8 PROBLEMS 1–30

1 RABBITS AND HUTCHES

NUMERICAL

There are some rabbits and some rabbit hutches. If seven rabbits are put in each rabbit hutch, one rabbit is left over. If nine rabbits are put in each rabbit hutch, one hutch is left empty.

Can you find how many rabbit hutches and how many rabbits there are?

2 FACTORS

GENERALISATION REQUIRED, NUMERICAL

number	factors					
1	1					
2	1	2				
3	1	•	3			
4	1	2	•	4		
5	1	•	•	•	5	
6	1	2	3	•	•	6

This table shows the numbers from 1 to 6 with their factors. The number 4, for example, has factors 1, 2 and 4, with no number in the space for 3.

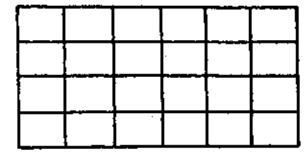
Make your own table for the numbers up to 30. See what you can find out about numbers with only two factors. Can you guess the next one? What sort of numbers are they?

Now try three factors and then four factors. See if you can guess the next number with three factors and then with four, and explain why. What sort of numbers have an odd number of factors? Can you find other patterns in your table?

3 THE MILK CRATE

SPATIAL

A milk crate holds 24 bottles and is shaped like this:



The crate has four rows and six columns. Can you put 18 bottles of milk in the crate so that each row and each column of the crate has an *even* number of bottles in it? Is there only one way to do it?

4 CROSSING THE DESERT

NUMERICAL, LOGICAL

A man has to deliver a message across a desert. Crossing the desert takes nine days. One man can only carry enough food to last him 12 days. No food is available where the message must be delivered. Two men set out.

Can the message be delivered and both men return to where they started without going short of food? (Food may be buried on the way out and used on the way back.)

5 MAKING TOAST

GENERALISATION REQUIRED, NUMERICAL

There is room in a grill to toast one side of two slices of bread in two minutes. How long does it take to toast both sides of three slices of bread? Is that the quickest way?

Work out the quickest way to toast four slices of bread on both sides. Then try five slices of bread and then six. Can you predict the quickest time for any number of slices? How would you do it? Try again with a grill that holds three or more slices of bread.

6 MAKING TRIANGLES

SPATIAL

Using 12 rods of varying lengths how many different triangles can you make?

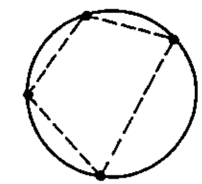
What *types* of triangle can you make? Can you make a triangle with *any* three rods? What about 2, 3 and 5 or 2, 2 and 3?

7 NO TRIANGLES ALLOWED

GENERALISATION REQUIRED, NUMERICAL

Draw four dots on a circle and join each dot to the next dot with a straight line. Can you draw any more straight lines, joining up the dots *without* making any triangles?

Try the same thing for five dots, six dots, seven dots and so on, getting in as many lines as you can. Can you fill in a table of the number of dots and number of lines? Predict how many lines can be drawn without making triangles when there are 13 dots. What sort of number never appears in the second column of the table?



8 VILLAGE STREETS

NUMERICAL, SPATIAL

In a village there are three streets. All the streets are straight. One lamp-post is put up at each crossroads.

What is the greatest number of lamp-posts that could be needed? Now try four streets and five streets. Predict the answer for six streets then check it. Can you see a pattern? Why does the pattern work?

9 AFTERNOON TEA PARTY

NUMERICAL

Some people had afternoon tea in a cafe which only sold tea and cakes. Tea cost 3p a cup, cakes cost 5p each. Everyone had the same number of cakes and the same number of cups of tea. The bill came to £1.33.

Can you find out how many cups of tea each person had?

10 PERFECT NUMBERS

NUMERICAL

What numbers go into 6 exactly? 6, 3, 2 and 1 do. Do you notice that $6 = 3 + 2 + 1$? 6 is called a *perfect* number because it is equal to the sum of the other numbers which go into it exactly.

What is the next perfect number?

11 CHANGING FIFTY PENCE

NUMERICAL

The other day I was asked if I could change a 50 pence piece. I had more than 50 pence in coins in my pocket but I could not make exactly 50 pence.

Can you find several ways this could happen? What is the largest amount I could have had in my pocket?

12 BADMINTON GAME

LOGICAL

Janet, Sangita, Anne and Margaret like to play badminton together but cannot all be free to play on the same day. Janet is unable to play on Tuesdays, Wednesdays and Saturdays. Sangita is free to play on Mondays, Wednesdays and Thursdays. Anne has to stay at home on Mondays and Thursdays. Margaret can play on Mondays, Tuesdays and Fridays. None of them play on Sundays.

Can each pair find a day on which to play? Are there any days when no games can be played? Are there any days when more than one game can be played?

13 TWO TOWERS

NUMERICAL

A child has a set of 10 cubes. One cube has 1 cm edges, one cube has 2cm edges, one cube has 3cm edges and so on until the largest cube which has 10cm edges.

Can she build two towers of the same height using all the cubes? Show how, or explain why, it can't be done. What difference does it make if another cube with 11 cm edges is used as well? Show how to do it in as many ways as possible.

14 A RECTANGLE

NUMERICAL, SPATIAL

The perimeter of a rectangle is 28cm. The length is 10cm more than the width.

What is its length? What is its area? What if the difference were 8cm?

15 PACKING BOXES

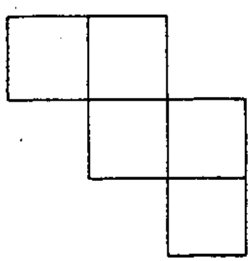
SPATIAL

Place some boxes with bases 8cm by 6cm on a tray 50cm by 35cm.

If all the boxes have to be placed the same way round, what is the greatest number of boxes which can be placed on the tray? Does it make any difference if the boxes don't all have to be placed on it the same way? What is the greatest number of boxes which can be placed on the tray?

16 PUT THE LID ON

SPATIAL



Make a large copy of this shape. Cut it out and fold it up; it will make a box.

Where could you add another square onto the shape so that when you cut it out and fold it up you will get a box with a lid?

Find as many shapes as you can which will fold into a box with a lid.

17 MARY'S MUDDLE

SPATIAL

Mary had two pieces of squared paper, 16 by 8 squares and two pieces 8 by 6 squares.

a) How many more pieces does she need to make a box? What size will the pieces be? What size will the box be?

b) Now begin with two pieces of squared paper 4 by 8 squares, one piece of paper 6 by 8 squares and one piece 4 by 6 squares. What else will you need to make a box?

c) If you have one piece of paper 5 by 9 squares and one piece 9 by 12 squares, what more will you need to make a box? What will be the size of this box?

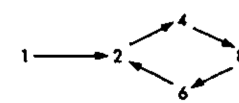
d) Can you make a box from two pieces of 5 by 10 squares, two pieces of 10 by 12 and two pieces of 7 by 12 squares? Why?

18 DOUBLE DIAMOND

NUMERICAL

Start with 1, double it, double it again and so on. You will get 1, 2, 4, 8, 16, 32, 64, 128, 256 ... Look at the units digit of the numbers: *1*, *2*, *4*, *8*, 1*6*, 3*2*, 6*4*, 12*8*, 25*6* ... Can you see a pattern?

This 'diamond' diagram shows the pattern:

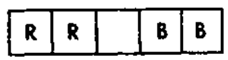


Start with 3 and keep doubling: 3, 6, 12, 24, 48, 96 ... Do you notice the pattern in the units digits? Make a diamond diagram for starting with 3. Can you see how to combine this new diamond with the diamond for starting with 1?

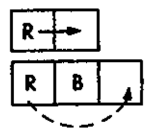
19 LEAPFROG

LOGICAL

On a row of five squares, two red counters and two black counters are placed like this:



The red counters can move one place to the right or they can hop over a black counter. The black counters move in the same way but to the left.

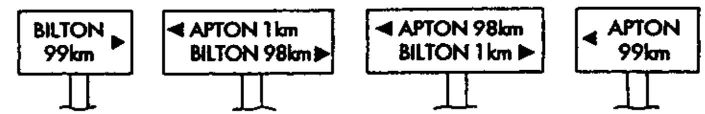


Can you finish with the black counters where the red ones were and the red counters where the black ones were? What would be the least number of moves needed?

Now try with three counters of each colour on a row of seven squares.

20 SIGNPOSTS

NUMERICAL



Apton and Bilton are 99km apart. There are 98 signposts between them, one every kilometre.

How many of the signposts are made up using only two digits?

This signpost is one, it only uses the digits 3 and 6:

21 FREDDY FROG

NUMERICAL, SPATIAL

Freddy Frog is at the bottom of a well 10m deep. Each hour he climbs up 1m and then falls back 0.5m.

How long is it before Freddy is out of the well?

22 FUNNY FOURS

GENERALISATION REQUIRED, NUMERICAL

Can you make all the numbers from 1 to 20 using only the number 4? You can make 8 by 4 + 4. How can you make 1?

23 CROSSING THE RIVER

LOGICAL

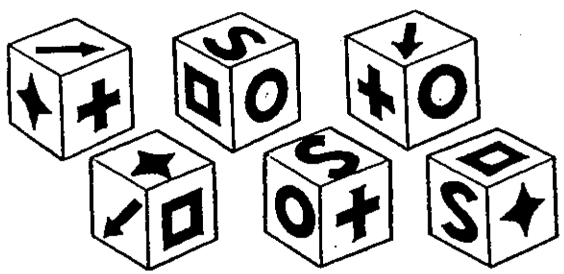
Two men and two boys want to cross a river. None of them can swim and they only have one canoe. They can all paddle but the canoe will only hold one man or two boys.

How do they all get across?

24 CUBE PUZZLE

SPATIAL

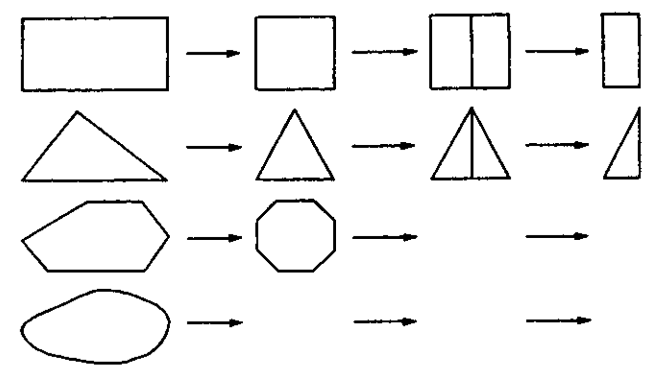
Here are six views of the same cube. Which pattern is opposite which?



Make up a cube puzzle like this.

25 SHIFTING SHAPES

GENERALISATION REQUIRED, SPATIAL



Fill in the gaps and invent one more shifting shape.

26 FUNCTION FINDING

GENERALISATION REQUIRED, NUMERICAL

16 →	A	→ 20 →	B	→ 40 →	C	→ 30 →	D	→ 15
7 →	A	→ 11 →	B	→ 22 →	C	→ 12 →	D	→ 6
20 →	A	→ →	B	→ →	C	→ →	D	→
101 →	A	→ →	B	→ →	C	→ →	D	→
→	A	→ →	B	→ 100 →	C	→ →	D	→
→	A	→ →	B	→ →	C	→ →	D	→

Fill in the gaps of these functions.

27 BUILDING BOXES

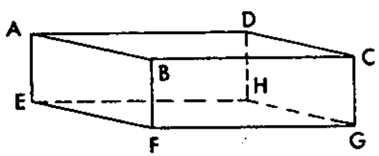
GENERALISATION REQUIRED, NUMERICAL, SPATIAL, COMBINATORICS

How many different rectangular boxes can be made with:
1 cube?
2 cubes?
3 cubes?
4 cubes and so on?
How can you be sure you have found them all?

28 THE FLY ON THE BOX

SPATIAL, LOGICAL

A fly walks along the edges of this box, starting at A and finishing at G.



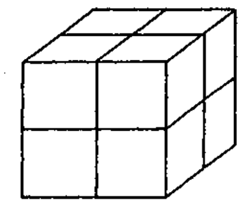
The fly never moves upwards and never walks along the same edge twice on any one journey.

How many different routes can the fly take? Sort out the routes according to the number of edges he uses. See what you can find out about the distance he walks.

29 THE COLOURFUL CUBE

SPATIAL

Use small cubes in four different colours to make a bigger cube with one of each colour on every face. Each of the small cubes must be painted in one colour only.

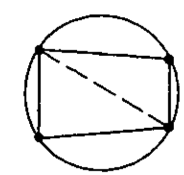


Is there only one way to do it?

30 DIAGONALS

GENERALISATION REQUIRED, NUMERICAL, SPATIAL

Draw four dots on a circle. Join them up. Choose one point only and see how many diagonals you can draw in from it.



The diagram shows the results. On a new circle try the same thing with five points. Then try six points, seven points and so on.

Make a table like this:

number of points	number of diagonals
4	1
5	2
6	3

Can you see a pattern?

If you had 100 points on a circle, how many diagonals could you