elaborate definition, the following is given: “to retain in the memory: remember the facts until the test is over.” As defined, the concept of retention is one that definitely has to do with keeping things in your mind or memory, but what is interesting is the example given to clarify the meaning of remember, “remember the facts until the test is over.” Although this example has nothing to do with the theory of how long knowledge obtained from school should be held for, the popular assumption made in the example is that students need retain knowledge until the day of the test and this is where variance on what the term retention refers to arises in mathematics education.

For some mathematics educators, the ability of students to do well on an exam given right after a unit review shows good retention of material. Others might argue that this is not the case and instead if they can show that they can remember the knowledge gained from one grade level and then apply it to the next, that this shows good retention of material. Boaler (2016) writes:

If you learn something deeply, the synaptic activity will create lasting connections in your brain, forming structural pathways, but if you visit an idea only once or in a superficial way, the synaptic connections can “wash away” like pathways made in the sand. (p. 1)

Engagement → Relational Understanding → Retention

The level of retention a student has of a concept depends on the process in which that concept was learned. As Piaget argued, it is important to give students the opportunity to create new mental maps, thus, similar to a puzzle, all the pieces of knowledge should be connected for a student to create solid understanding. In his research article, Gordon Moore (2002) emphasizes this fact by citing the research of Craik and Lockart from 1972. These researchers believed that how memory is represented in the brain depended on the amount of time that was taken to make connections with the knowledge that was to be understood. As Moore (2002) writes, “The more processing that the material received, the deeper and stronger the memory” (p. 42). Secondly, he states that there are three things which affects the memory of something: how a memory is acquired, how the memory is maintained in the mind and how it is retrieved (p. 42). He states if one of these areas is neglected, this will impact the overall memory, or forgetting the knowledge acquired. Lastly, Moore argues that acquiring the memory in

a unique setting can also help retain it: “Making material distinctive also makes it memorable” (p. 43).

Knowledge is what mathematics educators have students work with in each of their lessons, but it is building student relational understanding that is the goal of each lesson and retention of this understanding, which becomes the long term goal. One of the ways that teachers can foster relational understanding of mathematical knowledge is focusing on how students acquire their memory of knowledge, specifically, using methodology in the classroom that is engaging for students and does not promote memorization of concepts. For instance, when teaching the Pythagorean Theorem in grade 8, utilizing geoboards and digital technology with students so to explore the theorem may allow for a stronger and more vivid memory in comparison to just writing the theorem on the board and working through procedural examples. Boaler (2016) supports this idea in her findings from teaching, with her graduate students, a five-week summer school program to four classes of grade 7 and 8 students, who were dissatisfied with their regular mathematics classes. She focussed on engaging these classes by using the following principles: believe the students are active and capable learners; teach the class through questioning, representing, generalizing, reasoning and justifying; and try to create a collaborative community. The results of using these interventions were positive and Boaler reported that students had an increase in content knowledge and enjoyed the framework of this class compared to their regular mathematics class. These three principles that Boaler (2016) used in her study are foundational to what is found in Liljedahl’s *thinking classroom* framework. By believing that students are capable of thinking and understanding and utilizing VNPS and VRG for creating a collaborative work environment in the classroom, educators are essentially allowing students to build their connections with the mathematical content and create understanding.

Once students have had enough time to process the knowledge in their mathematics classes through engaging teaching methods and create relational understanding of the content, the level of retention of this understanding will depend on the level of connections that were created by the student. John Piel and Michael Green (2010) discuss the problem found in mathematics classrooms of having to spend time to review mathematics from the previous year before looking at new concepts from the current class. They suggest an alternative to this by looking at a charter school in North Carolina, Socrates Academy, which uses a Comprehensively Applied Manipulative

Mathematics Program (CAMMP). Equivalent to elementary education, the Socrates Academy does not spend time having children try to memorize basic facts, but instead, through manipulatives, have them try to concretize their understanding. As Piel and Green (2010) write of Socrates Academy, "They seldom spend time reviewing the previous year's content simply because they do not need to; they have learned with understanding" (p. 73). Thus, it can be hypothesized that if students learn through engaging teaching methods which help increase relational understanding, then the level of retention of the mathematics should increase for the student as solid connections have turned basic knowledge to relational understanding.

2.4. Research Questions

Although research has not directly established the connection between the use of engaging teaching methods in the classroom and retention of mathematical knowledge, I believe there is a connection between these two concepts and what connects these two ideas is relational understanding. By changing a traditional classroom environment to one that is engaging for students, which allows students to collaborate with their peers will not only make the class more enjoyable for students, changing how a memory is acquired, but it will also allow them to understand the content of the class better, and with this new found understanding, I believe it will help students retain the knowledge they have gained. In my research study, I would like to focus in on use of engaging tools found in Liljedahl's *thinking classroom*, mainly the use of mindful notes, VNPS, VRG, and I also plan to incorporate digital technology. My main research inquiry will focus on: Is there a relationship between student perception of their retention of mathematical knowledge and the use of engaging teaching methods such as vertical, non-permanent surfaces, visibly random groupings, mindful notes, and digital technology? Some of the questions I intend to explore include: Will the use of such methodology improve student academic performance in a positive or negative way depending on if knowledge is retained longer? Was there any changes in student perception of their engagement with course content and retention of mathematical knowledge due to the change in class methodology?

Chapter 3.

Methodology

“Research is creating new knowledge.”

- Neil Armstrong

In order to explore the relationship between students' perception of their retention of mathematical knowledge and the use of engaging teaching methods such as vertical, non-permanent surfaces (VNPS), visibly random groupings (VRG), and digital technology, I used my Pre-Calculus 11 classes as the primary setting to implement these teaching tools. My primary method to determine level of mathematical retention was to ask students their thoughts on their retention at different intervals from learning the mathematical content. I also used some of the learning experiences of my former Pre-Calculus 11 students and interviewed them after they had completed both Pre-Calculus 12 and Calculus 12. This chapter will examine: the classroom dynamics of both Pre-Calculus 11 blocks who participated in this study, the changes implemented in teaching style that led to this study, the types of data that were gathered in the study, and the means used for the analysis of this data.

3.1. Setting and Participants

The setting of my research investigation was at King's Cross Secondary School¹, which is an inner-city school located in the lower mainland of British Columbia. Having a population of approximately 1500, the students come from a wide range of socio-economic backgrounds, as the school has a French Immersion Program, which results in a small student enrollment that may not be from the general catchment area of the school. Being an inner-city school, some of the challenges teachers face from students include absenteeism, tardiness, and lack of motivation, leading to poor student participation in classroom activities. One may conclude that a high absenteeism rate may result in students having gaps of knowledge in many subject areas. Since

¹ Pseudonym for secondary school where research occurred

mathematics is a discipline that builds knowledge on previously learned concepts, this may make mathematics a challenging subject for students with high absenteeism rates.

The main participants in my research study were the students in my two blocks of Pre-Calculus 11, whom I taught from September 2016 to January 2017. Additional data was also gathered from my former students who have now completed Pre-Calculus 12 and Calculus 12, but were in my Pre-Calculus 11 class from February 2016 to June 2016. Pre-Calculus 11 is a pathway of mathematics intended for those students who are hoping to pursue mathematics, sciences, or engineering in university, or any other field which may require calculus. Curricular topics in Pre-Calculus 11 include: Radical Operations and Equations, Quadratic Functions and Equations, Rational Expressions and Equations, Trigonometry, Absolute Value and Reciprocal Functions, Systems of Equations, and Linear and Quadratic Inequalities. The course topics have a natural continuity to them and most topics build on previous learned concepts, therefore retention of previously taught topics is one of the keys to student success in the course.

My two Pre-Calculus 11 classes were composed of 25 students each. Block B consisted of 10 male students and 15 female students and Block C consisted of 14 male students and 11 female students. Although the class size for both blocks was the same, the class dynamics were very different. Block B was a very social class, and from my observations enjoyed understanding mathematics in a discussion-based setting. Although most of the students actively participated in the activities in class, many of them did little outside of class in terms of review or homework questions. Conversely, Block C was a quiet class, consisting of more academic students who were used to attaining high marks, and initially expressed they would prefer a “traditional” classroom environment where a teacher delivered the curricular content rather than a cooperative learning environment which was discussion-based. Compared to Block B, they did a lot of work outside of class in terms of reviewing class material and doing homework questions. See Table 1 for a summary of research participants.

Table 1. Characteristics of two blocks of Pre-Calculus 11 classes taught September 2016 – January 2017

	Block B	Block C
Number of Students	25 Students	25 Students
Males:Females	10:15	14:11
General Characteristics	-social class -enjoyed discussion-based mathematics -little done outside of class	-quiet class -preferred traditional mathematics classroom -highly marks oriented

3.2. Shift in Teaching Methods: Is there a relation to retention?

When I initially started teaching Pre-Calculus 11, the course was designed around a traditional mathematics teaching model. Each lesson would consist of a notes portion where students would fill in a skeleton of notes which I pre-made. The skeleton of notes consisted of three main parts: portions where I would go over a mathematical definitions and concepts, “you try” questions for students, and one or two application questions which we would work through together as a class. Throughout the notes portion of the class, I encouraged students to ask questions, and interrupt me if they were confused about what was being discussed or needed me to slow the pace of the lesson. The lesson would end with approximately 20 minutes of class time for students to work on an assignment or homework questions from the text. During this time, I would try to individually meet with students to clarify concepts from the lesson, but due to time constraints it would, at times, be difficult to meet with all the students in the class. Being an academic mathematics class, I always mapped out the lessons for the semester before classes even started, always feeling the pressure of not having enough time to cover all the topics in depth. Summative assessments consisted of quizzes, unit tests and a final exam. Homework question nor assignments contributed to summative assessment but did aid with formative assessment.

This traditional teaching model which I used in most of my classes changed after I took Education 847, Teaching and Learning Mathematics, in the spring of 2016 with Dr. Peter Liljedahl, a course requirement for my master of science degree at Simon Fraser University. It was here I was introduced to teaching mathematics through VNPS, VRG, and mindful notes. Dr. Liljedahl stressed that VNPS, VRG, and mindful notes were tools that could aid a teacher in creating a *thinking classroom* environment. As mentioned in

Chapter 2, VNPS could be any vertical surface that students could write on while standing and have the ability to erase with ease. This could consist of an actual vertical whiteboard, windows in the classroom, or a laminated piece of paper on the wall. VRG consisted of putting members in a class into groups of three or four in a completely randomized process using a deck of cards or any other means of random sorting. It was important that students realize the groups were not pre-selected by the teacher. Students were to use the VNPS and their random group members as tools for mathematical exploration and the teacher was meant to be a guide to monitor the “flow” of the classroom, giving thoughtful hints to those groups that were stuck, and challenging those students who had completed the activity. It was through these explorations that students were to learn and understand various mathematical concepts for themselves rather than just being told by a teacher, step by step, what to do through the skeleton notes. Through discussions with their peers, students were said to be more mindful of the material they were learning as now they were taking ownership for their learning rather than being told exactly what to do by the classroom teacher. Hence the notes they had from the class would be more mindful and would consist of pictures of their work on the VNPS or mini lessons that the teacher would have as a class.

One of the tasks assigned to us in Education 847 was to implement these teaching tools in one of our own classes and observe any changes to our classroom environment. I was initially worried about how well my lessons would go after implementing VNPS and VRG in my classroom. I was concerned that this teaching methodology might not allow me to cover all the content of the course within the set amount of time I had in the semester and I was sceptical of how students would discover concepts on their own without having a teacher show them how the mathematics worked. I felt that I was already doing a good job of creating a mindful classroom, which had an atmosphere of respect where we had active classroom discussions, explorations, and questions were encouraged during notes. I was also very hesitant about using VRG for this discovery process as I felt certain students would not work well together. Whenever I implemented cooperative learning in my classes I tried to make the groups so students of a variety of skill levels would form a group and conflicting personalities were placed in different groups. I felt if a group was formed of only low-achieving students, they would get easily frustrated with challenging tasks, thus not engage in the mathematics. I felt a stronger student was necessary in each group to form the

motivation factor to keep each student engaged in the mathematics. With complete random groupings, I would have no control over creating these groups, thus I felt the motivation to learn the mathematics would decline.

Nonetheless, with some initial reluctance, I implemented VNPS, VRG, and mindful notes in my two Pre-Calculus 11 classes from February 2016 to June 2016. From my very first class, each student would be greeted by me with a playing card. I explained to students that the number on the card would indicate the group they would be a part of for that class. On the front board I would have a series of questions that I took from my skeleton of notes. The questions would get progressively more challenging, and would end off with an application question. At the bottom of the questions I would also write the recommended assignment for the day, hence each group could work at their own pace and when they finished could work on the assignment. Even before the bell rang, students would get into their groups, not waiting for a cue from me, and attempt to work together in their groups to generate a solution to the problems. I would only interrupt the whole class to briefly introduce a new concept, or a question that came up in a group, which I thought the whole class could benefit from hearing the explanation of. I also made a point of reviewing the problems on the board, either myself or with the help of a student, but at various intervals in the class when I know that each group has had a chance to try the problem and found success with it.

I was surprised with how much more engaged with the mathematics my students became compared to the traditional lessons using notes, which I had presumed to be engaging. I was also amazed to see how much mathematical knowledge students were coming into my classroom with. I remember how in the traditional setting I would always review background mathematical knowledge in my notes, which I just assumed students had forgotten from their previous mathematics class. Not only would this take up a large portion of class time, but many students who did not need this extra review were most likely disengaged. Even my thoughts of VRG proved to be false as most students found a means of working well together. It was surprising to see personalities who I thought wouldn't work well together to be engaged in mathematical discussions and, in most cases, groups who got stuck on a problem felt comfortable approaching other groups for hints or suggestions, hence this helped create a more cohesive classroom culture. With a good problem, all students, regardless of skill level, will work together to create a

solution. Some groups may take longer than others, but in most cases they realize that the learning process is not a race.

Along with VNPS and VRG, I also utilized Desmos in my redesigned Pre-Calculus class as a means of enhancing student understanding of the mathematical topics covered, particularly with the graphing component of the course, and to help with student engagement with the mathematics. Desmos, is an online graphing calculator program which I was introduced to by Dr. Sean Chorney in my Education 845 class, another requirement of my MSc. program. In Pre-Calculus 11, Desmos allowed students to utilize technology to not only understand transformations of graphs, but also understand what it means to solve various types of equations as they can use graphs to visually see what solving equations means. In lieu of a traditional final exam, we ended the semester with a final project where students had the option of using Desmos to create an image of their own or import an image and then trace it using various equations. The equations students could use to create their image were not limited to what they had learned in mathematics class that year, they could also use the knowledge they had acquired in previous years or do research on their own to learn new equations that resulted in shapes they did not yet know how to make. My main goal with this project was to have students attain a better understanding of transformations and reinforce domain and range, a mathematical concept my students really struggled with this year. This project was also meant to be an introduction to Pre-Calculus 12 material as some of the equations students experimented with would be formally introduced to the in Pre-Calculus 12.

What I learned from this project was there is definite value in the use of technology to enhance student understanding of mathematics. By struggling though the process of creating their images, students were going through the process of personalizing knowledge on transformations and domain and range. The technology allowed for this struggle to occur without frustration occurring since students could easily experiment with the equations without it becoming a time-consuming process, which it could have become by hand. When I met with individual groups they were able explain the transformations of new graphs, something students usually struggle with. I also liked how the technology allowed students to work at their own pace as each project was unique.

There was a shift in classroom dynamics that resulted in my Pre-Calculus 11 classes due to the implementation VNPS, visibly random groupings, and digital technology. Through this more mindful learning environment which allowed for students to have more ownership for their learning, students were becoming more engaged with the mathematics. I now became curious about the relationship of student perception of their retention of mathematic knowledge and the use these more engaging teaching methods of VNPS, VRG, and technology. Hence, the next semester, September 2016 to January 2017, I decided to continue with the same teaching strategies, and collect data to see if a relationship did exist.

3.3. Data Collection

Data was collected from three main sources in both my Pre-Calculus 11 classes to see if a relationship did exist between student perception of their retention of mathematical knowledge and the use of engaging teaching methods. First, in both classes I staggered my assessments, waiting a few weeks from the completion of each unit before giving the unit test. The results of these staggered unit test data were inconclusive, showing neither a positive or negative correlation. Thus, I then gathered qualitative data, focussing on how students perceived their experiences with VNPS, VRG, mindful notes, and digital technology. Surveys were given to both blocks at the end of the semester to get student opinions of whether they thought VNPS, VRG, mindful notes, and technology helped them retain more of the mathematics that they learned that semester. From these surveys, five students from Block B and nine students from Block C selected for an interview to elaborate on their responses. Furthermore, three former students who had now completed Pre-Calculus 12 and Calculus 12 were chosen for interviews to reflect on their experiences in mathematics over the past two years.

3.3.1. Staggered Unit Tests

In most mathematics classes I have taught or observed, unit tests are given a few days after a unit is completed and a review of the unit is done. I have always questioned what we, as teachers, are teaching students by engaging in this process. Were we suggesting that mathematical concepts only had to be retained for a short period of time, just long enough for the unit test, after which concepts learned can be forgotten? Even when a

final exam is given at the end of a semester, generally a few days are spent reviewing all the concepts taught in the course, and then the final exam is given. Even though I did not agree with this practice, there are two main reasons I did not previously stagger my assessments. Firstly, I found that generally student achievement declined with the number of days that passed between the completion of a unit and the date of a unit test. Secondly, I found students always complained if more than a few days went by before the end of a unit and the date of a unit test. They expressed that they felt that they would forget everything and that it would not be fair to them if the test was given after a few days had passed.

With the implementation of teaching methods that promoted a *thinking classroom* environment, students were becoming more engaged with the content of the class, and, consequently should be making more connections with the mathematics as they were now given the opportunity to explore the concepts on their own and readjust their own “mental maps.” As Van de Walle (2019) suggested, increased connections could lead to increased retention of mathematics concepts. Furthermore, with students now being in a more engaging setting compared to traditional mathematics classroom, students could be acquiring a stronger memory, which is a crucial element of retention as Moore (2002) explained in his work discussed in Chapter 2. If students were retaining more mathematical knowledge by being more engaged with the mathematics, students would not only feel more comfortable with having staggered assessments, as they would feel like they would not forget everything but should also do well on these unit tests. There were seven-unit tests that I gave students throughout the semester and I gradually started to increase the number of days between the end of the unit and the day of the unit test as outlined in Table 2.

As stated earlier, the units in Pre-Calculus 11 have a natural progression to them. Most units depend on knowledge that should be retained from previous units. Several of my unit tests had questions which required students to apply prior knowledge, thus I thought I could also use this as a measure of retention of knowledge over a longer period of time. Nonetheless, as will be discussed in Chapter 5, the results of these tests were inconclusive, showing neither a positive nor negative correlation with student retention of mathematical knowledge. Despite the variance in staggering, the averages of most unit test scores fell into the range that I would have normally had in my traditional classroom setting. Hence, I could not utilize these test results alone to draw

any conclusions on whether the use of VNPS, VRG, mindful notes and digital technology had any affect on student retention of mathematical knowledge. However, student perceptions of what they felt as to be their level of retention could lead to evidence which could assist in inferring solid conclusions. It was here I shifted my data collection to the subjective, focusing now on how students perceived their experiences of VNPS, VRG, mindful notes, and digital technology in relation to their engagement and retention of the mathematics they were learning in the classroom.

Table 2. Number of days between end of a unit and unit test

UNIT	START DATE	END DATE	TEST DATE	DAYS STAGGERED
Unit 1: Factoring Review	Sept. 7	Sept. 14	Sept. 19	5
Unit 2: Radical Operations & Equations	Sept. 15	Oct. 3	Oct. 14	11
Unit 3: Quadratic Functions & Equations	Oct. 4	Nov. 2	Nov. 16	14
Unit 4: Rational Expressions & Equations	Nov. 7	Nov. 24	Dec. 6	12
Unit 5: Trigonometry	Nov. 25	Dec. 9	Dec. 16	7
Unit 6: Absolute Value & Reciprocal Functions	Dec. 12 Jan. 3	Dec. 15 Jan. 9	Jan 19	10
Unit 7: Systems of Equations & Linear and Quadratic Inequalities	Jan. 10	Jan. 17	Jan. 25	8

3.3.2. Student Surveys

Since, the results of the staggered unit tests were inconclusive, my focus changed to obtaining student perspectives on the implementation of VNPS, VRG, mindful notes and digital technology and its relation to their retention of course content through student surveys. The surveys were given the last day of class in the semester, after all assessments were complete. Student were read the Ethics Assent Script before they were given the survey to complete. Students were given an option of opting out from participating in the survey, knowing it would have no affect on their mark. They were also aware that they could leave any of the questions they were not comfortable answering blank on the survey. All 50 students in both classes participated in the surveys which took approximately 15 minutes to complete. The survey consisted of the following questions:

1. How do you best learn mathematics so that you can understand and retain the material?
2. What did you like or dislike about the use of vertical, non-permanent surfaces and random grouping as the main teaching tool in the class?

3. Do you think the vertical, non-permanent surfaces and random grouping helped you retain information for a longer period of time?
4. Was it difficult for you to not write the assessment right after we were finished a unit?
5. How did you feel about having a final project using Desmos rather than a final exam? What did you learn from this project?
6. What was your most memorable experience from Pre-Calculus 11?
7. What would you change about the course?

At the bottom of the survey, students were asked to check a box if they gave consent to be re-contacted for an interview. In Block B seven students consented for interviews while in Block C 13 students consented for interviews.

3.3.3. Interviews

The goal of conducting the interviews was to give students an opportunity to elaborate on their survey responses. Students were only considered for an interview if they had given consent to participate on their survey. When selecting students for interviews, I tried to get a representation of students with a wide range of mathematical skill sets and motivational levels. My reasoning for doing this was to see if there were any common experiences with retention that students with similar or different skill sets and motivational levels experienced based on the use of mindful notes, VNPS, and VRG. More specifically, I wanted to see if there was a shift in knowledge retention, whether positive or negative, based on the use of these new instructional tools. Five students from Block B were selected and nine students from Block C were selected. One student in Block B and five students in Block C were selected for an interview based on the fact that not only were they were actively engaging in classroom activities using the VNPS, but they were also using the VNPS to do their homework or assignment questions. These five students in Block C were interviewed as a group. I wanted to investigate this further to see why students were choosing to do this with no prompting on my part and to see if there was a link to retention.

Interviews were conducted at the end of June 2017, a full semester after the end of Pre-Calculus 11, and took approximately two weeks to finish. The interviews were recorded using the Notability Application on my iPad mini and the length of the

interviews ranged from approximately six minutes to twenty-one minutes. The interviews were conducted in a semi-formal setting, giving me the opportunity to add questions to my original interview script as the interview progressed. Students participating were aware that participating in the interview would have no affect on their mark for the course and they were welcome to say pass to any of the questions they did not feel comfortable answering. Upon completion of the interviews, they were sent to my Dropbox where they were transcribed for me by Cabbage Tree Solutions. It took them approximately one week to transcribe the 11 interviews for me. The following questions were used as possible interview questions at the start of the interview:

1. What do you think retention refers to?
 - 1.1. What is the purpose of retaining mathematical knowledge?
2. How did you feel about having very little notes in Pre-Calculus 11?
3. What were some of the things you liked about the use of vertical, non-permanent surfaces, random groupings, and mindful notes?
 - 3.1. Why did you like these things?
4. What were some of the things you did not like about the use of vertical, non-permanent surfaces, random groupings, and mindful notes?
 - 4.1. Why did you not like these things?
5. How does digital technology help understand mathematics?
 - 5.1. Can you give examples from the course?
6. How did you experiences from Pre-Calculus 11 help in Pre-Calculus 12?
7. Could you explain how you would change $y = 5x^2 + 30x + 41$ to standard form?

To get a better perspective of retention over a long period of time, I also interviewed three former Pre-Calculus 11 students, as a group, who had just completed Pre-Calculus 12 and Calculus 12 in a traditional mathematics setting, as their current teacher taught primarily using pre-made notes which students had to fill out. At the time of the interview it had been a year since they had completed Pre-Calculus 11 and I wanted to see which setting, if any, proved to be better for them in terms of retention of mathematical knowledge. I also wanted to hear there thought of how well prepared they felt they were

for Pre-Calculus 12 after completing Pre-Calculus 11 in a setting that was not traditional. Please see Table 3 for a summary of interview participants.

Table 3. Number of students selected for interviews

	Block B	Block C	Former Students Feb. 2016 – June 2016)
Number of Students Interviewed	5	9	3

3.4. Data

Data utilized in this study came from the following sources: student test papers, student survey responses, recordings of student interviews, and transcripts of student interviews. Test papers came from the seven main units covered in the course, which resulted in 350 test papers. Although the results from these test papers were inconclusive, the examination of these tests helped direct the research study to utilize subjective data. Furthermore, individual test scores were studied to see if a change in student results occurred as the semester progressed and the student became more comfortable with the non-traditional methodology. There were 50 completed surveys, 11 audio recordings of interviews, which resulted in 11 transcripts of interviews.

3.5. Data Analysis

Formal analysis of my data occurred in three parts. First, I broke down the unit test results by letter grade percentages for each class, to get an idea of how well students were doing on these staggered assessments. I wanted to see if the grades declined if there was a greater number of days between the final day of the unit and the unit test or if the marks stayed consist. Although some units' concepts may be more difficult than others, I wanted to see in general if marks were staying consistent and if there was any variance between the two Blocks. From my observations Block B seem to enjoy using the VNPS and working with their visibly randomized groupings more than Block C. Since the results from the staggered unit tests were inconclusive, I then focussed on the student perceptions of their engagement and retention of the mathematical content by analyzing both student surveys and interviews.

I used analytic induction to see if any common themes emerged as I examined the surveys for both blocks. Although responses were interesting for all the questions, I focussed on questions two, three, and five for the analysis:

2. What did you like or dislike about the use of vertical, non-permanent surfaces and random grouping as the main teaching tool in the class?
3. Do you think the vertical, non-permanent surfaces and random grouping helped you retain information for a longer period of time?
4. Was it difficult for you to not write the assessment right after we were finished a unit?
5. How did you feel about having a final project using Desmos rather than a final exam?

From this examination, students for interviews were selected and I next went through the recordings to see if students were voicing similar thoughts between mathematical retention and the use of engaging teaching methods. In particular I wanted to see their thoughts on using the VNPS, working with random class members, using technology and how this effected their retention of the mathematics we learning that year.

Chapter 4.

Results – Cases that Emerged

“The purpose of education is to enlighten the student as to how to use their own mind, rather than to fill it with facts for a test.”

- Unknown

To derive a relationship between the use of engaging teaching methods, such as VNPS, VRG and digital technology, in the classroom and student perception of their retention of mathematics, student experiences in the classroom were explored. As stated in Chapter 3, data on student experiences were gathered primarily through student surveys and interviews. Before analyzing their experiences, I will look at each participant’s personal characteristics, background and feelings towards mathematics.

4.1. Ava, Mena, and Sara

I have been privileged to have Ava in my mathematics classes for three years now. A keen mathematics student, she was promoted to Mathematics 9 from elementary school, thus always a year ahead of her peer group since beginning high school. From my experiences with Ava, her goal in my classes has always been to understand the mathematical content rather than just receive high marks, although she is extremely studious, homework and assignments are always complete, and she has received 100 percent or close to 100 percent in her mathematics classes since grade 8. Nonetheless, Ava is extremely quiet and will rarely contribute to classroom discussion without prompting, which may be partially due to the fact that she was not in her peer group in mathematics class.

Mena, is another strong mathematics student who started her studies in Canada in grade 4 from Sri Lanka. Instilled with a strong work ethic from her early years of study, Mena stated that she felt the expectations for students in Canada were much lower than what she experienced in her classrooms in Sri Lanka. Although Mena enrolled a week late in my Pre-Calculus 11 class, she was able to catch up with the work she missed in the week over a weekend. Being only a grade 10 student, Mena’s main goal for taking the course was to get ahead in her mathematics classes so that she could ease her

course load for her senior years. As she told me coming into the class, she was nervous enrolling into a class of mostly grade 11 students, being an extremely shy and quiet student.

Whereas Ava and Mena were both in my Block B class, Sara was in my Block C class. Another hardworking student, Sara struggled with mathematics in her junior mathematics classes, and took Foundations and Pre-Calculus twice to improve her mark so that she would be recommended to Pre-Calculus 11 rather than Foundations 11. I taught Sara in Mathematics 8 and she was a shy student who always hesitated to ask for help or clarifications on material she did not understand. Initially, when I saw her name on my class list, I was concerned that she might struggle with the course and find the content of the course to be difficult. Nonetheless, I knew she would be committed to doing her best in the class.

The characteristics that all three girls had in common were they were conscientious, serious students, who were not very social and contributed very little to class discussions in a traditional classroom setting. However, Ava and Mena were mathematically strong students, whereas Sara did struggle in mathematics in the past. I was curious how the girls would be in an environment based on mindful notes, VNPS, VRG, and digital technology. Especially in the case of Ava and Mena, as they were not with their peer group, so they might find the class setup to be intimidating. Furthermore, they would not have their direct teacher instruction and meticulous notes to rely on, hence would their understanding and retention of the material be affected in a negative or positive way? Would they now come in for help outside of class, study the material in their own time, or embrace the new methodology?

4.1.1. Acquiring Content through Active Learning

Although with initial reluctance, all three girls embraced the activities of mindful notes, VNPS, and VRG and emerged as natural leaders in the class. These three girls who were always reluctant to share their knowledge during class discussions in a traditional classroom setting, were now working with their classmates, often enthusiastically guiding them towards understanding the problem. Not only were they using these tools to work on class activities, but also homework assignments. Instead of relying on the teacher to go over difficult assignment questions, the girls would work with their peers to see if they

could come up with a solution. These engaging teaching methods were actually helping all three girls with their confidence in working with their classmates. Furthermore, for Sara, her understanding of the mathematical content also improving considerably and she ended off the term with 87%. Ava had a mark in the A range, ending with 98%, as did Mena, finishing with 94%. So it appeared as the girls were able retain their mathematical knowledge, with the staggered assessments combined with the changes made to the teaching model, but how did they feel about writing these staggered assessments? Responding to the survey question “Was it difficult for you not to write the assessment right after we were finished a unit,” they wrote:

Ava: It was quite difficult to not write the assessment right after a unit was finished. This really challenged me to remember everything we had learned from past weeks.

Mena: It was not difficult to write an assessment right after because it was fresh in my mind.

Sara: A bit, but, after it was useful in having extra time to finish our work, and review.

I was surprised to read that Ava, although she did well on her assessments (see Table 4 for full test results), especially with the unit tests that had overlapping course content from previous units (for example there was overlap between Quadratic Functions and Equations and Absolute Values and Reciprocal Functions), found the staggering tests to be challenging. Mena and Sara spoke positively of these staggered assessments pointing out that the extra time allowed for the material to be absorbed, or the memory of the knowledge to be maintained, from further practice, which Moore(2002) indicates to be an important factor for retaining a memory. The rest of their survey responses and interviews brought further insight as to their perspectives on their personal retention and the use of engaging teaching methods in the class.

Table 4. Ava's unit test results

Unit	Days Test Staggered	Test Results
Factoring Review	5	19.5 / 20 = 98%
Radical Operations & Equations	11	24 / 25 = 96%
Quadratic Functions and Equations	14	25 / 25 = 100%
Rational Expressions and Equations	12	25 / 25 = 100%
Trigonometry	7	23 / 27 = 85%
Absolute Value and Reciprocal Functions	10	25 / 25 = 100%
Systems of Equations and Linear and Quadratic Inequalities	8	11 / 11 = 100%

When questioned on the surveys if the VNPS and visibly random groupings helped them retain information for a longer period of time, the girls responded as follows:

Ava: I believe it did, because we were forced to be more active and engaged in our learning. By being engaged in learning, I feel that this has helped us retain information for a longer period of time.

Mena: Yes doing these activities helped me remember the most because I was practicing my skills and at the same time I got different ideas on how to do it as my group members think differently than me.

Sara: Yes, i(t) has helped me a lot. I will remember the information that I had done on the non-permanent vertical surface longer than the notes we do with the teacher. Random grouping has help(ed) a lot because you learn from others mistakes.

The common theme I saw emerging from their responses was the role active learning was playing in the retention of mathematical knowledge. The VNPS and VRG were providing the girls with an environment where they were active during the process of acquiring their memories of course content, having to work through the problems themselves in their groups rather than relying on the teacher to reach a conclusion for them through direct instruction. Although Ava did find the staggered assessments difficult, she did believe that being actively involved in the learning process helped with retention of course content. Eliminating the notes that all three girls initially depended on for leaning mathematics was actually allowing them to understand the material at a different level. Sara summarizes it eloquently in her interview response:

Nikki: How did you feel about having very little notes in Pre-Calc 11?

Sara: I actually loved it...

Nikki: Really? OK

Sara: ...because I felt like I taught myself and for that reason that I kept a lot in my brain rather than just looking over the notes several times.

Nikki: Yeah

Sara: It just kind of... I looked back and I was like oh OK. I did it like that.

Nikki: Yeah

Sara: And then I did notes I'm just listening... I'm not even listening to the teacher. I'm just copying.

Perhaps this is why Sara had struggled with mathematics up to this point in high school. She was a hardworking student, but she had not been given an opportunity to engage with the mathematics in an active manner so that she could really understand the content and retain the memory of how she acquired this knowledge. Now, as she stated, she was teaching herself, thus making personal connections.

Another way the girls were being active in their learning through this new class structure, was they were given an opportunity to relay their understanding of the mathematics to their group members in a safe environment which in turn solidified their understanding of the mathematical content. As Ava writes on her survey:

I also liked being put in random groups because when my group members didn't understand, I could explain everything. This not only deepens the understanding of my group members, but also myself by explaining the process.

Not only were they relaying their own understanding, but the VRG was also providing them with different perspectives of approaching the mathematics through their group members, hence allowing them to make connections with the material using the method that best connected to their prior mathematical skills. As Van de Walle (2019) states, "When learning in mathematics establishes a rich set of connection, there is much less chance that the information will deteriorate; connected information is more likely than disconnected information to be retained over time" (p. 32).

The final Desmos project also gave the girls an opportunity to continue to be actively involved in their learning, something a final exam might not have allowed. As Mena discusses in her interview, Desmos allowed for a visual representation of

transformations. By allowing her to plug in various parameters to see how the graph would shift, she became more actively involved in the learning process and understand the graphing process better:

Mena: That's the only thing but I really liked using Desmos because it helped us visualize the graphs even for test practice. When we plugged it in, we could see that this is what the graph would look like and then it's like more visual learner because I'm more oral and visual learner.

Nikki: Right.

Mena: I like to see stuff that I learn.

Ava found the digital project allowed her to use what she had learned in class for a longer period of time rather than on a typical test setting, hence a longer period to obtain validity that concepts were being understood:

I enjoyed having Desmos as a final project... we were using our knowledge for a long period of time, rather than a short period of time on a test.

4.2. Sabina and Isabelle

Although in opposite blocks of my Pre-Calculus 11 classes, Sabina and Isabelle shared many similar qualities. Both girls strived towards attaining high marks in all subject areas including mathematics, and were extremely nervous of achieving anything less than an A in any class. It appeared they felt a lot of pressure from home to attain high marks, especially in the case of Sabina whose brother Ali, whom I taught Foundations and Pre-Calculus 10, was in the same block as her and was an extremely strong mathematics student. Despite the pressure to achieve perfection, both girls were quite social and well respected by their peer group. I had taught Isabelle, a French Immersion student, in Mathematics 8, but had never taught Sabina, and when I explained my teaching methodology to the class of very little direct instruction, only mindful notes, and the onus of learning falling upon students, both girls appeared nervous.

Although both Sabina and Isabelle appeared to engage with the activities with VNPS and VRG, what I noticed in my initial observations of them was their desire to achieve a high percentage in the class came at the expense of fully understanding a topic at times. They were not always striving to achieve relational understanding. They

were not comfortable with making mistakes and learning from them, which took away from the purpose of using VNPS and VRG as learning tools. On their first assessment based on material in Pre-Calculus 11, radical operations and equations, Sabina received 66% and Isabelle received 48%. For two students who were not used to receiving anything less than an A, both girls were shocked. When Isabelle received her test she asked to speak with me outside of class and when she spoke she immediately started crying as she had never failed a mathematics exam. It was here where I saw the pressure she was putting herself under. She was not enjoying the process of learning but aiming at achieving a number. Similarly, I received a call from Sabina's mom who reinforced that she was a good student just like Ali, and urged me to help Sabina. To ease the pressure of the numbers game, I told both girls not to worry about this one assessment and how we would focus on the bigger picture and to see how the term progressed with the remaining assessments.

4.2.1. Changing Mental Maps through Creating Connections

As the term progressed, Isabelle started to work more actively with her group members towards understanding the mathematics, rather than memorizing the mathematics, which I felt both girls had been doing up to this point. On her next assessment on quadratic functions and equations, she received 88% despite the unit having more content than the previous one and the fact that I waited 14 days instead of 11 before the end of the unit and the date of the unit test. For the rational expressions and equations test, I staggered the test approximately the same as the quadratics test, 12 days, and she achieved an even higher score, 92%. The final three tests were staggered with fewer days, and surprisingly Isabelle's achievement went down a bit compared to the tests that were staggered with a greater amount of time. This shows that the number of days in between the test did not necessarily decrease her retention of the material. Isabelle ended the term with 82% (see Table 5 for full test results), and it appeared that by embracing relational understanding of mathematics through VNPS and VRG rather than memorizing it, her retention of the content was increasing.

Table 5. Isabelle's unit test results

Unit	Days Test Staggered	Test Results
Factoring Review	5	17 / 20 = 85%
Radical Operations & Equations	11	12 / 25 = 48%
Quadratic Functions and Equations	14	22 / 25 = 88%
Rational Expressions and Equations	12	23 / 25 = 92%
Trigonometry	7	22 / 27 = 81%
Absolute Value and Reciprocal Functions	10	20.5 / 25 = 82%
Systems of Equations and Linear and Quadratic Inequalities	8	11 / 11 = 100%

When asked if the VNPS and VRG helped her retain information for a longer period of time Isabelle writes on her survey:

Yes. The memory of doing the work and working through it stays in my head longer than it would have if I were to do the work on paper or if I were simply just given the information.

Isabelle makes an important point here. As was discussed in Chapter 2, Moore (2002) argued that how a memory is acquired has a direct impact on how a memory is retained. Isabelle clearly indicates that through utilizing VNPS and VRG the memory was staying with her for a longer period of time. Isabelle elaborates in her interview:

Nikki: So, how do you feel about having very little notes in Pre-Calc 11

Isabelle: It is very different, very different from what I was used to. It's kind of hard, like I know it was easier to remember when it was on the board, but sometimes I kind of forget that I needed to have it on the cover, to be able to look back on, but there were something new, but the photos really helped as well.

Nikki: Yeah.

Isabelle: And like being able to do it yourself kind of helped me, personally, to remember how to do some stuff...

Nikki: Yeah. So, my next question what were some of things that you like about using the vertical, non-permanent surfaces or the whiteboards? And what were some things you did not like and explain why and you can be completely honest.

Isabelle: So, okay one thing I liked is that how we were like doing it ourselves not you teaching us. Sometimes, we kind of get side tracked when you are kind of teaching us like you

know play with our pencils, erasers, but when we are on the board doing it ourselves we kind of like do it like, what do you call, I forgot the word. One sec. I forgot Ms. Mann.

Nikki: Okay, that's okay. But, you feel like you were doing it yourself so that helps?

Isabelle: Mmmhmm.

As her assessments were clearly showing, Isabelle was also able to reflect on the experiences of working with VNPS and visibly randomized grouping and realize that the retention of the material was occurring because she was not focussed on memorizing a correct way to get the answer but instead was creating her personal connections with the material. Her response to whether it was difficult to write staggered assessments clearly indicates this:

It was a little bit but I've learned to do it that way. It also helped me not lose the information I just learned.

Isabelle was also embracing learning mathematics for the sake of learning rather than achieving a certain mark, as she indicated on her survey response on using Desmos for a final project rather than a final exam:

I think the final project is so much better than the exams because it still shows what was learned throughout the course and doesn't restrict us from learning even more.

Sabina's experience with this new classroom methodology was not as positive as Isabelle. Although Sabina ended up with 83%, her assessment results were not as consistent as Isabelle's test results. See Table 6 below for full results. Sabina was still focussed on using her experiences with VNPS and visibly randomized groupings to memorize rather than understand the content, hence not allowing for retention of the material and the variance in test results. As she expressed in her interview:

So yeah, I like notes. I like to like study notes and memorize it so I can know how to do or solve problems.

Table 6. Sabina's unit test results

Unit	Days Test Staggered	Test Results
Factoring Review	5	12.5 / 20 = 63%
Radical Operations & Equations	11	16.5 / 25 = 66%
Quadratic Functions and Equations	14	23.5 / 25 = 94%
Rational Expressions and Equations	12	15.5 / 25 = 62%
Trigonometry	7	25 / 27 = 93%
Absolute Value and Reciprocal Functions	10	21 / 25 = 84%
Systems of Equations and Linear and Quadratic Inequalities	8	11 / 11 = 100%

Despite the fact mindful notes were only given throughout the term, and having success on some of her assessments, Sabina still favoured having traditional notes which she could memorize as she felt this helped her learn and retain the material. Furthermore, although she was participating in the class activities, the pressure of doing well was still not allowing her to just work towards understanding with her group members:

Nikki: How did you feel about using the whiteboards and working in groups and learning math that way rather than the traditional way with notes?

Sabina: It depends on the group because sometimes when I group with like average people like below average so like when we're doing like a problem then maybe they give you like wrong answers and stuff and we don't... I don't really learn from that so I like the old way.

Nikki: OK. What were... So that's one of the negative things about the vertical boards. Was there anything positive about working on the boards with, or the groups?

Sabina: Like teamwork, teamwork was really big thing and then also like you have a voice so if you like don't get this then you guys can like discuss it and like come up with a solution for it.

Even though Sabina realized that value of working in groups and utilizing her own thinking to derive a solution, she contradicts herself by suggesting that working with "average" or "below average" was affecting her learning and retention of the material. She contradicts herself yet again responding on to the survey whether VNPS and random grouping her retain information for a longer period of time:

Yes, it was a good way to remember and retain the information.

Consequently, it can be concluded that the value of retention gained for utilizing VNPS and visibility randomized groupings can only be seen if the student is willing to value creating relational understanding.

4.3. Jake and Sonia

Jake was a French Immersion student in my more academic Block C class and was generally an A or B student. I had taught Jake in Foundations and Pre-Calculus 10 and mathematically he was a quick learner and did well in the traditional mathematics class setting. He had a busy schedule, as he was in a competitive sailing program and was attempting to graduate a semester earlier, therefore taking Pre-Calculus 12 the semester after completing Pre-Calculus 11. Furthermore, he relied on his class time to learn most of the mathematics as he had little time outside of class to practice the concepts. Sonia was in my Block C class and, like Jake, did well in her academic classes. I had not taught her before, but from the first few classes I could tell she took ownership for her learning and immediately asked for help if she was confused with a concept and also came to afterschool math tutorials to attain extra help.

When placed in VRG, Jake always emerged as a natural leader. He would always take lead with the pen in his group, and always slow the pace down so that everyone in his group could follow, something he did struggle with initially as he would always want to finish the problem quickly. Sonia was similar, though a little more hesitant taking the leadership role, but always helped organize her group to keep them on track. What I noticed about both students was they were okay with wandering to other groups and getting help and also giving hints when they could see a group was stuck or struggling. They were both okay making mistakes and learning from them.

4.3.1. Affordances VNPS and Mindful Notes for Acquiring a Strong Memory

Jake and Sonia's survey and interview responses showed their perspectives of the affordances that VNPS and VRG have to offer to the retention of mathematical knowledge. In response to whether VNPS and random groupings helped retain information for a longer period of time both students responded as follows:

Jake: At the beginning of the course I would memorize material, after learning this was not an effective method I started trying to get a deeper understanding of the work which was greatly helped by the whiteboard and groups.

Sonia: The fact that we got to work on a bigger surface helped as well as the fact that it was easier to erase mistakes on the whiteboards. Random groupings helped at times when I was grouped with people who actually worked and people who knew what they were going.

As suggested earlier, Jake verbalized one of the key affordances of using these tools for acquiring a stronger memory, which was enabling understanding rather than promoting memorization of mathematics. Furthermore, as Sonia writes, the VNPS give students a larger surface where students can work out solutions and erase as necessary. If using pencil and paper, the flow of ideas cannot be written as easily as erasing is not as simple and the size of the paper confines the amount of writing one does. As Sonia explains in her interview:

I found out that I like being able to easily change my answers compared to paper. It's easier for me to learn on something that's not as – Because when you erase mistakes on paper like it's there, but then on the whiteboards it's really easy to erase and it's big. I'll be able to focus on like one question at a time.

Jake had a mutual feeling towards this large work place:

Yeah I really like getting to write on the boards. I think that it was really helpful to visualize and you have a lot of space to draw stuff on. You can always exit or not exit but like walk away and come back and it will still be there and you could erase it and use that room very well. I wasn't a big fan of the random grouping, but I understand the concept because it's definitely kids who needed the extra help from the different groups. So I think that that was good instead of having the same groups every time over and over.

Here Jake highlights the role collaboration between groups played in this atmosphere. The knowledge students attained from collaborating with other groups could easily be applied to their own solution with the VNPS, hence concretizing their own understanding. Although Jake and Sonia did not seem to account VRG to assist in the retention process as much as VNPS, stating negative point of grouping to be that some people did not contribute as much to the group, both know there is value in collaboration. As Sonia explained to me:

The part that I like was that I was able to get different insights from everyone, and everyone had different ways of solving the problems... good seeing different perspectives and seeing different ways to solve it because not everyone solves it the same way. And the part that I didn't like was that some people didn't contribute as much.

Having only mindful notes in class also had its affordances for retaining mathematical knowledge. As Jake rationalizes in his interview, having no notes forced him to create his own notes which he was more likely to understand and retain:

Nikki: Okay how did you feel about having very little notes in Pre-Calc 11?

Jake: It was a difference from what I'm used to, but I think that it was good. It was definitely made you visualize and think about it more and go home and write down my own notes if I have the time.

Nikki: Okay so you actually did create your own notes.

Jake: Yeah when it was difficult to understand some of the concepts, I would go home and do that.

Nikki: Okay and how has your experience in Pre-Calc 12 been in comparison? Are you doing similar things or has it been different?

Jake: It's more lecture-based.

Nikki: Okay.

Jake: My teacher will talk and we'll take notes, and we receive all of our notes at the beginning of the year so we just flip through the booklet.

Nikki: Okay.

Jake: And we do our homework with 20 minutes in class whatever you don't finish you hand in after.

Nikki: Okay and how has that experience been compared to what we did in Pre-Calc 11?

Jake: I feel that it's easier to go back on my notes, having the notes there. But I preferred having my own notes to write because I found that easier to follow along with and easier to remember when I had to go back and find the information.

Sonia also held similar views. Being an immigrant, she points a key difference she noticed in mathematics, which was in her schooling in Canada, she felt as she was just

given the answer. However, using the new methodology, she was learning from her mistakes and retaining that experience so she would not make similar mistakes in the future:

Nikki: How do you feel about having very little notes in Pre-Calc 11? So compared to traditional math classes we had very few notes, how do you feel about that?

Sonia: It was definitely a big change because I'm used to like a lot on the notes because I noticed especially since I'm emigrating from another country, everything here is kind of spoon-fed. They give you a lot of notes, and I was like it's a new way for me to learn. It took kind a bit to adjust, but it was a good learning process because it's more like hands-on and it actually got me to think and then every time I messed up, I remember it so I will not make the same mistake again.

4.4. Shawn

Shawn's experience with VNPS and VRG brings an interesting perspective to the role these tools play in the retention of mathematical knowledge both through how mathematical knowledge is acquired and maintained. Having a quirky personality, anytime there was direct instruction in the class, Shawn would always lose focus unless that direct instruction happened while the class was standing up near the boards. His perspective on the mindful notes used in my class was:

I... honestly, I don't usually look at notes anyways so. It's kind of convenient for me.

Shawn was not an academic student, but what I witnessed with the use of VNPS and VRG was that he valued learning for the sake of creating relational understanding. He was always actively participating, questioning, giving his opinion and striving to understand the "why" behind everything. It was quite remarkable the change in personality that occurred with Shawn when he was at the desk compared to when he was working on the VNPS. When asked about his experiences using VNPS and VRG, he highlighted this change he felt:

Nikki: OK. And how did you feel about the use of the whiteboards in my class? And you can be completely honest. What did you like about it? What did you dislike about it? And completely honest.

Shawn: Alright, I really liked the fact that it was so easy to erase and all the work was shown up there and we can just take pictures

of it if needed. What I didn't like is how some of them were like really like broken down like they were just like... they were used a lot and...

Nikki: Yeah

Shawn: ...the ones that... the best ones were to get were the windows or the actual whiteboards because like the post it note things, they were a little sketchy but that's like the only complaint. I found that working on them because it's so easy, just be like oh, I made a mistake here and wipe half the thing off.

Nikki: Right

Shawn: It's not like... and it's just fun. You're just like standing... it's a different air than just sitting down with a pencil and just writing it.

Like Sonia and Jake, Shawn saw the affordances of being able to work at a large space which was easily erasable, but he also mentions how he enjoys learning in this way. By the simple act of standing, the classroom dynamics changed for him. Now learning had become “fun,” whereas, before he struggled to focus on the material. Jakes memories of mathematics class were now being acquired in a positive way, which could enable him to retain them for a longer period of time, but this did not occur.

4.4.1. Maintaining a Memory

Despite Shawn actively engaging with the new teaching methodology, his final mark in the class was only 50%. Why was Shawn having difficulties retaining the material, despite his high engagement with the material? On his survey when asked if the VNPS and VRG helped him retain the material for a longer time, Shawn responds:

Quite possibly, however the lengthy periods between the end of the unit and the test made the retention disappear.

Hence, Shawn felt the length between the assessments was affecting how well he was doing on the assessments and not the actual teaching methods. He elaborates further on the survey:

Once a unit is done, I focus more so on the current and pay little mind to the past. Trying to juggle both was a challenge and I believe this is why I failed.

However, there may have been other reasons, besides the introduction of new material before the assessment that may resulted in him doing poorly. When it came time to practice the concepts learned, Shawn usually did very little, unless again he was with a group and at the board. As Moore (2002), stated, how a memory is maintained will affect the retention of the memory. Shawn takes ownership in his interview:

Shawn: With Pre-Calc 11, I think... what really got to me was the fact we had to wait that two weeks before writing...

Nikki: Yeah

Shawn: ...or a week or two. But I think that's just me and refusing to study because I'm really bad when it comes to studying.

Nikki: Right

Shawn: I'm super lazy. I didn't even study for the final.

Nikki: Right

Shawn: Yeah. And the trigonometry. I'm like how do I do all this.

Nikki: Yeah

Shawn: So, yeah, that was like honestly other than that, I don't think. It's just me being stubborn and refusing to take ownership. I think more so rather than anything wrong with the class.

Nikki: Yeah

Shawn: The class was pretty good looking back on it. And...

Nikki: So, you did like the class?

Shawn: Yeah, yeah, I did, yeah, looking back on it.

Although Shawn did very little outside of class, based on his activities in class he was successful in retaining enough material to pass the class. Furthermore, Shawn, similar to Isabelle, by the end of the course found value in exploring mathematics for the sake of learning new mathematical concepts, as he expressed was an affordance of utilizing Desmos for a final project rather than having a final exam:

I love it. I feels as if things are coming back to me. It also allows us to explore new ideas.

4.5. Kam, Lila, Mark

Kam, Lila, and Mark were among the first students that experienced the changes in teaching style in my Pre-Calculus 11 classes, and, at the end of the 2016 – 2017 school year, had just completed both Pre-Calculus 12 and Calculus 12 in a more traditional classroom together. All three students were strong academic students and were driven to excel. Not only did they excel academically, but they were well rounded, pursuing extra-curricular activities both in school and in the community, along with being leaders of their graduating class. All three students had received acceptance in university programs, Lila and Mark in the Lower Mainland and Kam in Ontario, and planned to take Calculus as one of their first year university courses. As stated earlier, when describing the methodology used for this research study, I was hoping to compare these students' experiences in Pre-Calculus 11 with mindful notes, VNPS, VRG, digital technology to the more traditional atmosphere they experienced in their grade 12 year to see their perspectives on whether they felt one methodology lead to better mathematical retention compared to the other.

4.5.1. Acquiring a Memory in a Traditional Class versus a *Thinking Classroom*

I interviewed all three students together and I was quite surprised to discover that Kam and Lila had initially wanted to withdraw out of my Pre-Calculus 11 class as they felt my teaching methodology, which was new to all three students, would affect their understanding of the course content. Kam and Lila explain their feelings towards VNPS, VRG initially, and as time progressed:

Nikki: Yeah. Thinking about — It's been a while. It's been a year since you guys had Pre-Calc. 11. Thinking back to your experiences in Pre-Calc 11, what were your thoughts on the use of vertical surfaces and the random grouping? How did you feel at the time? You can talk about the positive aspects, negative aspects, but make sure you tell me a little bit of why you feel that way. You guys can start in any order. It doesn't matter who starts.

Kam: At first, I actually hated the idea. I was honestly thinking of switching out of the class. I'm like 'It wasn't what I was used to.' But I think, after a while, I actually really liked it more than just...

Nikki: You liked it after.

Kam: More than just what we do. What we're doing now is just the teacher does notes and we copy them down and stuff, and then I just get really bored especially now. Before, I used to like it because I used to just copy down and listen to lessons but I like just learning it by myself now after being in this class in Pre-Calc. 11. I think it'll be really good for university because that's kind of how they work. It's more you kind of do your own kind of thing - independent study.

Nikki: Okay. That's an interesting point you raised. Initially, you didn't like it and you didn't want to — You wanted to switch out of the class. But now, you're finding that you're bored in Pre-Calc. 12. Are you finding this too easy or is it because you're not actually, you find, using your mind?

Kam: I feel like I'm not very active. I'm just prescribed what to do is the...

Lila: I feel like we're less engaged.

Kam: ...not learning. I still learn the concepts.

Kam: I still learn them but I don't retain them as well as I did. I feel like I remember more from Pre-Calc. 11 than I do from Pre-Calc. 12 and Calculus 12 now.

Nikki: Really? Because —

Kam: Even though Pre-Calc (12) was just a couple of months ago in the fall and Pre-Calc. 11, we finished almost a year ago.

Nikki: ...Do you feel the same way?

Lila: Yeah. In Pre-Calc. 12 test — In Calculus test, I feel like every — The date that we learn the lesson, I feel like I'm not retaining the knowledge. I'm just going along with the notes. But then, after every test — The day before the test, 2 days we were reviewing for the test, I'm basically doing what we learned in grade 11, the same technique where we're learning on our own basically.

Both Kam and Lila bring up some interesting outcomes of mathematical learning and retention without direct teacher instruction. Although both students were reluctant at the start of Pre-Calculus 11 without direct instruction, they found it difficult to go back to traditional teaching in Pre-Calculus 12 and Calculus 12 after their experiences in Pre-Calculus 11. Lila even mentions using the methods she learned in Pre-Calculus 11 before tests so that she can learn the material the teacher was delivering in the direct notes. Furthermore, Kam mentions, he felt bored being “prescribed” what to do whereas Lila mentions being less engaged. With the lack of engagement both students felt they

were now retaining less, which was surprising considering Kam ended up with 100% in Pre-Calculus 12, the exact same mark he achieved in Pre-Calculus 11 and Lila achieved 94%, compared to 86% in Pre-Calculus 11. Thus, it can be concluded that how students were acquiring their memories of their lessons in mathematics class had a direct impact on how they perceived they were retaining the course content. Being more engaged while creating connections with the knowledge impacted how students felt they were retaining the knowledge from the course.

Based on Kam and Lila's experiences it seems retention might not always have a direct correlation to assessment results in mathematics education. The students provide insight as to why this may be:

Nikki: It's interesting though. Your marks, I would say are still pretty high because I've asked her your marks. But you're feeling like you haven't learned as much? Is that true or is that...?

Mark: It's just that it's harder to kind of memorize — It's just kind of harder to retain stuff rather than in Pre-Calc. 11 where it's easier because you have different ways. You can see different ways of doing the question whereas this, you kinda just have one and then you stick to that.

Nikki: That stuff? The same stuff?

Kam: We just memorized the way a teacher does it for the test that we get perfect on the question and then, after that, we just forget.

Lila: It's more doing it just for the test instead of doing it for ourselves.

All three students highlight an important affordance of utilizing mindful notes, VNPS, VRG, and digital technology which was not always possible in the traditional classroom. By allowing students to explore different solutions to questions rather than just giving one method, this enabled student to create their own connections and learn mathematics for the sake of creating personal connections rather than just memorizing the content for an assessment, which does not always enable retention. In the case of the traditional classroom, all three students were learning for the test and achieving a mark which doesn't always result in retaining the material for a long period as the students voiced:

Kam: No. We don't use the boards at all. We just use our papers. She gets us the notes and then if we have questions, we go ask her and stuff but it's not really... It's more like we're not really learning. We're just 'How did you do it?' to the teacher.

Lila: Yeah.

Kam: I'm going to copy the same way and you do it.

Nikki: You don't learn?

Kam: I don't learn my own way. I just learn whatever the way the teacher thinks on that and then, I just memorize and use it on the test and forget it afterwards.

Lila: I forget everything after each test. My mind just wipes it out.

To see how much the three students retained from Pre-Calculus 11 I had the students utilize the VNPS to graph the equation, $y = 5x^2 + 30x + 41$ during the interview. Students had created their own methodology on how to change a quadratic equation from standard form to vertex form in class, rather than just being "prescribed" how to complete the square. All three students worked well together and were successful in creating a solution, even though a year had passed since they were in Pre-Calculus 11. See Figure 2 below.

I had students reflect on this experience, upon solving the problem:

Nikki: How did it feel after a year working on the board as a group? Was it weird?

Kam: Like it was refreshing.

Lila: Yeah. I think it helps.

Kam: It felt really easy.

Lila: Yeah. It was a lot easier.

Mark: Completing the square. That process was kind of embedded into us, how to do it and it was used throughout the whole course kind of so we kind of retained it really good.

Nikki: You retained it extremely well.

Lila: It's actually how to do it ourselves at first, right? Because the whole whiteboard kind of thing is just like learning by ourselves. First, figuring it out. And then, I think doing that, we retained it a lot more.

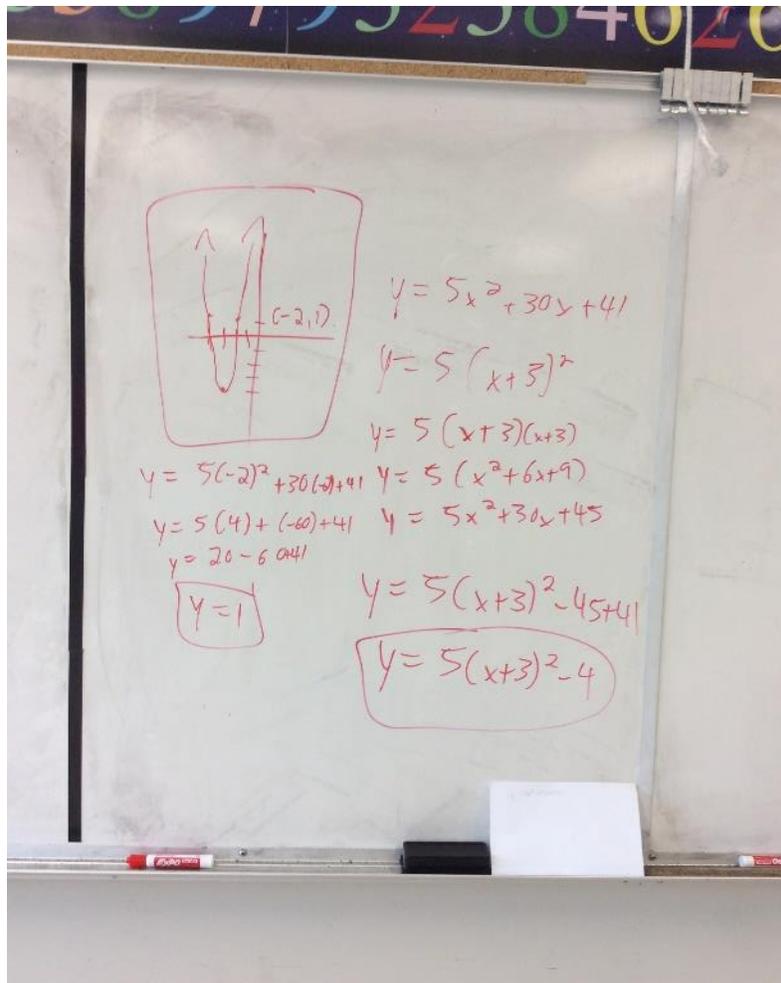


Figure 2. Kam, Lila, and Mark's solution to changing a quadratic to vertex form from standard form

Again, all three students not only enjoyed the experience of working on the VNPS, but realized the retention was a result of making the personal connections with material, which was reinforced throughout the entire course. All three students agreed at the end of the interview that although initially intimidating, utilizing mindful notes, VNPS, VRG, digital media increased engagement with the mathematical content. Through this engagement came a memory that was retained longer because the manner in which it was created. It allowed for greater personal connections and consequently greater relational understanding.

Kam: I think this format of teaching will be very valuable for Math ... I think for the STEM fields, this would be a really valuable way of teaching and I think the profession should go more towards that. I think at first, students will be a little bit scared of it like I was.

Lila: I agree. Yeah.

Mark: That's what I found too.

Kam: I still remember, I think Lila and I went to see Ms. L

Lila: Yeah. We did.

Kam: ...on the first day and we were like 'This teacher's crazy. I don't know what kind of idea she has.'

Lila: We should probably leave this class.

Kam: We're committed to your class now. But we were forced to. But I'm happy that we were forced to because we were able to learn a lot and we really enjoyed the class.

Chapter 5.

Results – Cross-Case Analysis

“... a freeman ought not to be a slave in the acquisition of knowledge of any kind”

- Plato, The Republic (2016, p. 282)

In the previous chapter, selective students' experiences with the introduction of such engaging teaching methods as VNPS, VRG, mindful notes, and digital technology in the classroom were explored. This was the first time most of the students in the classroom were experiencing mathematics in a non-traditional and more mindful manner. Student experiences were examined to see if there was a relationship between the use of engaging teaching methods and student perception of their retention of mathematical knowledge. In Chapter 4, students' experiences highlighted key factors that may have led to a greater perception of retention by students as a result of learning in this manner. Those factors included: engagement, collaboration, active learning, creating personal connections and relational understanding, and maintaining mathematical understanding through practice. Although a majority of the students in each of the cases varied in skill and motivational level, it is important to examine if there were any common experiences with perceptions of retention of mathematical ideas that these students experienced across the different cases.

5.1. Active Learning in Mathematics: Vertical, Non-Permanent Surfaces and Mindful Notes

In many traditional mathematics classes students rely on teacher directed notes as a tool to foster understanding of mathematical content. It is expected by many teachers as well as, surprisingly, students themselves, that all the content displayed on a projector or whiteboard must be copied down by the student, with little alterations, for mathematical understanding to be achieved (Boaler, 2016). The method demonstrated by the teacher is considered to be the correct approach to attaining a solution to a problem, hence can be considered by many students to be the only method to obtain the solution. At times, students become so stressed in the classroom at getting everything copied down, they become unaware of the actual mathematical content that is being discussed (Boaler,

2016). Utilizing such a teaching strategy in the mathematics classroom gives few opportunities for students to actually engage with the mathematical ideas for themselves, and, instead, it becomes a practice of memorizing the mathematical content by the student primarily for assessments. In contrast, VNPS is a tool which promotes active learning and, consequently, allows for a stronger memory to be created in comparison to a traditional classroom setting. This section will explore the main characteristics of VNPS which may allow for a greater level of engagement and consequently a stronger memory.

5.1.1. Vertical Space: Standing Versus Sitting

Initially, students were reluctant about the use of VNPS as a learning tool. This was because it forced them to leave the comfort of their desk, chair, and, in many cases, friends, and stand at a vertical surface with a random group of three or four students, whom they may or may not work with normally. This can be perceived as a daunting task for many students as not only are you standing, but you are also expected to contribute to active mathematical discussions, which can be intimidating, as some students may feel their mathematical skill levels may not be at par with their peers. Nonetheless, after implementing this teaching strategy in my Pre-Calculus 11 classes, student interview and student survey responses alluded to numerous affordances of the use of this strategy in the classroom.

Usually when students are standing in the classroom there is a general feel of greater energy in the room compared to if they are sitting at their desks. The energy does not necessarily change if they are sitting in groups at their desks and expected to engage in group discussions. This shift in energy can be especially witnessed during morning classes, when students are generally more tired and inclined to allow their mind to wander from the mathematical ideas being discussed. Moe who was in my block C class and Ranjit from my block B elaborate on this idea in their surveys:

Moe: The use of the vertical surfaces makes me less drowsy since you have to stand and help your groupmate/s solve a question.

Ranjit: I liked using the non-permanent vertical surfaces because they require me to get up and think and work with others to solve questions.

Hence, by simply using a vertical space, the “drowsy” feeling which can inhibit thinking can be reduced, and students are instead out of their seats, working with others with the goal of attaining mathematical understanding. Shawn articulates it best in his interview:

Shawn: ... it's just fun. You're just like standing... it's a different air than just sitting down with a pencil and just writing it.

As discussed in Chapter 4, Shawn was a student who was not usually actively engaged in mathematics class, but once he was out of his seat, he instantaneously became interested in the content and was contributing to his group. Thus, it can be concluded the vertical aspect of VNPS is an important feature which contributes to student engagement, and subsequently a stronger memory of knowledge discussed in class.

5.1.2. Permanent versus Non-Permanent Surfaces

In many traditional mathematics classes students demonstrate most of their learning on permanent surfaces such as lined paper, graph paper or even chart paper. By utilizing such materials, students are able to keep a permanent record of their learning and the new ideas they have learned in class. By having a permanent record, it may be assumed by both the teacher and students, that this will aid in the retention of the new material as students can refer back to this record periodically to make personal connections and attain relational understanding. Nonetheless, continuously re-reading and memorizing a procedure, especially prior to a test, is not a good example of how to retain mathematical knowledge (Boaler, 2016). This could measure how well you can memorize mathematics ideas rather than try to achieve relational understanding of the mathematical content. There is value in writing down mathematical ideas, nonetheless, a student should write down their understanding of the content once they have had the opportunity to engage with the material for themselves. It is during this engagement process that students have the opportunity to make personal connections, which may lead to greater retention of the material. As Van de Walle (2019) writes, “... constructing knowledge requires reflective thought – actively thinking about or mentally on an idea” (p.29).

By utilizing a non-permanent surface such as a VNPS, students are given a tool to engage with mathematics without the “fear” of making mistakes as the surface is easily erasable and, consequently perceived as a space where one can brainstorm. When using pencil and paper many students may, at times, be reluctant to try different

approaches as it is more difficult to erase and leave a clean surface, especially if the student is a perfectionist. By not having to worry about keeping your thoughts neat and tidy, it allows students to have the freedom to truly engage with the mathematics and establish their personal understanding with the material. Students share their perceptions in their surveys:

Erick: I liked that it was easy to erase and the space we had to write.

Garry: I liked using the non-permanent vertical surfaces because don't waste paper and can take pictures of it.

Ava: I liked it a lot, even when we didn't have to work on vertical surfaces. I work on them anyways. I think it's because it's easier to erase things like you make a lot of mistakes that's easier to just wipe it with one stroke and not erasing for a really long time.

Furthermore, the size of the VPNS gives students enough space to organize their thoughts in a manner which is easily visible by all students that are collaborating in a group. Having this visibility is not always possible in a paper-and-pencil environment, thus making it difficult for all members in a group to share their ideas:

Sid: I liked using them because we can write everything we need/know on the boards. It gives us lots of room and we can get a chance to write on it.

Jan: It was also helpful being able to space out the work on a larger surface.

Mike: I liked non-permanent vertical surfaces as it gave me and my classmates a wider vicinity in order to share and compare work.

At times, having a space where work can be organized in a clear and concise manner allows an environment to be created which enhances deeper understanding and if one is able to understand the material fully they are more likely able to retain the material for a longer period of time. Based on student feedback, the space available on VNPS encourages this environment.

5.1.3. Affordances of Mindful Notes

Utilizing VNPS in the classroom to understand mathematical content allows for students to have an opportunity to experience an environment without mathematical notes, and the opportunity to decide if no notes compared to formal notes has greater affordances

for understanding and retaining mathematical content. Even through the use of VNPS students still have an opportunity of creating a permanent record of the ideas discussed through the use of digital technology; the only difference being that students would create this record upon having the chance to discuss the ideas rather than just copying something they may or may not fully understand. There are two main affordances that the two Pre-Calculus 11 classes witnessed through the use of VNPS rather than formal, teacher-led notes: active engagement with content and learning and making connections at one's own pace.

VNPS promotes active engagement by being a large enough vertical space which takes students out of their desks and allowing them to share and expand on ideas for themselves rather than being teacher led. As shown in Chapter 4, Isabelle summarizes this in her interview:

Isabelle: So, okay one thing I liked is that how we were like doing it ourselves not you teaching us. Sometimes, we kind of get side tracked when you are kind of teaching us like you know play with our pencils, erasers, but when we are on the board doing it ourselves we kind of like do it like, what do you call, I forgot the word. One sec. I forgot Ms. Mann.

Nikki: Okay, that's okay. But, you feel like you were doing it yourself so that helps?

Isabelle: Mmmhmm.

Part of the reason that students may get side-tracked when copying notes led by teacher is the pace of the lesson may be too fast or too slow to engage the student to make personal connections. In a normal class of 30 students it is difficult for a teacher to keep the pace of the lesson suitable for every single student. However, by utilizing VNPS, the pace and flow of the lesson can be changed based on students in individual groups.

Here are excerpts of student survey responses that further emphasize student perceptions of the benefits of utilizing VNPS in the classroom:

Mani: I found it extremely helpful in learning / remembering concepts

Liam: Writing on a white board is generally more engaging than sitting in a desk.

Mary: I've never understood the material better than with the vertical surfaces method. It made me think for myself and really understand what I was doing and why it worked.

Erika: I felt that working on the non-permanent vertical surfaces got me more engaged.

Jim: I liked the use of the whiteboards and working in groups because it made me more motivated to actually do the work.

Mani and Liam are high achieving students, Mary and Erika are average students and Jim, who did very little outside of class, received a minimal pass in Pre-Calculus 11. What is interesting is regardless of academic achievement, a common theme emerged in response to the use of VNPS in the mathematics classroom, which was being engaged. Both Liam and Erika responded that working on the boards got them more engaged. Mary's response emphasised that through the VNPS she was able to understand the material which led to learning and remembering, which Mani wrote in his response. Finally a student like Jim, who usually did very little in mathematics class was motivated to actually try to make connections with the material when he was working at the VNPS. Therefore, students of all skill level can benefit from its use in the classroom.

5.2. Visibly Random Grouping: Role of Collaboration

Group work is something that can be found in most classrooms, but in many traditional mathematics classrooms, its use can be very limited. Students may be put in groups to attempt "you try" questions during a lesson, once a teacher has explained a concept, or after a lesson to reinforce concepts discussed. The main goal of collaboration in both situations is to practice concepts for which a "proper" mathematical methodology has already been discussed by the classroom teacher. Students might be under the misconception in these groups that there is only one correct way to approach a problem, which was the "proper" method demonstrated by the teacher. Furthermore, when placing students in groups, a teacher might strategically decide on these groups by dividing students up so that each group has a representation of each skill level. The logic for using such a dividing system might be with the goal of having stronger students helping students who are struggling, alleviating some of the pressure off of the teacher to be in multiple places at once. Students may also be divided by behaviour so that students that normally don't work well together are not placed in the same group.

In comparison, implementing VRG in the mathematics classroom has a different use of collaboration in comparison to the traditional mathematics classroom. Firstly, this type of collaboration is based on the idea of the groups being “visibly random.” As discussed in Chapter 3, with VRG, students are aware that the groups they have been placed in are based on a random process where the teacher had no role in determining which students would be working together. This has many advantages for the collaboration process as each member of the group knows that they have not been chosen to be part of the team based on their perceived skill set. Hence, when it comes to working on a mathematical concept or idea together, each person in the group will be more inclined to participate as they will not feel they are being judged on their previous mathematics performance. Secondly, when utilizing visibly random groups with VNPS and mindful notes, the goal is for students to work together in their groups to discover or understand a mathematical concept for themselves, as well as become aware of the multiple approaches to understand the concept. Hence, students are not collaborating to understand a concept that was already explained to them, but instead are working together with different students to make personal connections with a topic for themselves. As Rebecca writes in her survey:

Rebecca: ... each time you're working with someone new, everyone's knowledge or understanding of math is different. By being with different people each time gives you a better and different learning experience.

There were specific things within this type of collaboration, which helps students create personal connections with content, allowing for a stronger student perception of retention of mathematical knowledge.

As stated previously, VRG allows each student to have an opportunity to share their mathematical knowledge and understanding of a problem since no discrimination has been made due to skill level, making one more inclined to share their thoughts. Fakar, is a student who normally did not participate in large group discussions in the class, perhaps as he was always doubting his mathematical ability. However when placed in random grouping he would always attempt to contribute to the group and discuss his thoughts, consequently making his own personal connections with the material:

Fakar: When it's your turn in the group you sort of have to do the question which gives you really good practice.

Because the group size would never exceed four people, Fakar might have also felt more comfortable participating. Nonetheless, the random grouping gave Fakar the opportunity to practice his skills and, as Shawn, established in the Chapter 4, with practice come the ability to retain the knowledge one gains in mathematics.

VRG also gives students a chance to learn something from the students in their group and gain different perspectives of the problem. These different perspectives allow students to see the mathematics in different ways and then choose a path to approach the problem that makes the most sense to them. Students explain their perspectives in their surveys:

Ravi: I liked working in random groups on some days because I learned something new from every student.

Neeta: I liked the fact that I got to learn how my fellow students went about answering a question. At times their ways were much simpler than mine.

Moe: Working with random people also expands my horizons since I get to know or see their own perspective and how they approach a specific task.

Having students see mathematics in different ways promotes critical thinking rather than memorization. Furthermore if something makes more sense to a student, the probability of them retaining this information increases. This was demonstrated by Kam, Lila, and Mark. Recall in Chapter 4 when these students changed the quadratic function from standard to vertex form during their interview. They were easily able to remember how to do this process a year later as they had created a unique way to approach this task.

VRG also gave students an opportunity to get help or clarification from their peers, but this was done in manner which was not based on perceived skill set. At times, a student who is considered to excel in math might receive clarification from a student who isn't, as this new environment promotes equity and that student who was considered to poorly in math might have done so as they did not have the opportunity to make their own connections with the material:

Migel: I liked the idea of working in groups since your able to communicate with other classmates and get help when you're unsure.

Jan: The other people in the group would have different ideas or be able to point out mistakes I made.

Ritek: It helps you ask for help from others in the group if you do not know something.

Naam: It helped sometimes if one of your group members could explain it well.

Migel and Jan were both considered to be high achieving students in mathematics; whereas, Ritek and Naam were considered to struggle with concepts. Regardless, when looking at the survey results all four students wrote about the value of getting help from their peers. If students are more willing to get help in such a setting, this again leads to greater retention as understanding of the material once again increases.

Another way students perceived that VRG helped increase understanding and retention of mathematical material is that they had an opportunity to explain material to their classmates. As Mani writes:

Mani: Random groupings allowed for me to be paired with those who needed further explaining, and in doing that, I gained further knowledge.

Ava also had similar thoughts as discussed in Chapter 4:

Ava: I also liked being put in random groups because when my group members didn't understand, I could explain everything. This not only deepens the understanding of my group members, but also myself by explaining the process.

Students who normally are looked as leaders in a group because they may be strong mathematics students may feel a lot of pressure in a group setting. Nonetheless, by randomizing the grouping process this stress is lessened and these students are more inclined to explain their understanding. This is demonstrated by both Mani and Ava who are strong mathematics students, and found value in helping others in their group in this setting. Regardless of skill level, VRG provides every student the opportunity to solidify for their understanding by explaining concepts to their peers.

5.3. Digital Technology: Visual Tools that Promote Engagement

As discussed in Chapter 3, one of the changes implemented in both Pre-Calculus classes was using digital technology as both a visual tool and a way to get students engaged with the mathematics. The main digital tool used throughout the course was Desmos. It was used not only to solve quadratic and absolute value equations but also as a graphing tool. Instead of having the traditional cumulative final exam, a final project which utilized this technology was assigned. In the project, students were to use Desmos to create or trace an artistic image, either individually or collaboratively with a partner, using various equations and restrictions on the domain and range of the equations. The minimum requirements for the project were to use at least 25 different equations and have at least one component of shading in the image. Students could use any of the equations that they had learned about up to now in high school mathematics, including linear, quadratic, absolute value, piecewise, and reciprocal, but no review of these equations were done in class. They could also research other mathematical equations that would create certain shapes they needed for their image, but this would be an individual or group undertaking.

What the project showed was how creative students can be in mathematics if they are given an open and engaging task. In both classes students were not focused in just meeting the minimum requirements, and handing the assignment in, but were genuinely interested in creating a certain image and the mathematics involved in this process. Many groups did research on their own, without teacher prompting, to discover new equations they hadn't learned in class, but were necessary in creating their image. They also learned how to manipulate these equations and to create certain transformations:

Mani: It was much more engaging and refreshing. It also forced me to go outside of the curriculum and learn things on my own.

Some of the equations used included circles, trigonometric functions, inverse functions, material that would normally be taught in Pre-Calculus 12. Nonetheless, as Mani stated, engagement levels are increased if students are given the opportunity to explore on their own without being confined to the curriculum. Furthermore, marks were not a motivator for their hard work, but it was genuine interest in learning about graphing and

transformations, and this interest, in turn, would lead to greater retention of the material learned earlier in the semester since students were expanding on this knowledge, thus reinforcing concepts they had previously learned. As Fakar and Mary wrote in their surveys:

Fakar: It is more interesting and lets us expand the knowledge we have retained this semester in a fun way.

Mani: I surprisingly really enjoyed using Desmos. It helped to further expand my knowledge of restrictions and all other aspects of the course

Below is a screenshot showing Mani and Fakar's project which used a total of 320 equations in total, something a teacher cannot expect from a student using only a paper-and-pencil environment:



Figure 3. Mani and Fakar's Desmos project

At the end of the course, one of the questions I asked students on the survey was, "How did you feel about having a final project using Desmos rather than a final exam? What did you learn from this project?" Ten students from Block B and eight students commented on how the project was less stressful than a final exam. Ten students from Block B and eight students from Block C commented about learning to graph new functions on their own. When stress is taken away from a learning task, and students are given an opportunity to discover concepts on their own, whether through VNPS or digital technology, this will lead to greater engagement with the mathematical concepts, a more memorable experience and retention of the content:

Ranjit: Doing the project was less stress than having a final. I learned that graphs and lines are what make an image. It is incredible how much we can create with math equations.

Ritek: ...I find that projects like this engage the brain more than a written test

Furthermore, by having students work on a project where they can see mathematics in everyday life can help increase student engagement because then mathematics will not simply be viewed as a discipline where facts of little value outside of class have to be memorized:

Jenny: I love the final project because you get to use the things you learned from the course. Most people don't think they will ever use what we learned in real life. But in the project we aren't solving some random question we are making art.

5.4. Active Learning, Engagement, Collaboration: Role in Acquiring a Memory

Plato theorized that genuine knowledge is acquired in a setting that allows individuals to make personal connections with the material. He stated in *The Republic*, "... a freeman ought not to be a slave in the acquisition of knowledge of any kind" (2016, p. 282). This idea can still be applied to our mathematics classrooms today. In many traditional mathematics classrooms, the main teaching structure is still teacher delivered notes, which students attempt to copy, in most cases not questioning the delivery of the material nor engaging in any discussion. Although it might seem intimidating for both a teacher and student at first, but by eliminating notes from a mathematics classroom and utilizing tools such as VNPS, VRG, mindful notes, and digital technology, a learning environment where students take ownership for their learning can be created. Students are given an opportunity to actively engage with the mathematical content for themselves, making personal connections, creating a stronger memory of the knowledge presented and consequently should retain this knowledge for a longer period of time.

One of the questions students were asked at the end of Pre-Calculus 11 in their survey was, "Do you think the VNPS and visibly random groupings helped you retain information for a longer period of time?" The results are summarized in the table below:

Table 7. Responses to survey question # 3

	YES	NO	SOMETIMES
BLOCK B 25 Students	13	4	8
BLOCK C 25 Students	11	6	8

Based on the responses, only four students in Block B and six students in Block C felt that these engaging teaching methods had no effect on the retention of mathematical content. Even the students who felt it only helped sometimes, only thought so as they placed more value on VNPS over VRG or vice versa, felt it worked better for some units compared to others, or felt a component of practice or writing notes for oneself later would enhance the retention process. Since, a majority of students thought that these changes in the classroom had a positive effect on what they retained from the course it is important to look at why they believed this might be the case.

VNPS, VRG, mindful notes, and digital technology help in creating classroom environments where students can play with mathematics through social interactions with their peers enabling opportunities for rich mathematical discussions. Social interaction can be seen as a foundational element to mathematics education that allow students to assimilate mathematics knowledge and develop a deeper understanding. As students stated in their surveys and interviews, VRG gave them opportunities to discuss mathematical content with different people allowing for different perspectives to arise. Being exposed to these different perspectives allowed for deeper connections to be made with the material and a higher level of relational understanding and retention. Furthermore, the learning experiences become more memorable as you are working with your peer group and it is more engaging than having a teacher tell you what you should do with the mathematics:

Hannah: Since in the past, some teachers would just teach us their method, and if I didn't understand their method it wouldn't stay for long. With this, I could discover my own methods of solving.

Resham: I really liked the aspect of learning with others and writing on the whiteboards because of how immersive and social this method of learning.

Noora: What I liked about this teaching tool is that we get to share our ideas and knowledge to others, increasing support and reducing "shyness."

Students placed a lot of value in learning for the sake of learning and understanding for themselves. This can be seen from the work students put into completing their Desmos project and how well they worked in their random groupings on the VNPS. As Kam stated in Chapter 4, it was difficult to go from being an active participant in understanding in Pre-Calculus 11 to being told step by step what to do with little emphasis placed on the why, in Pre-Calculus 12:

Kam: ... What we're doing now is just the teacher does notes and we copy them down and stuff, and then I just get really bored especially now. Before, I used to like it because I used to just copy down and listen to lessons but I like just learning it by myself now after being in this class in Pre-Calc. 11.

Similarly, Shawn, another student examined in Chapter 4, although received 50% in my Pre-Calculus 11 class and retook Foundations 11 the next semester, felt he got more from the Pre-Calculus class than his Foundations class even though he was getting a better mark. He valued the process and took ownership of the fact he was not practicing concepts in Pre-Calculus 11, which was also necessary in retention.

As mathematics educators it is important we create a classroom environment which nurtures this desire to gain knowledge for the sake of attaining relational understanding rather than teach to an exam. Table 8 shows the results of all major unit exams for Pre-Calculus 11 for both blocks. What can be noticed is despite the staggered assessments, most exam results were considered to be in the normal range for a Pre-Calculus 11 class. What is interesting is that the less academic class of Block B ended up having a higher average compared to the more academic Block C. Although these results did not affect my research question in a positive or negative way, they are important results. What they show is despite changing the classroom structure to a non-traditional one where students had the freedom to explore the mathematics at their own pace and with different people, summative assessments are in the most part not affected. Hence, by changing how students are acquiring their memory of mathematical knowledge to a methodology which utilizes engaging teaching methods such as VNPS, VRG, mindful notes, and digital technology, will most likely not affect the student's class mark. More importantly, what it will change is the experience students attained from the class and their perception of the knowledge they were retaining:

Ranjit: I think this does help rather than having the teacher provide notes for the whole class. It allows me to think and challenge

myself. When I solve a question by myself and with the help of others, I am able to understand more.

Mike: Yes it did help me retain information for a longer period of time. Why, is because it not only made us feel more comfortable, yet, more aware of other's styles of work.

Table 8. Unit test results

Unit	Number of Days Test Staggered	Class Average	≥ 86%	73% - 85%	67% - 72%	60% - 66%	50% - 59%	0% - 49%
Unit 1 (B)	5 days	71%	9	2	1	4	6	3
Unit 1 (C)	5 days	69%	5	8	3	2	4	3
Unit 2 (B)	11 days	65%	5	5	1	5	3	6
Unit 2 (C)	11 days	58%	1	3	3	5	6	7
Unit 3 (B)	14 days	76%	12	4	1	3	3	2
Unit 3 (C)	14 days	66%	7	3	1	2	4	8
Unit 4 (B)	12 days	64%	7	4	1	4	1	8
Unit 4 (C)	12 days	56%	4	3	1	1	4	12
Unit 5 (B)*	7 days	67%	9	3	1	0	5	6
Unit 5 (C)	7 days	61%	2	6	3	1	6	7
Unit 6 (B)*	10 days	67%	7	2	2	4	2	7
Unit 6 (C)	10 days	65%	6	4	1	4	4	6
Unit 7 (B)*	8 days	78%	10	6	0	3	3	2
Unit 7 (C)	8 days	70%	6	9	0	3	2	5

Class Average (B): 70%

Class Average (C): 64%

See Table 2 for Unit Content

Chapter 6.

Conclusions

“By learning you will teach; by teaching you will learn.”

- Latin Proverb

How is it that there are certain things you will remember for life and other things you will forget in an instant? I have always been curious about retention and what makes things memorable for us, especially in education. For example, ask a student the details of a play in drama class from a week ago, and usually they can give you an account of all the experiences it took to make the play. However, ask them the details about the mathematics lesson from yesterday, and many students will give you a perplexed look. What is it about the play and drama class that makes it unforgettable even after a week, but the mathematics lesson so easily forgettable? The structure of a drama class is quite different than that of a traditional mathematics class, drama class being more high energy and collaborative compared to a traditional mathematics class, which usually consists of teacher directed notes followed with an assignment, which is usually done individually. Some other key characteristics of most drama classes include interactive lessons where there is room for independence and creativity for students, which again differs from traditional mathematics classes. One of my own most memorable experiences as a student in a mathematics class was Dr. Peter Liljedahl's Education 847, Teaching and Learning Mathematics. It was here I was introduced to the use of VNPS VRG, mindful notes and how these engaging tools can be used in the exploration of mathematics. After successfully implementing this teaching model in one of my Pre-Calculus 11 classes, I wanted to explore to see if using such a non-traditional methodology in a mathematics class, one that is more engaging, might change the level of retention of mathematical content by students. In my research study I looked at: *Is there a relationship between student perception of their retention of mathematical knowledge and the use of engaging teaching methods such as vertical, non-permanent surfaces, visibly random groupings, mindful notes, and digital technology.* I wanted to know if the use of such methodology would improve student academic performance in a positive or negative way depending on if knowledge is retained longer. Also would there

any changes in student perception of their engagement with course content and retention of mathematical knowledge due to the change in class methodology?

6.1. Responding to the Research Questions

At first, changing the expectant structure of a mathematics class can be difficult for students to accept. As students from my first Pre-Calculus 11 class in which I initially implemented VNPS, VRG, mindful notes, and digital technology stated in their interview:

Kam: ...on the first day and we were like 'This teacher's crazy.' I don't know what kind of idea she has.

Lila: We should probably leave this class.

Although students in both my Pre-Calculus 11 classes were reluctant in accepting the changes made to the methodology of the class at the beginning of the semester, as now the burden of thinking fell upon them, and they were simply not told what to do, by the end of the course they could appreciate these changes, and what they learned from the course by being more mindful of their learning. Even though student academic performance did not change in a positive nor negative way with the implementation of these engaging teaching methods, as could be seen from the results of student assessments (Table 8), there were other changes both students and I noticed in the classroom, which impacted student perception of their retention of mathematical content.

What VNPS, VRG, mindful notes and digital technology allowed for was a learning environment which promoted active learning of mathematics rather than the simple memorization of mathematical content, which can be found in many traditional mathematical classes. Sabina and Isabelle showed this tension in Chapter 4 that exists between memorizing and understanding. Both were used to doing well in mathematics through memorization and although Isabelle did adapt to an environment which required understanding, Sabina did struggle with this idea. With the active learning environment came student engagement with mathematical content and there were different aspects of each tool that encouraged this engagement. As discussed in Chapter 5, from student feedback on surveys and interviews, what students liked about the VNPS was first, it was a surface that allowed them to get out of their desks and be more engaged with the mathematical content compared to when they were just sitting at their desks. Secondly, because it was a non-permanent surface, students were more comfortable taking risks,

as the surface was easily erasable, whereas, on a permanent surface, at times, students can be more worried about how their notes look rather than if they understand the mathematics. Finally, working on the VNPS allowed students to have an opportunity to create their own mathematical understanding at their own pace rather than copying notes, prescribed by the teacher, which they may not fully understand. As discussed in Chapter 2, Piaget stated this process of making connections allows students to experience disequilibrium and then gain their own equilibrium when the knowledge they are receiving is understood because changes have been made to their “mental maps.

The next tool, VRG, provided the collaboration to occasion understanding of mathematical knowledge. By having an opportunity to discuss problems and possible solutions with their peers, students have an opportunity to clarify questions they might have about content, and gain different perspectives to the same problem. Furthermore, because students are aware that they have not been put in groups due to skill level or behavioural reasons, they are more inclined to share their understanding with their groups and become a more active member in the discussion process. With this collaboration and discussion comes the strengthening of knowledge to relational understanding as with the discussions and mistakes that students make along the way comes deeper connections with the material.

Finally, utilizing digital technology, mainly Desmos, allowed students to have a visual tool to deepen their understanding. Not only was Desmos engaging, but it gave students the much needed independence needed in mathematics to build understanding. Through their Desmos project, students had an opportunity to explore various mathematical functions to create artistic images, hence were given an opportunity to be creative with their mathematics. When students have an opportunity to create something, such as a play in drama, students are more inclined to remember this experience rather than if they were just shown the graphs and their corresponding transformations through a lecture style. Furthermore, through this process of creation, knowledge is again strengthening to relational understanding.

A common theme that emerged with the use of such engaging teaching methods as VNPS, VRG, mindful notes, and digital technology in my Pre-Calculus 11 classes was that students were more actively involved in their learning, hence making more connections with the knowledge being taught, and hence creating relational

understanding. With this relational understanding comes a stronger web of connections with previously learned mathematical content and the new knowledge students are learning, and hence, with greater connections comes a greater level of retention of mathematical knowledge. Even though marks did not change with this implementation, student experiences of their mathematics class changed and it will be these experiences that students will remember. They will remember creating understanding on their own. Rather than being told what the quadratic equation was, they created it on their own. Instead of being told how rational expressions and equations worked, they connected it themselves to what they learned about fractions in grade 8. Instead of being told what transformations did to graphs, they understood it on their own through technology. There is great value in giving experiences in a classroom which strengthens understanding, retention being one such by product. As Moore (2002) enforced in his research, how a memory is acquired had direct impact on the strength of the memory.

6.2. Implications for Research and Teaching

This research study which examined the role of the use of engaging teaching methods in mathematics and its affect on student perception of retention of mathematical content was done in only two Pre-Calculus 11 classes over a period of a semester, or five months, hence it is a very small scale study. Regardless, there is still value in its findings, which were based on student interviews, surveys, and my own personal classroom observations. These findings can be used for further research in the fields of student engagement and retention in mathematics. The methodology utilized in the study can also be used in various mathematics classrooms with the goal of increasing student engagement and retention of mathematics.

6.2.1. Research

Research is a field which allows for continuous growth and development. The findings of one study can lead to further research opportunities. This study focused on the use of engaging teaching methods, mainly, VNPS, VRG, mindful notes, and digital technology, and student perception of retention of mathematical knowledge. There was a clear correlation that these engaging teaching methods created opportunities for students to transmit knowledge to relational understanding, which could lead to better retention of

the mathematical knowledge. Nonetheless, the study could be modified to consider further questions such as:

- What other engaging teaching methods could create opportunities for connections to be made? Would things such as manipulatives, maker-spaces have the same effect as VNPS, VRG, and technology?
- This study was done in a Pre-Calculus 11 class where students are generally pretty motivated to achieve high marks. How would the same study fair in a Mathematics 8 or 9 class? Could the same results be achieved in an Apprenticeship and Workplace class where the level of motivation is very low, but there are also behavioural issues?
- The measurement of retention is something that can be difficult to do. This study was based on qualitative data such as teacher observations, student surveys and interviews. Is there other qualitative or quantitative measures that can be utilized to measure retention? What kind of results would come out of using these new measurements?

6.2.2. Teaching

The Latin proverb, “by learning you will teach; by teaching you will learn,” has an important message for teachers. True, in the classroom, teachers are considered to be the experts in the room, thus, their manner of teaching is usually not questioned. Being experts, they should know the best atmosphere for learning. Nonetheless, as teachers, it is important to grow with research otherwise we cannot be progressive in creating the best learning environment for our students. As teachers, we are lucky that we not only have access to research studies to improve our practices, but we also have access to a wealth of knowledge we can learn from each day with the students in our classes. Simply by observing what helps students understand material and making adjustments in our teaching can change the classroom experience for both students and teachers.

What I hope my research does for teaching is that it may have a teacher who uses only teacher directed teaching in their classroom, reflect on this practice. Dr. David Pimm said to our Education 844 class, Issues in Mathematics Education, that there was value in every type of teaching practice if utilized properly. I definitely agree there is value to having mindful teaching moments. However, they need to be embedded with a *thinking classroom* that Liljedahl describes in his research which allows students the

opportunity to make their own connections with the mathematics they are to be learning. If building connections through the use of mindful teaching methods allows for a greater retention of mathematical knowledge by our students then we, as teachers, should build these opportunities within our practice. If students retain nothing from our classes, but their letter grade, then what is the purpose of mathematics education and how are we helping our students realize their human potential?

6.3. Personal Growth

One of the best aspects of the MSc. program at SFU was each class had a component of self-reflection which promoted personal growth both as a mathematics teacher and, in many cases, as a researcher. My whole purpose of undertaking this research was motivated by how I could improve my practice so that students were not only gaining more from my mathematics classes, but also retaining more. Here is how I have grown a mathematics teacher and mathematics education researcher.

6.3.1. Teacher

As a mathematics teacher I always hoped my students would gain an appreciation for mathematics at the conclusion of my class. I always thought I was doing a good job building a connection with my students and teaching mathematics in my mostly teacher directed classes. I usually got positive feedback from my kids, however, I was still noticing the troublesome pattern of students forgetting so much of the knowledge I was hoping they would gain from my classes at both the conclusion of a unit and / or at the end of the class. Upon taking Dr. Liljedahl's class and implementing VNPS, VRG, mindful notes, and digital technology in my Pre-Calculus 11 classes, I saw a clear shift in my classes and what was happening with the knowledge I was hoping they would gain. Because I was allowing them the opportunity to learn at their own pace and make their own personal connections, they were gaining more understanding from my class. There were other positive changes in my classroom as well.

Instead of struggling with including everyone during class discussions and helping every single student that was struggling with a concept at the same time, these teaching tools allowed me to see more students involved in the discussion process, and it gave me more individual time with students that were struggling, as in many cases

groups collaborated to answer each group members concerns. I was also surprised with how much my 11's learned on their own while enjoying the process. For example, these classes were able to derive how to go from standard form to vertex form for quadratic functions with no help from me and were successful in retaining their method. This was something many would not have understood if I showed them how to complete the square. I am amazed with how quickly they were able to learn things with less structure.

I appreciate the changes that have incurred in my teaching practice (as due my students!) and I hope to continue to effectively use the tools that I have been taught. What remains unchanged in my classroom is my relationship with my students and my classroom expectations, the sense of respect being the most important component. What is interesting is that my classroom becomes complete chaos during problem-solving using VNPS and VRG, something that I initially thought hindered learning, but students are actually learning and retaining more from these experiences.

6.3.2. Researcher

As a researcher I have grown so much by undertaking this research study. I now understand the various layers found in a research study and how important it is to undertake such reflections in our teaching practices. Change is fundamental and important and by creating our own research questions and noticing the changes they bring to our classrooms can help us grow immensely as an educator. As a researcher I would like to continue my exploration of utilizing engaging teaching methods such as VNPS, visibly random groupings, and digital technology and its relation to retention. Having seeing the positive effects in my Pre-Calculus 11 class, I would like to see how other class compositions would do with these changes.

At the conclusion of this school year I will be moving to a new school in the same district, which is built around a 21st model of learning. Student collaboration will not just be stressed in one subject field, but the hopes are that there will be cross-curricular explorations. I am interested to see how these engaging teaching methods will help create connections in such an environment. I am excited to see if retention and student experiences will remain the same with this added dimension of cross-curricular studies. I still believe what it means to be human is still to develop our minds to gain worthwhile knowledge and understanding, but this does not stop when we become the teacher in

the classroom responsible for relaying the knowledge. To realize our human potential we too must continue to grow in our fields of study as we teach the next generation of students.

References

- Barrow, R., & Woods, R. (1988). *An Introduction to Philosophy of Education*. (3rd Ed.). New York: Routledge.
- Barrow, R., & Milburn, G. (1990). *A Critical Dictionary of Educational Concepts*. (2nd Ed.). New York: Teachers College Press.
- Boaler, J. (1998). Open and Closed Mathematics: Student Experiences and Understandings. *Journal for Research in Mathematics Education*, 29(1), 41-62. doi:10.2307/749717
- Boaler, J. (2015). *Whats math got to do with it?: How teachers and parents can transform mathematics learning and inspire success*. NY, NY: Penguin Books.
- Boaler, J. (2016). *Mathematical mindsets: Unleashing students potential through creative math, inspiring messages, and innovative teaching*. San Francisco, CA: Jossey-Bass & Pfeiffer Imprints.
- Boaler, J., & Sengupta-Irving, T. (2016). The many colors of algebra: The impact of equity focused teaching upon student learning and engagement. *The Journal of Mathematical Behavior*, 41, 179-190. doi:10.1016/j.jmathb.2015.10.007
- Boaler, J. (2016). Designing mathematics classes to promote equity and engagement. *The Journal of Mathematical Behavior*, 41, 172-178. doi:10.1016/j.jmathb.2015.01.002
- Clark, K. (2015). The Effects of the Flipped Model of Instruction on Student Engagement and Performance in the Secondary Mathematics Classroom. *The Journal of Educators Online*, 12(1), 91-115. doi:10.9743/jeo.2015.1.5
- Corso, M. J., Bundick, M. J., Quaglia, R. J., & Haywood, D. E. (2013). Where Student, Teacher, and Content Meet: Student Engagement in the Secondary School Classroom. *American Secondary Education*, 41(3), 50-61. Retrieved August 1, 2017.
- Earl, L., & Sutherland, S. (2003). Student Engagement in Times of Turbulent Change. *McGill Journal of Education*, 38(2), 329-343. Retrieved August 1, 2017.
- Fraser, V., & Garofalo, J. (2015). Novice Mathematics Teachers' Use of Technology to Enhance Student Engagement, Questioning, Generalization, and Conceptual Understanding. *Journal of Technology and Teacher Education*, 23(1), 29-51. Retrieved July 18, 2018.
- Gallenstein, N. L. (2005). Engaging young children in science and mathematics. *Journal of Elementary Science Education*, 17(2), 27-41. doi:10.1007/bf03174679

- Goldenberg, E. P. (2000). Thinking (And Talking) About Technology in Math Classrooms. *Issues in Mathematics Education*, 1-8. Retrieved July 18, 2018.
- Jowett, B. (2016). *Republic of Plato*: Translated into english with introduction, analysis marginal analysis, and... index (classic reprint). S.I.: Forgotten Books.
- Lahann, P., & Lambdin, D. V. (2014). Collaborative Learning in Mathematics Education. *Encyclopedia of Mathematics Education*, 75-76. doi:10.1007/978-94-007-4978-8_23
- Liljedahl, P. G. (2005). Mathematical discovery and affect: The effect of AHA! experiences on undergraduate mathematics students. *International Journal of Mathematical Education in Science and Technology*, 36(2-3), 219-234. doi:10.1080/00207390412331316997
- Liljedahl, P. G. (2014). The Affordances of Using Visibly Random Groups in a Mathematics Classroom. *Transforming Mathematics Instruction Advances in Mathematics Education*, 127-144. doi:10.1007/978-3-319-04993-9_8
- Liljedahl, P. G. (2016). Building Thinking Classrooms: Conditions for Problem-Solving. In P. Felmer, J. Kilpatrick, & E. Pekhonen, *Posing and Solving Mathematical Problems: Advances and New Perspectives*, 361-386. doi:10.1007/978-3-319-28023-3_21
- Liljedahl, P. G., & Allan, D. (2013b). Studenting: The case of "now you try one". (Vol. 3). In Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education.
- Modebelu, M. N., & Ogbonna, C. C. (2014). Reform-Based-Instructional Method and Learning Styles on Students' Achievement and Retention in Mathematics: Administrative Implications. *International Journal of Education and Literacy Studies*, 2(2), 48-52. doi:10.7575/aiac.ijels.v.2n.2p.48
- Moore, G. (2002). What Do You Mean, You've Forgotten!: An Investigation into Mathematics and Memory. *Mathematics in School*, 31(5), 42-44. Retrieved August 1, 2017, from <http://www.jstor.org/stable/30215568>
- Piel, J. A., & Green, M. (2010). Jump right in. *Teaching Children Mathematics*, 17(2), 72-76. Retrieved August 1, 2017, from <http://www.jstor.org/stable/41199597>
- Remember. (n.d.). Retrieved April 19, 2018, from <https://www.merriam-webster.com/dictionary/remember>
- Retain. (n.d.). Retrieved April 19, 2018, from <http://www.merriam-webster.com/dictionary/retain>

Skemp, R. R. (1978). Relational Understanding and Instrumental Understanding. *The Arithmetic Teacher*, 26(3), 9-15. Retrieved December 22, 2018, from <http://www.jstor.org/stable/41187667>

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2019). *Elementary and middle school mathematics: Teaching developmentally* (10th ed.). NY, NY: Pearson.