WHO TEACHES MATH FOR TEACHERS?

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Math for Teachers courses are specialised mathematics content courses whose nominal aim is to build conceptual mathematics understanding in prospective elementary school teachers. But who teaches these courses? This paper offers some insights into this question, exploring the views of two particular instructors whose interview responses reveal very different perspectives on their goals for this course with respect to knowledge-for-teaching, beliefs about mathematics, and the attitudes/emotions of their students.

Background

Prospective elementary school teachers are often expected to exhibit proficiency in mathematics by completing a university level mathematics content course before entering their accreditation programs. In an effort to provide this group of students with a course that is potentially more appropriate to their needs than a calculus or statistics course, mathematics departments at many colleges and universities have developed specialised "Math for Teachers" courses. Though they differ from institution to institution, these courses typically cover elementary school arithmetic and geometry topics, with an aim to help prospective teachers develop a strong conceptual understanding of the mathematics they will one day teach.

Relatively little research has been done to explore the role that these specialised content courses play in the development of teachers. As part of this larger project this qualitative study focuses on the instructors of these courses. Despite the surface similarities between Math for Teachers courses offered within and between institutions, given the autonomy that most post-secondary educators enjoy, the course-as-delivered has the potential to be significantly influenced by the individuals who teach the course. This study seeks to shed light on who the instructors of the Math for Teachers course are and how they attempt to contribute to the development of future teachers.

Theoretical Perspectives

Our inquiry is both informed by and informs a number of theoretical perspectives including classic and contemporary views on knowledge-for-teaching and sociomathematical norms.

Initial review of curriculum descriptions for Math for Teachers courses suggested that Shulman's (1986) framework for classifying knowledge for teaching might be helpful in understanding the types of knowledge that instructors of these courses hope to transmit to their students. Shulman identifies three major categories of knowledge: subject content, pedagogical content and curricular content. Given that the Math for Teachers course is intended to be a content course, we anticipated that in interviews instructors would address subject content knowledge, but not curricular content in their descriptions of their goals for this course. However, we were interested to see the extent to which pedagogical content issues would be addressed. Under this type of teacher knowledge Shulman includes

> *the most useful forms of representation of those [mathematical] ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of*

> *representing and formulating the subject that make it comprehensible to others.* (Shulman, 1986, p. 9)

He goes on to add

> *an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons.* (Shulman, 1986, p. 9)

The work of Ball and Bass (2003) also contributes to our perspective on the scope of pedagogical content knowledge. They identify an ability to unpack (or break-down) mathematical ideas, to understand the connectedness of mathematics concepts both at a particular level and across levels, and how students conceptions of mathematical concepts will evolve over time, as examples of mathematical knowledge required for teaching. Furthermore, they include knowledge of conventional mathematical practices, such as the role of definitions, and what constitutes an adequate explanation.

Though theories of knowledge-for-teaching influenced our initial interview questions, recurrent themes led us to broader and deeper considerations. Early interviews revealed that students' beliefs and attitudes are also of particular concern for instructors of this course. They can affect the students' ability to acquire knowledge content (Ball, 1990), and contribute to the development of the attitudes, beliefs and values that they will carry forward with them into their teaching careers. It also became apparent that any analysis of these affective issues would need to consider not only the aspects of their students' attitudes and beliefs which the instructors hope to influence, but also the instructors' beliefs and attitudes about mathematics and the teaching of mathematics that ultimately influence their own practice (Ernest, 1989).

Cobb and Yackel's (1996) notion of sociomathematical norms offers a lens through which to view this complex interplay between the beliefs and attitudes of the instructors and their students. Sociomathematical norms are normative understandings, negotiated through the interaction of teacher and students, which relate specifically to mathematical activity. Research supports the view that the development of these norms is closely integrated with the mathematics beliefs and attitudes of the teacher and students:

> *With regard to sociomathematical norms, what becomes mathematically normative in a classroom is constrained by the current goals, beliefs, suppositions, and assumptions of the classroom participants. At the same time these goals and largely implicit understandings are themselves influenced by what is legitimized as acceptable mathematical activity.* (Cobb & Yackel, 1996, p. 460)

Though the negotiation of norms involves participation of the students as well as the teacher, the teachers' various roles (as initiator, facilitator and validator) underscore the relevance of examining the beliefs and attitudes that the instructors bring to this negotiation. Integral to Cobb and Yackel's (1996) emergent perspective is the view that these social constructs are reflexively related to the individual's mathematical beliefs and values. From this perspective, references to beliefs about the nature or activity of mathematics can suggest the sociomathematical norms the instructors hope to establish in Math for Teachers courses, and vice versa.

Methodology

Interviews were conducted with instructors of Math for Teachers courses at post-secondary institutions in a metropolitan region. The Math for Teachers courses offered at these institutions are locally developed but are sufficiently similar that students can transfer credit for this course from one school to another.

The interviews, which lasted approximately one hour, began with a standard set of questions about the backgrounds of the instructors, their education, number of years of teaching, and number of years of teaching Math for Teachers. These questions were followed with questions about their initial orientation (preparation) for teaching the course, about what they do differently with this group of students compared to their other mathematics students, about their goals in teaching the course, and the outcomes they believe they achieve. As each interview was completed it was transcribed and coded for emergent themes through a process of constant comparative analysis (Cresswell, 2008; Corbin & Strauss, 2008). As part of this reflexive process the interviews were only semi-structured. This allowed the interviewer to incorporate additional and/or deeper questions to respond to new themes as they arose. To compensate for any bias that our own personal experience as occasional instructors of this course may introduce, the interview coding was corroborated by a more neutral colleague. Furthermore, narrative descriptions of the interview subjects were read and verified by the subjects themselves.

Through the analysis of the transcripts nine themes emerged: two related to teacher knowledge (subject content and pedagogical content), five related to sociomathematical norms (beliefs about mathematics, emotions/attitudes, aesthetics, communication, and community) and two others (teaching methods and tensions). Due to space limitations, in this report we will restrict our discussion to the two themes related to teacher knowledge, as well as two of the themes related to sociomathematical norms: beliefs about mathematics and emotions/attitudes. To facilitate readability, we will further limit ourselves to quotations from only two of our interview subjects, Harriet and Bob. Elaboration on these cases will permit us to illustrate two of the very different approaches that were revealed through our study.

We will begin our discussion of results with brief narrative descriptions of Harriet and Bob, and follow this with analysis of the selected themes.

Narrative Descriptions

Harriet

Harriet is an experienced Mathematics instructor who has been teaching for 22 years. She is relatively new to teaching Math for Teachers, but has taught the course six times over the last three years. She has neither taken any Mathematics Education courses, nor does she have a formal teaching designation. She has a Masters Degree in Mathematics, and has a special interest in the history of mathematics. Harriet was initiated into the teaching of this course by a colleague who has a Masters Degree in Mathematics Education, has taught Math for Teachers for many years, and has a particular passion for the course. This colleague provided information about course materials and the nature of the students and their difficulties. She also provided teaching resources, including suggestions for activities.

Harriet feels strongly about the need for good teachers of mathematics in the elementary schools, and has put a great deal of thought into what can be done in a Math for Teachers course. Her priority when teaching the course is to change students' attitudes towards mathematics and their own mathematical abilities. She hopes students will come to see mathematics as enjoyable, even when it is challenging, and will develop confidence, based on a solid conceptual understanding of elementary mathematics.

Bob

Bob has been teaching mathematics for 13 years and has taught the Math for Teachers course nine times over the last nine years. He has a Masters Degree in Mathematics, has not taken any Mathematics Education courses, and has not had any formal teacher training. Bob's first forays

into teaching the course were guided by the established curriculum, the textbook that had been selected by colleagues who had taught the course before, and through informal discussions with those colleagues.

Bob is passionate about mathematics. He enjoys its logic, its structure, and the challenges presented by a good problem. He cares about producing students who will be successful elementary teachers in the future, and to that end he hopes to equip them with a solid understanding of fundamental mathematics concepts, good communications skills, and a capacity to enjoy mathematics.

Analysis of Interviews

Knowledge for Teaching: Subject Content and Pedagogical Content

Harriet and Bob's interviews are illustrative of two quite different conceptions of the role of the Math for Teachers course in developing prospective teachers' knowledge-for-teaching.

Harriet's description of her goals and strategies for teaching the Math for Teachers course are permeated with comments coded under "pedagogical content knowledge". When describing the content of her course she mentions varieties of algorithms for arithmetic operations, along with models for their representation. Although these topics are part of the prescribed curriculum, her comments indicate that she goes beyond merely delivering this as subject content. She explicitly considers its relevance for teaching mathematics, noting "*We spend some time on the basic algorithms and different approaches to them, and how those can lead into different understandings of what you're doing when you're multiplying, or adding…*"

This notion of developing multiple understandings arises again when she is asked if there is anything that she teaches her Math for Teachers students about fractions that she wouldn't teach someone who just wanted to learn how to use fractions. She states: "*The fact that there are different models, there are different ways of picturing what's going on, and that they are appropriate for...what may work well for some situation, or for some student, may not work for some other one*". For Harriet, access to a variety of representations and approaches is mathematics content that is particularly relevant for her students as prospective teachers.

Connections between mathematical ideas also play a central role in Harriet's conception of the knowledge content of the course. She explains: "*…what you can do with a grade three student, and what you can do with a grade six student are quite different and I want them to see that it's all interconnected…*" Her evident appreciation for these connections echoes Ball and Bass's (2003) description of knowledge-for-teaching, addressing both the connections within and across grade levels. In her words:

> *I emphasize it [connections between topics] all the way through. I don't try to plan the course to start from the beginning and go through to the end with an obvious thread, because mathematics is way too big for that. […] But at all times I connect it, as far as I can, to what goes on at different levels. What you might do with a grade 1 class, how that connects to what they're going to see in, you know grade 4 or 5 or something like that, how that connects to what they might do in high school and how that connects to what I'm doing in Calculus. Because they've got to see how it's connected, and how we build bigger and bigger, you know, understandings of sets of numbers, or calculations, or whatever.*

Harriet does not just pay lip-service to these ideas. She describes assignments and activities for her classes that provide them with opportunities to exercise their pedagogical content knowledge: her students engage in analyses of pupil errors, as well as activities that allow them to compare alternative methods for solving math problems. While Harriet is concerned to ensure

that her students build proficiency in mathematics subject content, her interview stands out from the others in that she constantly returns to comments related to how the content would be used by her students in their future roles as teachers.

In contrast, Bob's interview stands out for its lack of statements that can be coded as pedagogical content knowledge. His emphasis is instead on the notions of developing both a strong understanding of fundamental mathematics and communication skills in his students. When comparing the Math for Teachers course with his other mathematics courses he notes *"...this one focuses on their ability to communicate and convey the ideas that they should, hopefully, be already familiar with and capable of doing.*" Bob describes teaching various algorithms and models as part of the course content, but does not specifically address any comments to consideration of how this information can be used differently at different grade levels.

Bob needed to be pressed by the interviewer to consider what aspects of the course content might be particularly relevant to prospective teachers as opposed to general learners of mathematics. Initially his comments revolve around his methodology, the use of group work and manipulatives (both coded under "teaching methods"), but he makes no reference to any special mathematics knowledge for teaching. Eventually an association between the mathematics that he teaches and the students' future role as teachers appears when he describes challenging his students to think about the kinds of questions that they will encounter as teachers:

> *…what kinds of questions will you encounter? And why is it important that you to be able to communicate your ideas effectively, […], why should you understand this material to the most, [...], fundamental and basic level, and understand all of the structure?*

Even then, this response seems to be a justification for developing strong subject matter content knowledge and communication skills. There appears to be a strong connection for Bob between knowing the mathematics subject content and being able to teach it. He goes on to note:

> *...when you get some of these obtuse questions, that are seemingly [...] obtuse, you have to be able to appreciate it and be able to differentiate whether that's something that can lead you into a teachable moment...*

His focus on subject content knowledge is echoed in his description of what his students leave the Math for Teachers course with:

> *…I think that they […] leave having had some sense of the structure of mathematics, because there's a sufficient amount of that in the course, and I think that they also leave the course feeling that they can solve problems, on their own. […] probably it's the technical skills that they have [...] solidified the most.*

Bob's views on the readiness for teaching of those who complete his course shed further light on this. Though he admits that they are not ready, he describes his students as still lacking maturity, confidence and communication skills. But in regard to their mathematics skills he relates:

> *...[for] my A's and high B students I wouldn't have any problem giving a recommendation in terms of how they're outfitted to [...] go into a classroom, and [...] I think other aspects of their [...] education career, will help fill out all of their [...] professional skills, in terms of knowing what's in the better interests of kids...*

It appears that for Bob, mastery of the subject content along with general pedagogical skills are sufficient for the teaching of mathematics—a traditional and not unfamiliar point of view (Hill et al, 2007).

Beliefs About Mathematics

Interview subjects commonly commented on the beliefs about mathematics the prospective teachers brought with them into the course, and about their efforts to influence those beliefs, though both the specific beliefs they addressed and their approaches differed.

Bob describes his students as believing that mathematics is arbitrary and incomprehensible: "*So many things seem magical to them*". He affirms that "*it's not your standard sort of math group, it's one that has encountered some challenges along the way, and it hasn't always left them with a positive impression of mathematics.*" More than once in his interview he describes the Math for Teachers course as a second start for these students, an opportunity to reshape their beliefs about mathematics and their own mathematical ability.

For Bob, this reshaping is attempted by providing his students with opportunities to see the logical structure of mathematics, which in turn will improve their ability and their conceptions of their ability to do and understand mathematics. In his words, the course "*focuses on a very sound fundamental ability to appreciate it [mathematics], in a theoretical way, why things work, as opposed to technical aspects of how do you do mathematics*". He tries to "*give them a sense that "yes, they can" but also ...force[s] them to dig a little bit deeper, so that they also know that they can understand this as well*". He also challenges their presumptions about the root of authority in mathematics, attempting "*to make sure that they know I'm not just saying this stuff because "It's the way it is" and Math isn't just something handed down by the gods, it's understandable*".

Harriet focuses on quite different aspects of her students' beliefs. Her comments indicate that her students believe that mathematics is rigid (allowing for only one right answer), that mathematics is arithmetic, and that at least some of it is irrelevant. She observes: "*It's never occurred to them that there's more than one way to do something*". In response to this, she takes the time to reinforce the existence of multiple representations and methods of solution. This is clearly a belief about mathematics that she values highly, as later in the interview she reiterates "*if I get across to them that they have to be aware that there are different ways to think about things, and they are all correct, even if they don't remember the details, then they've learned something*". Harriet also makes an effort, through course content, activities and assignments, to expand her students' conception of what mathematics is, noting that by the end of the course "*they're more open to the idea that geometry is a big part of mathematics than they might have been before.*" She also tries to "*explain why it is that fractions are important*", challenging beliefs that a portion of the elementary school mathematics curriculum is dispensable.

Harriet's comments about beliefs seem directed not only towards students' beliefs about the nature of mathematics, but also to beliefs about the teaching of mathematics. A nice example of this is the following comment which she makes in her discussion of teaching about division of fractions:

> *The students know the algorithm so well, they don't remember learning it. They don't remember a thing about how they learned to do these things. They just remember the rules. And their idea of helping somebody find their mistakes is to say "Oh! You're supposed to cross that out, and carry the one." Which isn't going to help anyone understand anything.*

This quote brings out the students' frequent belief that mathematics is a set of static rules that need to be memorised (Szydlik, Szydlik and Benson, 2003), as well as Harriet's contrary beliefs that not only can mathematics be understood, but it is the responsibility of the teacher to foster that understanding. Harriet provides her students with activities that allow them to analyse and discuss pupil errors. Through their discussion of these errors, she seeks to counter the belief that teaching mathematics merely involves showing someone how to execute the correct procedure.

It is worth noting that although the instructors are conscious of their students' beliefs about mathematics, and comment on how the activities and methods hope to contribute to a renegotiation of those beliefs, the development of positive beliefs and attitudes towards mathematics is not normally listed in the official course curricula for Math for Teachers courses.

Emotions/Attitudes

We bring our description to a close with a few comments related to the theme of emotions/attitudes that arose in the interviews. Once again this was a common concern, with instructors categorically describing their students as suffering from mathematics anxiety and lack of confidence in their ability to do mathematics. There were however, considerable differences in instructors' approaches to these negative feelings. Bob and Harriet represent two of the views expressed.

Bob describes his students as suffering from confusion and anxiety, which is closely linked to poor performance:

> *in many cases, some of the very elementary arithmetic operations are in fact, confused in their minds and so when they hit upon things, in particular when you hit rational numbers, as an example, that's one place where students have a great deal of anxiety and they would demonstrate poor understanding of ideas.*

For Bob the source of their anxiety is their lack of skill. It is his belief that improved skills will lead directly to increased confidence, however he confesses that the realities of the course conspire against this. At the beginning of the interview he expresses a wish that the students in his course develop a love of math, but when asked about whether this goal is accomplished, he admits: "*...in terms of the other goal, for love of math? Unfortunately, the course is so packed, that in some ways, I think they do get a little bit beaten by the end, and they're just tired.*" This begins to touch on the theme of "tensions" (not discussed here) however it illustrates Bob's realisation that the volume of subject matter content that he needs to cover in the limited amount of time he has with his students is at odds with his objective of instilling a "love of math" through building subject competence.

Harriet is also very concerned about her students' math anxiety: "*they are very anxious around problem solving. They are just terrified, most of them, of a problem they haven't seen before*". In contrast to Bob, her efforts to address this seem to be centred on changing their ideas of what the enterprise of mathematics is all about. She tries to convince them that "*we're supposed to have fun with this*" and tells her students that "*you may never have seen it; you might not get all the way through it. But what I'm looking for is how far did you get, and how well can you explain what it is that you got*", shifting the focus away from getting the right answer, towards less threatening goals. By the end of the course she hopes her students have grown in confidence and also "*they have more of a sense of play...I think they're more flexible. They think they're more flexible. They're not as scared if... that someone will ask them a question that they can't answer.*" It is not clear if Harriet needs to compromise on time spent on building mathematical proficiency in order to make time for students to approach the course in this way.

Conclusion

In her 2002 plenary address to the PME-NA, Ball noted that "we have not put in the foreground the "who" of teacher learning as often as we might" (Ball, 2002). The partial results of our study presented here provide a glimpse into the "who" of instructors of the Math for Teachers course, a course that is often the final mathematics content course taken by prospective elementary school mathematics teachers. We have shown that despite the course's nominal

focus on subject content knowledge, there is also potential for instructors to address pedagogical content, beliefs about mathematics and the teaching of mathematics, as well as attitudes.

The comments of the interview subjects reveal intentions for the course that go beyond the transmission of mathematical proficiency, providing examples of instructors' efforts to expand students' conceptions of mathematics and their relationship with it. However, we are aware that our methodology permits only a description of the instructors' espoused beliefs and intentions of practice, which may not be consistent with their actual practice (Liljedahl, 2008). Classroom observations, follow-up interviews, and examination of artefacts (such as assignments and tests) will be useful in providing validation for our conclusions. Furthermore it is our hope that the full report of this study will help inform future investigations into the contribution that mathematics content courses can and do make in the development of mathematics teachers.

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