MATHEMATICS KINDERGARTEN TO GRADE 9

INTRODUCTION

The Mathematics Kindergarten to Grade 9 Program of Studies has been derived from *The Common Curriculum Framework for K–9 Mathematics: Western and Northern Canadian Protocol*, May 2006 (the Common Curriculum Framework). The program of studies incorporates the conceptual framework for Kindergarten to Grade 9 Mathematics and the general outcomes and specific outcomes that were established in the Common Curriculum Framework.

BACKGROUND

The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions.

BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. Through the use of manipulatives and a variety of pedagogical approaches, teachers can address the diverse learning styles, cultural backgrounds and developmental stages of students, and enhance within them the formation of sound, transferable mathematical understandings. At all levels, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions provide essential links among concrete, pictorial and symbolic representations of mathematical concepts.

The learning environment should value and respect the diversity of students' experiences and ways of thinking, so that students are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. They must realize that it is acceptable to solve problems in a variety of ways and that a variety of solutions may be acceptable.

FIRST NATIONS, MÉTIS AND INUIT PERSPECTIVES

First Nations, Métis and Inuit students in northern and western Canada come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings, including urban, rural and isolated communities. Teachers need to understand the diversity of students' cultures and experiences.

First Nations, Métis and Inuit students often have a holistic view of the environment—they look for connections in learning and learn best when mathematics is contextualized. They may come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word, so oral communication and practical applications and experiences are important to student learning and understanding. By understanding and responding to nonverbal cues, teachers can optimize student learning and mathematical understanding.

A variety of teaching and assessment strategies help build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students.

Research indicates that when strategies go beyond the incidental inclusion of topics and objects unique to a culture or region, greater levels of understanding can be achieved (Banks and Banks, 1993).

AFFECTIVE DOMAIN

A positive attitude is an important aspect of the affective domain and has a profound impact on learning. Environments that create a sense of belonging, encourage risk taking and provide opportunities for success help develop and maintain positive attitudes and self-confidence within students. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

EARLY CHILDHOOD

Young children are naturally curious and develop a variety of mathematical ideas before they enter Kindergarten. Children make sense of their environment through observations and interactions at home, in daycares, in preschools and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, beading, baking, storytelling and helping around the home.

Activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in, and talking about, such activities as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs and building with blocks.

Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

GOALS FOR STUDENTS

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications

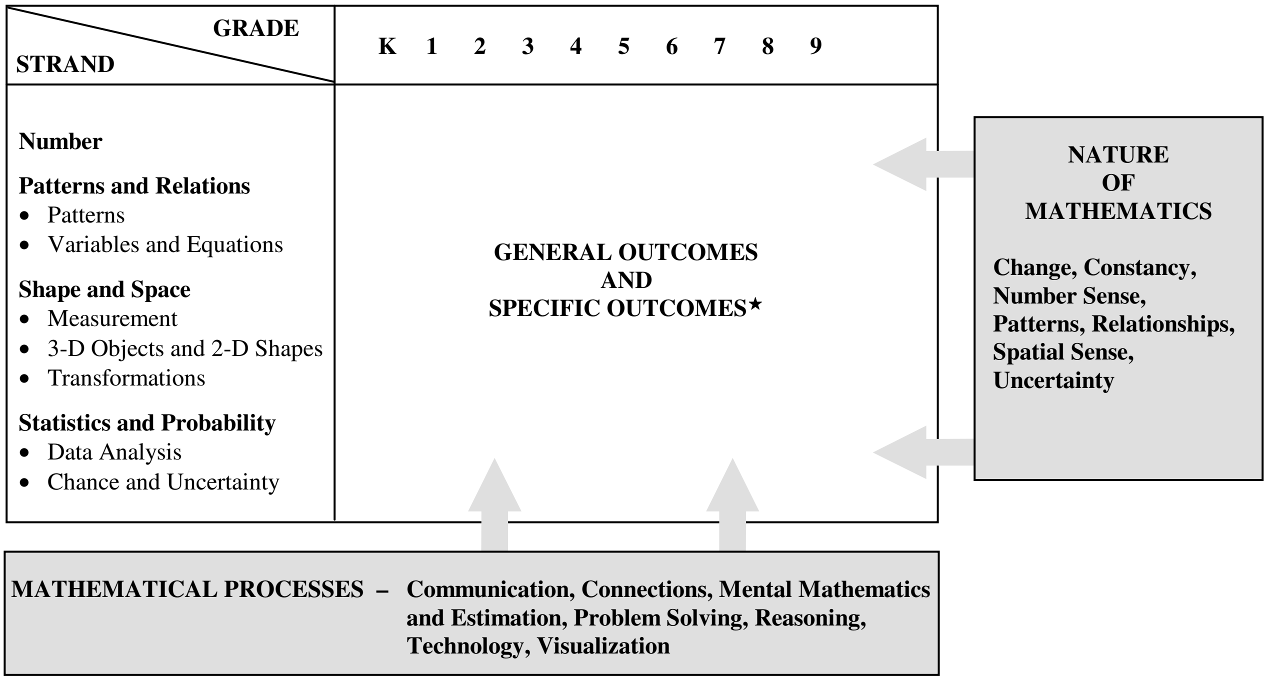
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity.

CONCEPTUAL FRAMEWORK FOR K–9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



* Achievement indicators for the prescribed program of studies outcomes are provided in the companion document *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*, 2007.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and embrace lifelong learning in mathematics.

Students are expected to:

Communication [C]	• communicate in order to learn and express their understanding
Connections [CN]	• connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
Mental Mathematics and Estimation [ME]	• demonstrate fluency with mental mathematics and estimation
Problem Solving [PS]	• develop and apply new mathematical knowledge through problem solving
Reasoning [R]	• develop mathematical reasoning
Technology [T]	• select and use technologies as tools for learning and for solving problems
Visualization [V]	• develop visualization skills to assist in processing information, making connections and solving problems.

The program of studies incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

COMMUNICATION [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication helps students make connections among concrete, pictorial, symbolic, oral, written and mental representations of mathematical ideas.

CONNECTIONS [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. This can be particularly true for First Nations, Métis and Inuit learners. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding.… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine, 1991, p. 5).

MENTAL MATHEMATICS AND ESTIMATION [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math" (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001, p. 442).

Mental mathematics "provides the cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers" (Hope, 1988, p. v).

Estimation is used for determining approximate values or quantities or for determining the reasonableness of calculated values. It often uses benchmarks or referents. Students need to know when to estimate, how to estimate and what strategy to use.

Estimation assists individuals in making mathematical judgements and in developing useful, efficient strategies for dealing with situations in daily life.

PROBLEM SOLVING [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type *How would you …?* or *How could you …?*, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

A problem-solving activity must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk takers.

REASONING [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for students to develop their ability to reason. Students can explore and record results, analyze observations, make and test generalizations from patterns, and reach new conclusions by building upon what is already known or assumed to be true.

Reasoning skills allow students to use a logical process to analyze a problem, reach a conclusion and justify or defend that conclusion.

TECHNOLOGY [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- create geometric patterns
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels.

VISUALIZATION [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and which estimation strategies to use (Shaw and Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this program of studies. The components are change, constancy, number sense, patterns, relationships, spatial sense and uncertainty.

CHANGE

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- the number of a specific colour of beads in each row of a beaded design
- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

CONSTANCY

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The ratio of the circumference of a teepee to its diameter is the same regardless of the length of the teepee poles.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

NUMBER SENSE

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p. 146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Mastery of number facts is expected to be attained by students as they develop their number sense. This mastery allows for facility with more complex computations but should not be attained at the expense of an understanding of number.

Number sense develops when students connect numbers to their own real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections to their own experiences and their previous learning.

PATTERNS

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands of this program of studies.

Working with patterns enables students to make connections within and beyond mathematics.
These skills contribute to students' interaction with, and understanding of, their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and nonroutine problems.

Learning to work with patterns in the early grades helps students develop algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

RELATIONSHIPS

Mathematics is one way to describe interconnectedness in a holistic worldview. Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves collecting and analyzing data and describing relationships visually, symbolically, orally or in written form.

SPATIAL SENSE

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes and to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of shapes and objects. Spatial sense allows students to make predictions about the results of changing these dimensions; e.g., doubling the length of the side of a square increases the area by a factor of four. Ultimately, spatial sense enables students to communicate about shapes and objects and to create their own representations.

UNCERTAINTY

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Strands

The learning outcomes in the program of studies are organized into four strands across the grades K–9. Some strands are subdivided into substrands. There is one general outcome per substrand across the grades K–9.

The strands and substrands, including the general outcome for each, follow.

NUMBER
- Develop number sense.

PATTERNS AND RELATIONS
Patterns
- Use patterns to describe the world and to solve problems.

Variables and Equations
- Represent algebraic expressions in multiple ways.

SHAPE AND SPACE
Measurement
- Use direct and indirect measurement to solve problems.

3-D Objects and 2-D Shapes
- Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Transformations
- Describe and analyze position and motion of objects and shapes.

STATISTICS AND PROBABILITY
Data Analysis
- Collect, display and analyze data to solve problems.

Chance and Uncertainty
- Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Outcomes

The program of studies is stated in terms of general outcomes and specific outcomes.

General outcomes are overarching statements about what students are expected to learn in each strand/substrand. The general outcome for each strand/substrand is the same throughout the grades.

Specific outcomes are statements that identify the specific skills, understanding and knowledge that students are required to attain by the end of a given grade.

In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are not requirements that must be addressed to fully meet the learning outcome.

Links to Information and Communication Technology (ICT) Outcomes

Some curriculum outcomes from Alberta Education's Information and Communication Technology (ICT) Program of Studies can be linked to outcomes in the mathematics program so that students will develop a broad perspective on the nature of technology, learn how to use and apply a variety of technologies, and consider the impact of ICT on individuals and society. The connection to ICT outcomes supports and reinforces the understandings and abilities that students are expected to develop through the general and specific outcomes of the mathematics program. Effective, efficient and ethical application of ICT outcomes contributes to the mathematics program vision.

Links to the ICT outcomes have been identified for some specific outcomes. These links appear in square brackets below the process codes for an outcome, where appropriate. The complete wording of the relevant outcomes for ICT is provided in the Appendix.

Summary

The conceptual framework for K–9 mathematics describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed in Kindergarten to Grade 9 mathematics. The components are not meant to stand alone. Activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between strands.

INSTRUCTIONAL FOCUS

The program of studies is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands.

Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using manipulatives and be developed concretely, pictorially and symbolically.
- Students bring a diversity of learning styles and cultural backgrounds to the classroom. They will be at varying developmental stages.